>

> with(plots) :

> with(Physics[Vectors])

[&x, `+`, `.`, ChangeBasis, ChangeCoordinates, Component, Curl, DirectionalDiff, Divergence, Gradient, Identify, Laplacian,∇, Norm, ParametrizeCurve, ParametrizeSurface, ParametrizeVolume, Setup, diff, int] **(1)**

> Setup(mathematicalnotation = true)

[mathematicalnotation = true] **(2)**

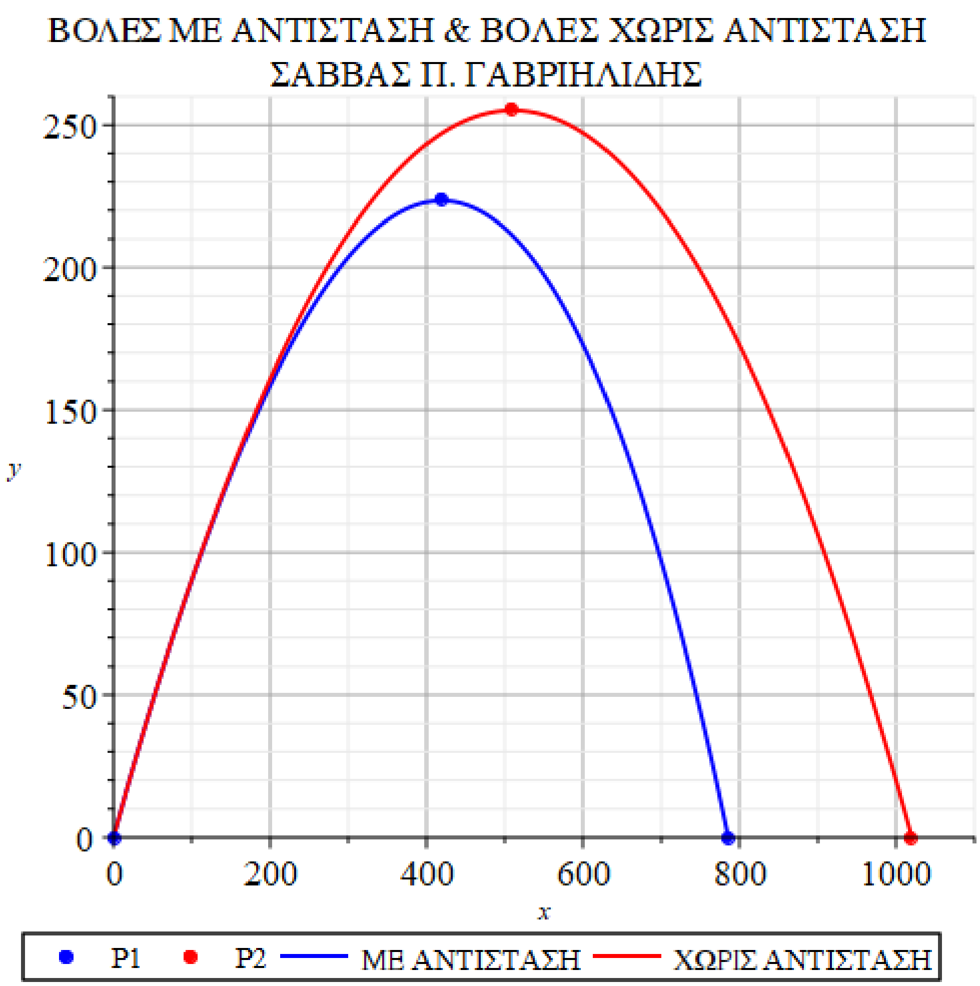
>

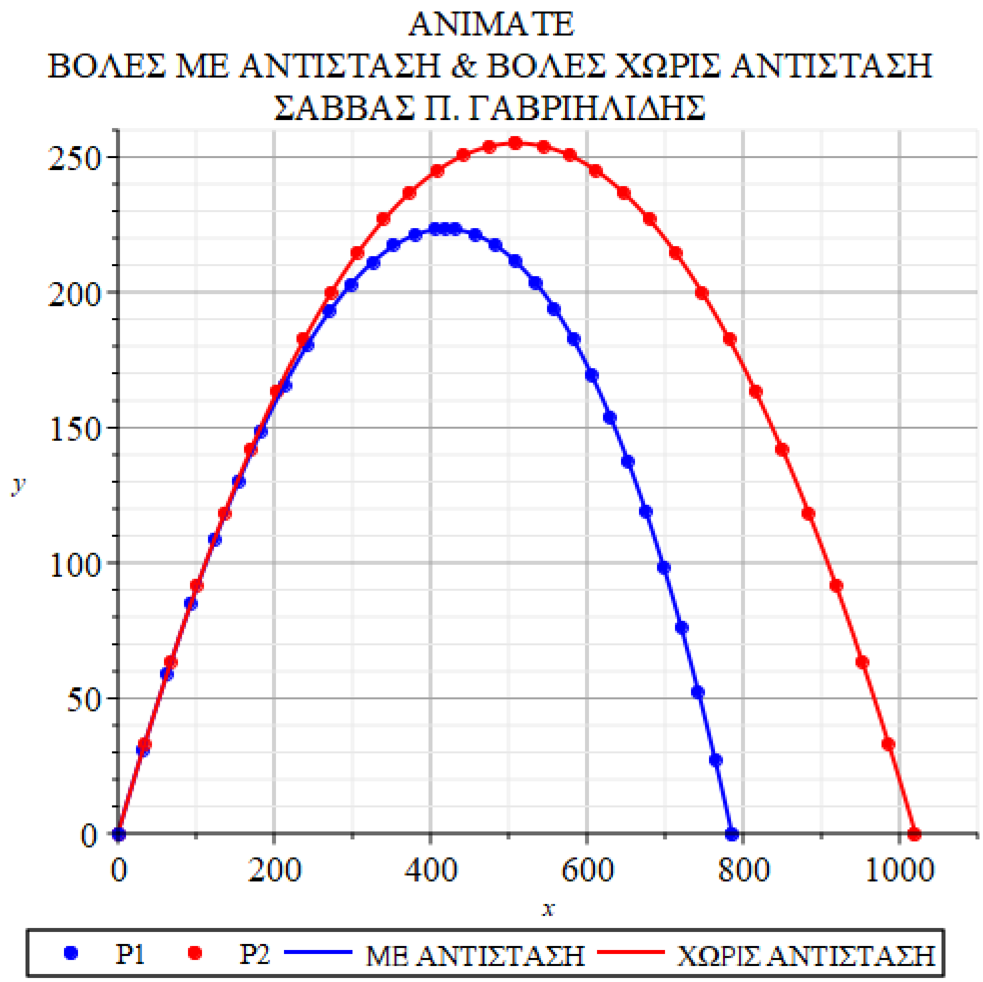
Θέμα : Πλάγια βολή με οπισθέλκουσα δύναμη Αντίστασης .

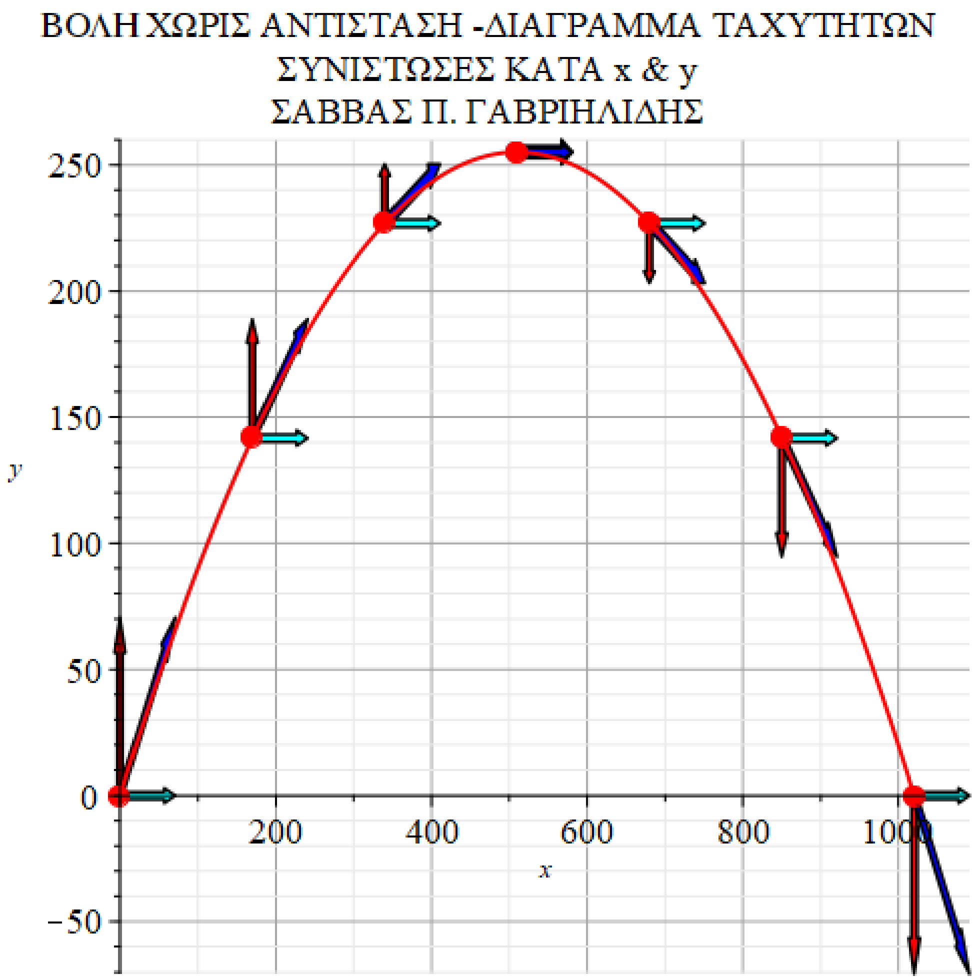
Να μελετηθεί η κίνηση σώματος που βάλλεται με αρχική ταχύτητα $\vec{v}_0$, υπό γωνία θ ως προς τον οριζόντιο άξονα , σε περιβάλλον με αντίσταση παράλληλη προς την ταχύτητα και αντίθετης φοράς .

>

>

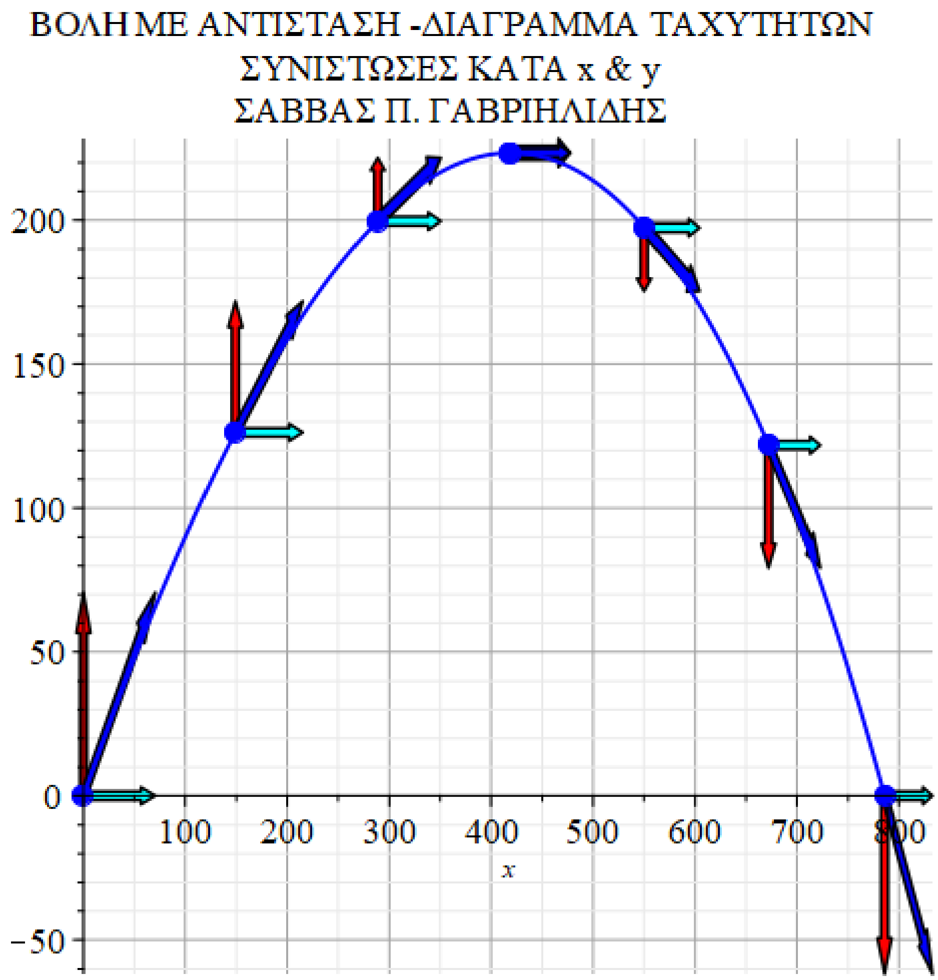



ANIMATE
ΒΟΛΕΣ ΜΕ ΑΝΤΙΣΤΑΣΗ & ΒΟΛΕΣ ΧΩΡΙΣ ΑΝΤΙΣΤΑΣΗ
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ
250
200
150
100
50
0
y
0
200
400
600
800
1000
x
P1
P2
ΜΕ ΑΝΤΙΣΤΑΣΗ
ΧΩΡΙΣ ΑΝΤΙΣΤΑΣΗ



ΒΟΛΗ ΧΩΡΙΣ ΑΝΤΙΣΤΑΣΗ :

Η οριζόντια συνιστώσα της ταχύτητας παραμένει σταθερή , ανεξάρτητη από τον χρόνο.!!!



ΒΟΛΗ ΜΕ ΑΝΤΙΣΤΑΣΗ :
Και οι δύο συνιστώσες της ταχύτητας μεταβάλλονται .!!!

\>

\>

\> $r\_ := x(t)\cdot\_i + y(t)\cdot\_j$

$$\vec{r} := x(t)\,\hat{i} + y(t)\,\hat{j} \tag{3}$$

\> $v\_ := diff(r\_, t)$

$$\vec{v} := \left(\frac{\mathrm{d}}{\mathrm{d}t}\,x(t)\right)\hat{i} + \left(\frac{\mathrm{d}}{\mathrm{d}t}\,y(t)\right)\hat{j} \tag{4}$$

\> $a\_ := diff(v\_, t)$

$$\vec{a} := \left(\frac{\mathrm{d}^2}{\mathrm{d}t^2}\,x(t)\right)\hat{i} + \left(\frac{\mathrm{d}^2}{\mathrm{d}t^2}\,y(t)\right)\hat{j} \tag{5}$$

\>

ΒΟΛΗ ΜΕ ΑΝΤΙΣΤΑΣΗ :
Έστω : b=0.30 (kg/s)

>

Μπορούμε να γράψουμε :

> $a\_ = -\frac{b}{m}\cdot v\_ - g\cdot\_j$

$$\left(\frac{d^2}{dt^2}x(t)\right)\hat{i} + \left(\frac{d^2}{dt^2}y(t)\right)\hat{j} = -\frac{b\left(\frac{d}{dt}x(t)\right)\hat{i}}{m} + \hat{j}\left(-\frac{b\left(\frac{d}{dt}y(t)\right)}{m} - g\right) \qquad (6)$$

> $Component(lhs((6)), 1) = Component(rhs((6)), 1)$

$$\frac{d^2}{dt^2}x(t) = -\frac{b\left(\frac{d}{dt}x(t)\right)}{m} \qquad (7)$$

> $Component(lhs((6)), 2) = Component(rhs((6)), 2)$

$$\frac{d^2}{dt^2}y(t) = -\frac{b\left(\frac{d}{dt}y(t)\right)}{m} - g \qquad (8)$$

> $ics1 := D(x)(0) = v[0]\cdot\cos(\theta), x(0) = 0$

$$ics1 := D(x)(0) = v_0\cos(\theta), x(0) = 0 \qquad (9)$$

> $ics2 := D(y)(0) = v[0]\cdot\sin(\theta), y(0) = 0$

$$ics2 := D(y)(0) = v_0\sin(\theta), y(0) = 0 \qquad (10)$$

> $dsolve(\{(7), ics1\}, x(t))$

$$x(t) = \frac{v_0\cos(\theta)\, m}{b} - \frac{v_0\cos(\theta)\, m\, e^{-\frac{b\,t}{m}}}{b} \qquad (11)$$

> $dsolve(\{(8), ics2\}, y(t))$

$$y(t) = -\frac{m\, e^{-\frac{b\,t}{m}}\left(v_0\sin(\theta)\, b + g\, m\right)}{b^2} - \frac{g\, m\, t}{b} + \frac{m\left(v_0\sin(\theta)\, b + g\, m\right)}{b^2} \qquad (12)$$

>

ΕΛΕΓΧΟΣ :

> $diff(lhs((11)), t) = diff(rhs((11)), t)$

$$\frac{d}{dt}x(t) = v_0\cos(\theta)\, e^{-\frac{b\,t}{m}} \qquad (13)$$

> $diff(lhs((12)), t) = diff(rhs((12)), t)$

$$\frac{d}{dt}y(t) = \frac{e^{-\frac{b\,t}{m}}\left(v_0\sin(\theta)\, b + g\, m\right)}{b} - \frac{g\, m}{b} \qquad (14)$$

> $dsolve(\{(13), x(0) = 0\}, x(t))$

$$x(t) = \frac{v_0 \cos(\theta)\, m}{b} - \frac{v_0 \cos(\theta)\, m\, e^{-\frac{b\,t}{m}}}{b} \tag{15}$$

> dsolve({(14), y(0) = 0}, y(t))

$$y(t) = -\frac{m\, e^{-\frac{b\,t}{m}} \left(v_0 \sin(\theta)\, b + g\, m\right)}{b^2} - \frac{g\, m\, t}{b} + \frac{m \left(v_0 \sin(\theta)\, b + g\, m\right)}{b^2} \tag{16}$$

> x(t) = int(rhs((13)), t = 0 ..t)

$$x(t) = -\frac{v_0 \cos(\theta)\, m \left(e^{-\frac{b\,t}{m}} - 1\right)}{b} \tag{17}$$

> y(t) = int(rhs((14)), t = 0 ..t)

$$y(t) = -\frac{m \left(\sin(\theta)\, e^{-\frac{b\,t}{m}}\, b\, v_0 - v_0 \sin(\theta)\, b + e^{-\frac{b\,t}{m}}\, g\, m + g\, t\, b - g\, m\right)}{b^2} \tag{18}$$

>

>

ΕΦΑΡΜΟΓΗ

>

>

> $$x(t) = \frac{v_0 \cos(\theta)\, m}{b} - \frac{v_0 \cos(\theta)\, m\, e^{-\frac{b\,t}{m}}}{b}$$

$$x(t) = \frac{v_0 \cos(\theta)\, m}{b} - \frac{v_0 \cos(\theta)\, m\, e^{-\frac{b\,t}{m}}}{b} \tag{19}$$

> $$y(t) = -\frac{m\, e^{-\frac{b\,t}{m}} \left(v_0 \sin(\theta)\, b + g\, m\right)}{b^2} - \frac{g\, m\, t}{b} + \frac{m \left(v_0 \sin(\theta)\, b + g\, m\right)}{b^2}$$

(20)

$$y(t) = -\frac{m\,\mathrm{e}^{-\frac{b\,t}{m}}\left(v_0 \sin(\theta)\, b + g\, m\right)}{b^2} - \frac{g\,m\,t}{b} + \frac{m\left(v_0 \sin(\theta)\, b + g\, m\right)}{b^2} \qquad \textbf{(20)}$$

>

>

$$\left\{t = DIARKEIA,\ b = 0.3,\ g = 9.80,\ m = 10,\ v[0] = 100,\ \theta = \frac{\mathrm{Pi}}{4}\right.$$

>

> $evalf\left(subs\left(\left\{b = 0.30, g = 9.80, m = 10, v[0] = 100, \theta = \frac{\mathrm{Pi}}{4}\right\}, rhs\left(y(t) = -\frac{m\,\mathrm{e}^{-\frac{b\,t}{m}}\left(v_0 \sin(\theta)\, b + g\, m\right)}{b^2} - \frac{g\,m\,t}{b} + \frac{m\left(v_0 \sin(\theta)\, b + g\, m\right)}{b^2}\right)\right)\right)$

$$-13245.91149\,\mathrm{e}^{-0.03000000000\,t} - 326.6666666\,t + 13245.91149 \qquad \textbf{(21)}$$

> $evalf\left(subs\left(\left\{b = 0.30, g = 9.80, m = 10, v[0] = 100, \theta = \frac{\mathrm{Pi}}{4}\right\}, rhs\left(x(t) = \frac{v_0 \cos(\theta)\, m}{b} - \frac{v_0 \cos(\theta)\, m\,\mathrm{e}^{-\frac{b\,t}{m}}}{b}\right)\right)\right)$

$$2357.022603 - 2357.022603\,\mathrm{e}^{-0.03000000000\,t} \qquad \textbf{(22)}$$

> $\boldsymbol{evalf\left(subs\left(\left\{b = 0.30, g = 9.80, m = 10, v[0] = 100, \theta = \frac{\mathrm{Pi}}{4}\right\}, rhs\left(\frac{\mathrm{d}}{\mathrm{d}t}\, y(t) = \frac{\mathrm{e}^{-\frac{b\,t}{m}}\left(v_0 \sin(\theta)\, b + g\, m\right)}{b} - \frac{g\,m}{b}\right)\right)\right)}$

$$397.3773446\,\mathrm{e}^{-0.03000000000\,t} - 326.6666666 \qquad \textbf{(23)}$$

> $\boldsymbol{evalf\left(subs\left(\left\{b = 0.30, g = 9.80, m = 10, v[0] = 100, \theta = \frac{\mathrm{Pi}}{4}\right\}, rhs\left(\frac{\mathrm{d}}{\mathrm{d}t}\, x(t) = v_0 \cos(\theta)\,\mathrm{e}^{-\frac{b\,t}{m}}\right)\right)\right)}$

$$70.71067811\,\mathrm{e}^{-0.03000000000\,t} \qquad \textbf{(24)}$$

>

>

Ο χρόνος πρόσκρουσης καθορίζει την ΔΙΑΡΚΕΙΑ πτήσης : (y(t)=0) .

>

> $DIARKEIA := fsolve(\textbf{(21)}, t, t = 5..20)$

$$DIARKEIA := 13.51937837 \tag{25}$$

> $BELHNEKES := evalf(subs(t = DIARKEIA, \textbf{(22)}))$

$$BELHNEKES := 785.856849 \tag{26}$$

>

Στο μέγιστο ύψος της βολής έχουμε : $\frac{d}{dt}\, y(t) = 0$

$$\frac{d}{dt}\, y(t) = 0 \tag{27}$$

>

> $tmaxYPSOS := evalf(solve(\textbf{(23)}, t))$

$$tmaxYPSOS := 6.531534555 \tag{28}$$

> $maxYPSOS := evalf(subs(t = tmaxYPSOS, \textbf{(21)}))$

$$maxYPSOS := 223.38798 \tag{29}$$

> $xMAX := evalf(subs(t = tmaxYPSOS, \textbf{(22)}))$

$$xMAX := 419.416629 \tag{30}$$

> $MAXPOINT := [xMAX, maxYPSOS]$

$$MAXPOINT := [419.416629, 223.38798] \tag{31}$$

> $PKROYSHS := [BELHNEKES, 0]$

$$PKROYSHS := [785.856849, 0] \tag{32}$$

> $P1 := pointplot([MAXPOINT, PKROYSHS, [0, 0]], symbol = solidcircle, symbolsize = 10, color = blue) :$

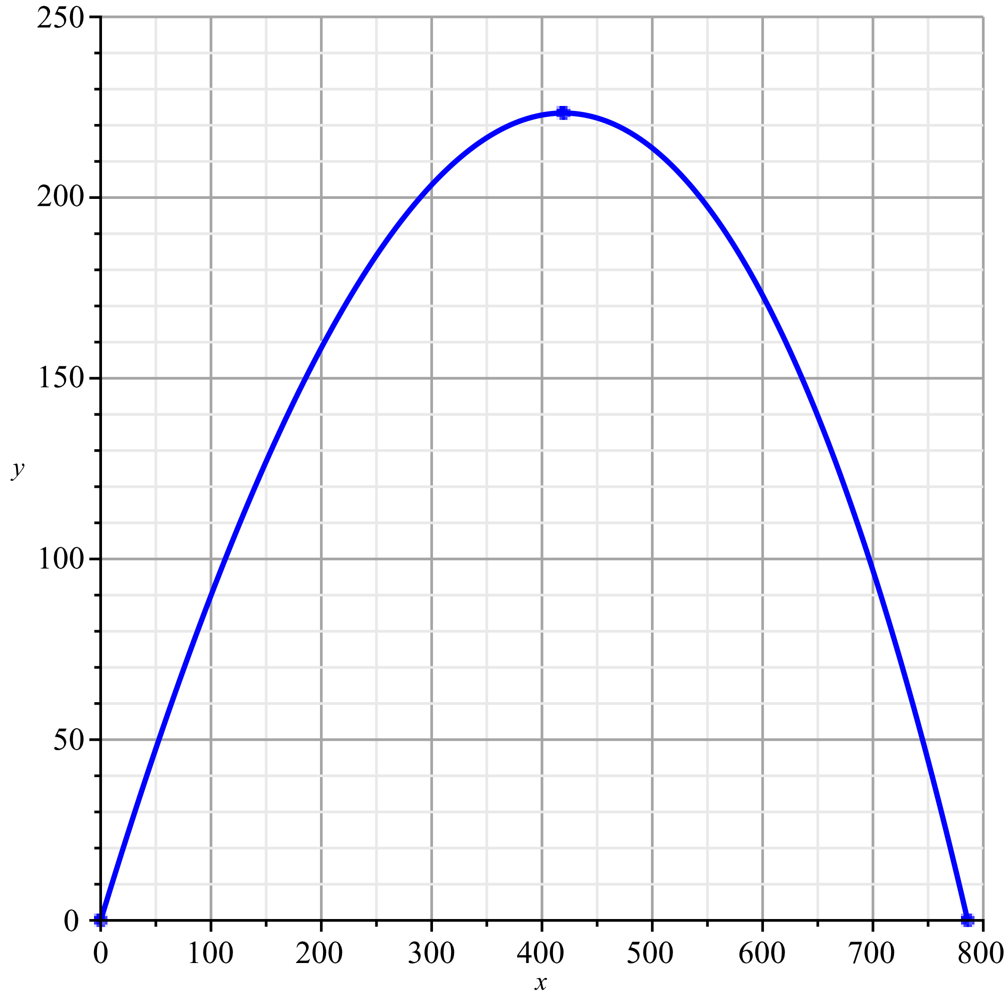
>

> $A := plot([\textbf{(22)}, \textbf{(21)}, t = 0..\textbf{(25)}], gridlines, labels = [x, y], linestyle = 1, thickness = 2, color = blue, view = [0..800, 0..250]) :$

>

>

> $display(P1, A)$



>

ΒΟΛΗ ΧΩΡΙΣ ΑΝΤΙΣΤΑΣΗ : b1=0

>

>

> $b1 := 0$

$$b1 := 0 \tag{33}$$

>

> $r\_ := x(t) \cdot \_i + y(t) \cdot \_j$

$$\vec{r} := x(t)\,\hat{i} + y(t)\,\hat{j} \tag{34}$$

> $v\_ := diff(r\_, t)$

$$\vec{v} := \left(\frac{d}{dt}\,x(t)\right)\hat{i} + \left(\frac{d}{dt}\,y(t)\right)\hat{j} \tag{35}$$

> $a\_ := diff(v\_, t)$

$$\vec{a} := \left(\frac{d^2}{dt^2}\,x(t)\right)\hat{i} + \left(\frac{d^2}{dt^2}\,y(t)\right)\hat{j} \tag{36}$$

>

Μπορούμε να γράψουμε :

> $a\_ = -\frac{b1}{m} \cdot v\_ - g \cdot \_j$

$$\left(\frac{d^2}{dt^2} x(t)\right) \hat{i} + \left(\frac{d^2}{dt^2} y(t)\right) \hat{j} = -g \hat{j} \qquad (37)$$

> $Component(lhs((37)), 1) = Component(rhs((37)), 1)$

$$\frac{d^2}{dt^2} x(t) = 0 \qquad (38)$$

> $Component(lhs((37)), 2) = Component(rhs((37)), 2)$

$$\frac{d^2}{dt^2} y(t) = -g \qquad (39)$$

> $ics1 := D(x)(0) = v[0] \cdot \cos(\theta), x(0) = 0$

$$ics1 := D(x)(0) = v_0 \cos(\theta), x(0) = 0 \qquad (40)$$

> $ics2 := D(y)(0) = v[0] \cdot \sin(\theta), y(0) = 0$

$$ics2 := D(y)(0) = v_0 \sin(\theta), y(0) = 0 \qquad (41)$$

> $dsolve(\{(38), ics1\}, x(t))$

$$x(t) = v_0 \cos(\theta)\, t \qquad (42)$$

> $dsolve(\{(39), ics2\}, y(t))$

$$y(t) = -\frac{g t^2}{2} + v_0 \sin(\theta)\, t \qquad (43)$$

>

ΕΛΕΓΧΟΣ :

> $diff\left(x(t) = v_0 \cos(\theta)\, t, t\right)$

$$\frac{d}{dt} x(t) = v_0 \cos(\theta) \qquad (44)$$

> $diff\left(y(t) = -\frac{g t^2}{2} + v_0 \sin(\theta)\, t, t\right)$

$$\frac{d}{dt} y(t) = -g t + v_0 \sin(\theta) \qquad (45)$$

> $dsolve(\{(44), x(0) = 0\}, x(t))$

$$x(t) = v_0 \cos(\theta)\, t \qquad (46)$$

> $dsolve(\{(45), y(0) = 0\}, y(t))$

$$y(t) = -\frac{g t^2}{2} + v_0 \sin(\theta)\, t \qquad (47)$$

> $x(t) = int(rhs((44)), t = 0..t)$

(48)

$$x(t) = v_0 \cos(\theta)\, t \qquad (48)$$

> $y(t) = int(rhs((\mathbf{45})), t = 0..t)$

$$y(t) = -\frac{g\, t^2}{2} + v_0 \sin(\theta)\, t \qquad (49)$$

> $subs\left(\left[g = 9.80, v[0] = 100, \theta = \frac{\mathrm{Pi}}{4}\right], rhs\left(y(t) = -\frac{g\, t^2}{2} + v_0 \sin(\theta)\, t\right)\right)$

$$-4.900000000\, t^2 + 100 \sin\left(\frac{\pi}{4}\right) t \qquad (50)$$

> $subs\left(\left[g = 9.80, v[0] = 100, \theta = \frac{\mathrm{Pi}}{4}\right], rhs\left(x(t) = v_0 \cos(\theta)\, t\right)\right)$

$$100 \cos\left(\frac{\pi}{4}\right) t \qquad (51)$$

> $subs\left(\left[g = 9.80, v[0] = 100, \theta = \frac{\mathrm{Pi}}{4}\right], rhs\left(\frac{d}{dt}\, y(t) = -g\, t + v_0 \sin(\theta)\right)\right)$

$$-9.80\, t + 100 \sin\left(\frac{\pi}{4}\right) \qquad (52)$$

> $subs\left(\left[g = 9.80, v[0] = 100, \theta = \frac{\mathrm{Pi}}{4}\right], rhs\left(\frac{d}{dt}\, x(t) = v_0 \cos(\theta)\right)\right)$

$$100 \cos\left(\frac{\pi}{4}\right) \qquad (53)$$

>

>

Ο χρόνος πρόσκρουσης καθορίζει την ΔΙΑΡΚΕΙΑ πτήσης : (y(t)=0) .

>

> $\boldsymbol{DIARKEIA1 := fsolve\left(rhs\left(y(t) = -4.900000000\, t^2 + 100 \sin\left(\frac{\pi}{4}\right) t\right), t, t = 5..20\right)}$

$$DIARKEIA1 := 14.43075063 \qquad (54)$$

> $\boldsymbol{BELHNEKES1 := evalf\left(subs\left(\left\{t = DIARKEIA1, v[0] = 100, \theta = \frac{\mathrm{Pi}}{4}\right\}, rhs\left(x(t) = v_0 \cos(\theta)\, t\right)\right)\right)}$

$$BELHNEKES1 := 1020.408163 \qquad (55)$$

>

Στο μέγιστο ύψος της βολής έχουμε : $\frac{d}{dt}\, y(t) = 0$

>

$$\frac{d}{dt}\, y(t) = 0 \qquad \textbf{(56)}$$

> $subs\left(\left\{g = 9.80,\ v[0] = 100,\ \theta = \frac{Pi}{4}\right\}, \textbf{(52)}\right)$

$$-9.80\, t + 50\sqrt{2} \qquad \textbf{(57)}$$

> $tmaxYPSOS1 := evalf(solve(\textbf{(57)}, t))$

$$tmaxYPSOS1 := 7.215375318 \qquad \textbf{(58)}$$

> $maxYPSOS1 := evalf(subs(t = tmaxYPSOS1, \textbf{(50)}))$

$$maxYPSOS1 := 255.1020407 \qquad \textbf{(59)}$$

>

>

> $xMAX1 := evalf(subs(t = tmaxYPSOS1, (\textbf{(51)})))$

$$xMAX1 := 510.2040815 \qquad \textbf{(60)}$$

> $MAXPOINT1 := [xMAX1, maxYPSOS1]$

$$MAXPOINT1 := [510.2040815, 255.1020407] \qquad \textbf{(61)}$$

> $PKROYSHS1 := [BELHNEKES1, 0]$

$$PKROYSHS1 := [1020.408163, 0] \qquad \textbf{(62)}$$

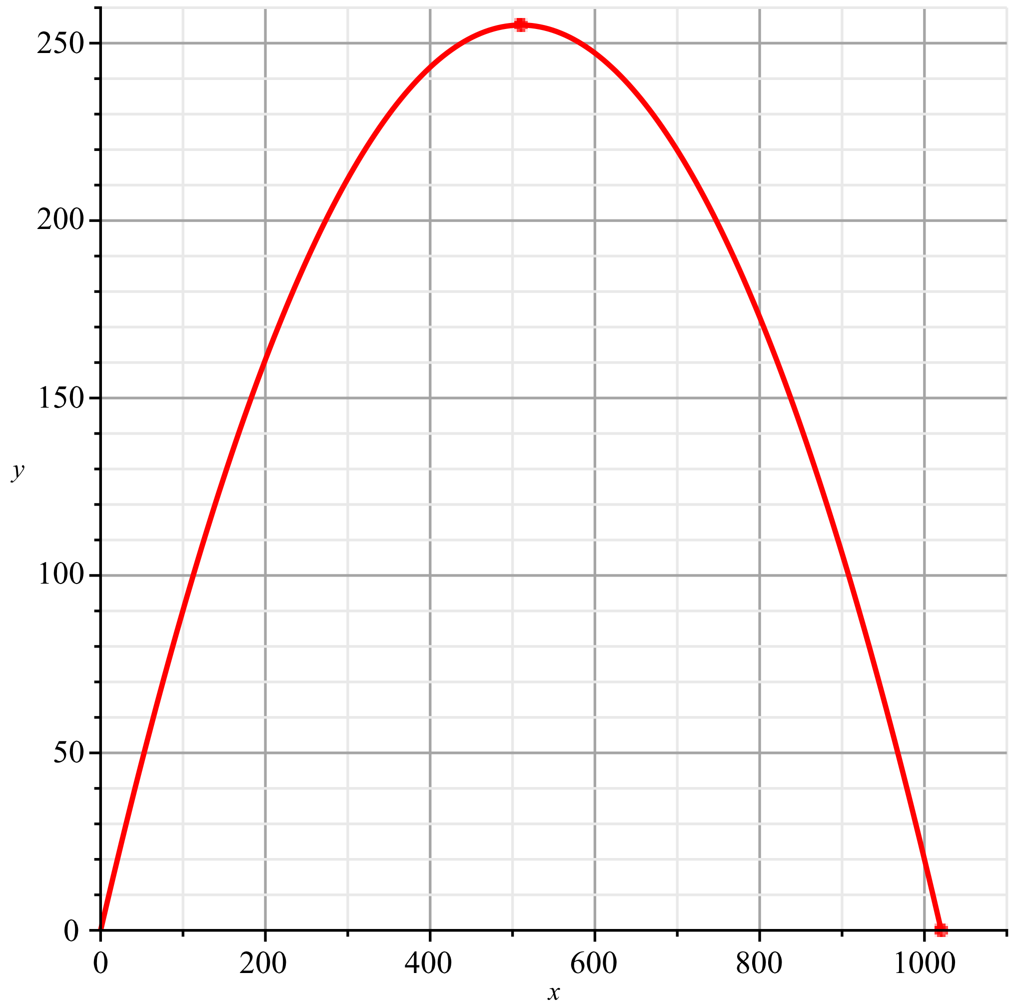
> $P2 := pointplot([MAXPOINT1, PKROYSHS1], symbol = solidcircle, symbolsize = 10, color = red) :$

> $B := plot\left(\left[rhs\left(x(t) = 100\cos\left(\frac{\pi}{4}\right) t\right), rhs\left(y(t) = -4.900000000\, t^2 + 100\sin\left(\frac{\pi}{4}\right) t\right), t = 0 ..DIARKEIA1\right], gridlines, labels = [x, y], linestyle = 1, thickness = 2, color = red, view = [0 ..1100, 0 ..260]\right) :$

>

>

> $display(P2, B)$



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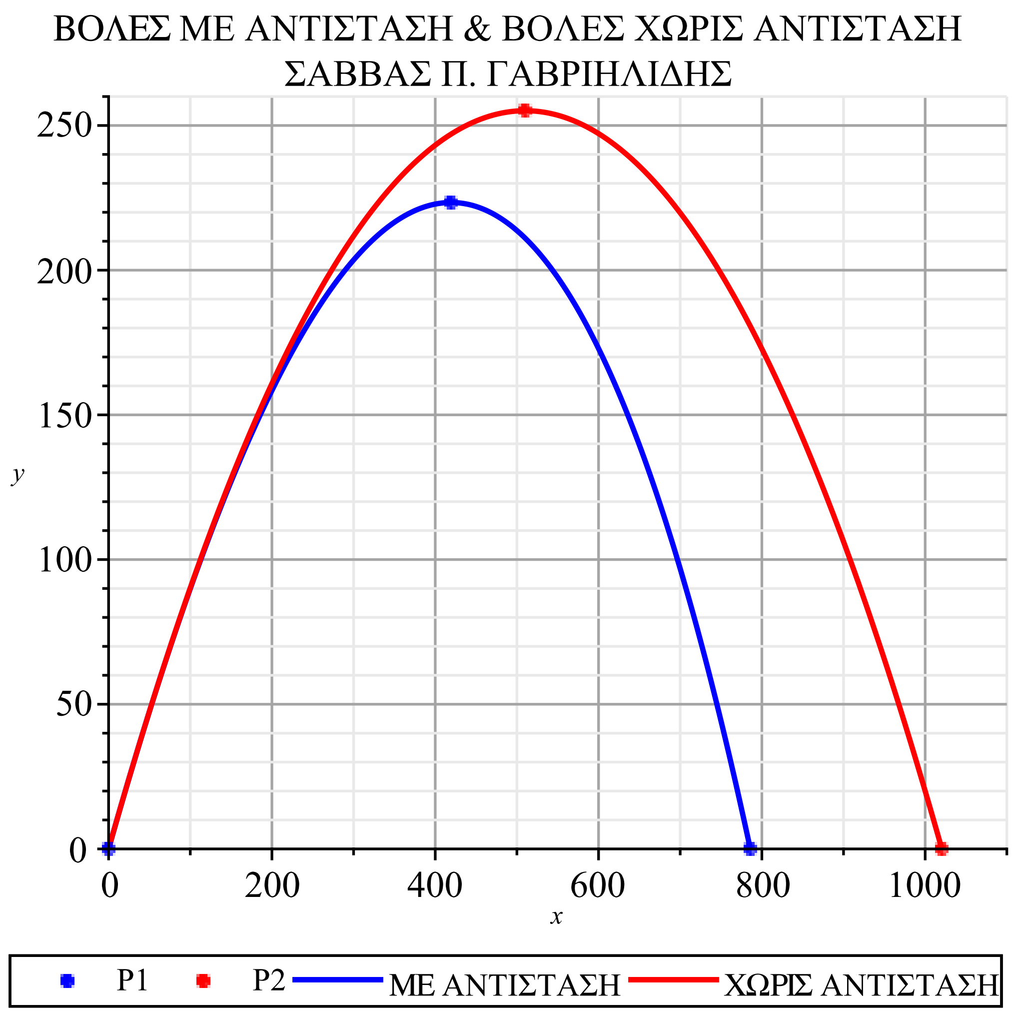
>

> *display*(*A*, *B*, *P1*, *P2*, *legend* = ["ΜΕ ΑΝΤΙΣΤΑΣΗ", "ΧΩΡΙΣ ΑΝΤΙΣΤΑΣΗ", "P1", "P2"], *title* = "ΒΟΛΕΣ ΜΕ ΑΝΤΙΣΤΑΣΗ & ΒΟΛΕΣ ΧΩΡΙΣ ΑΝΤΙΣΤΑΣΗ\nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ", *font* = [*arial*, *bolt*, 14])



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\>

\> [**(22)**, **(21)**]

$$\left[2357.022603 \quad 2357.022603\, \mathrm{e}^{0.03000000000\, t},\quad 13245.91149\, \mathrm{e}^{0.03000000000\, t} \quad 326.6666666\, t + 13245.91149\right] \qquad \textbf{(63)}$$

\> [**(51)**, **(50)**]

$$\left[50\sqrt{2}\, t,\quad 4.900000000\, t^2 + 50\sqrt{2}\, t\right] \qquad \textbf{(64)}$$

\>

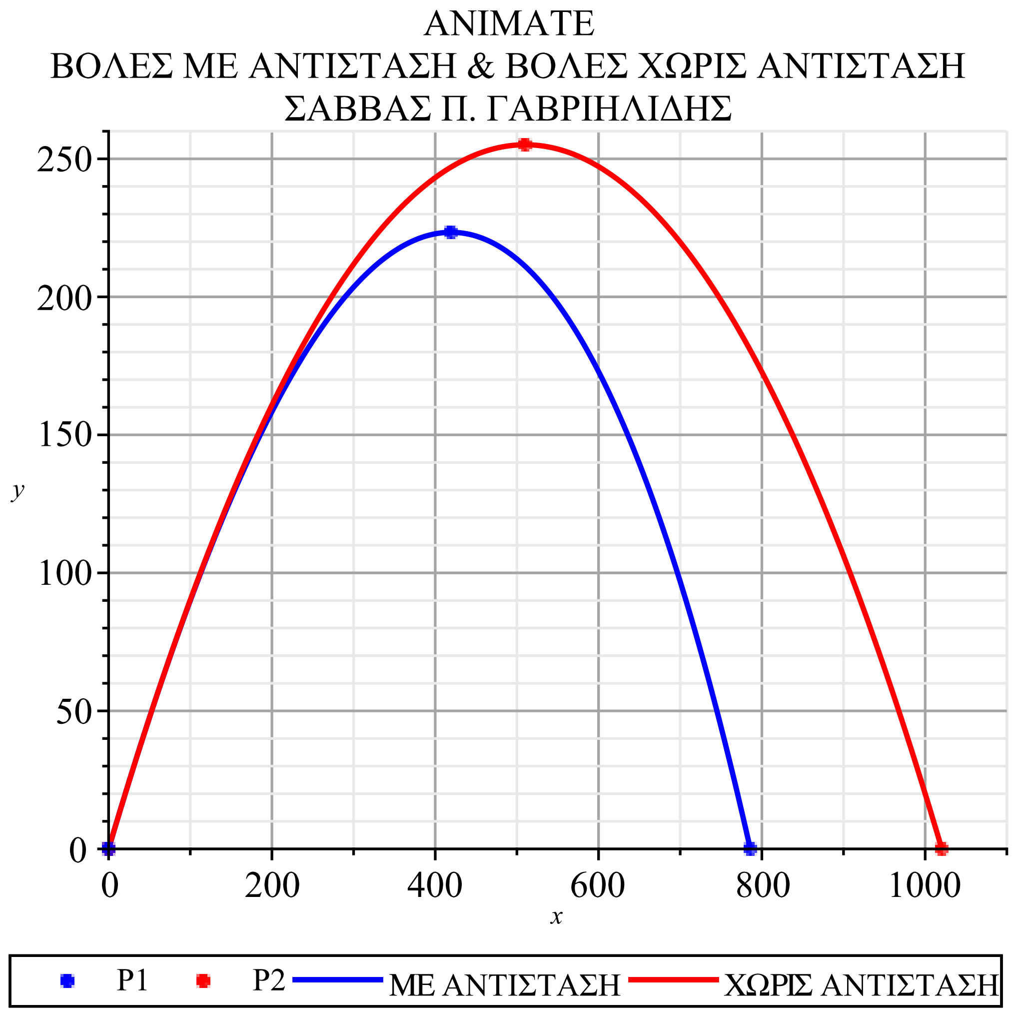
\> *ANIM* := *animate*(*pointplot*, [**(63)**, *color* = *blue*, *symbol* = *solidcircle*, *symbolsize* = 10], *t* = 0 .. 13.51937837, *frames* = 31, *trace* = 30) :

\> *ANIM1* := *animate*(*pointplot*, [**(64)**, *color* = *red*, *symbol* = *solidcircle*, *symbolsize* = 10], *t* = 0 .. 14.43075063, *frames* = 31, *trace* = 30) :

\>

\>

\> *display*(*A*, *B*, *P1*, *P2*, *ANIM*, *ANIM1*, *legend* = ["ΜΕ ΑΝΤΙΣΤΑΣΗ", "ΧΩΡΙΣ ΑΝΤΙΣΤΑΣΗ", "P1", "P2", "ANIM", "ANIM1"], *title* = "ANIMATE\nΒΟΛΕΣ ΜΕ ΑΝΤΙΣΤΑΣΗ & ΒΟΛΕΣ ΧΩΡΙΣ ΑΝΤΙΣΤΑΣΗ\nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ", *font* = [*arial*, *bolt*, 14])


ANIMATE
ΒΟΛΕΣ ΜΕ ΑΝΤΙΣΤΑΣΗ & ΒΟΛΕΣ ΧΩΡΙΣ ΑΝΤΙΣΤΑΣΗ
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ
250
200
150
100
50
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y
0
200
400
600
800
1000
x
P1
P2
ΜΕ ΑΝΤΙΣΤΑΣΗ
ΧΩΡΙΣ ΑΝΤΙΣΤΑΣΗ

>

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