>

> $with(plots)$

$[animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot, display, dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, implicitplot, implicitplot3d, inequal, interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra\_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions, setoptions3d, shadebetween, spacecurve, sparsematrixplot, surfdata, textplot, textplot3d, tubeplot]$ **(1)**

>

> $with(Physics[Vectors])$

$[\&x, `+`, `.`, ChangeBasis, ChangeCoordinates, Component, Curl, DirectionalDiff, Divergence, Gradient, Identify, Laplacian, \nabla, Norm, ParametrizeCurve, ParametrizeSurface, ParametrizeVolume, Setup, diff, int]$ **(2)**

> $Setup(mathematicalnotation = true)$

$$[mathematicalnotation = true] \quad \textbf{(3)}$$

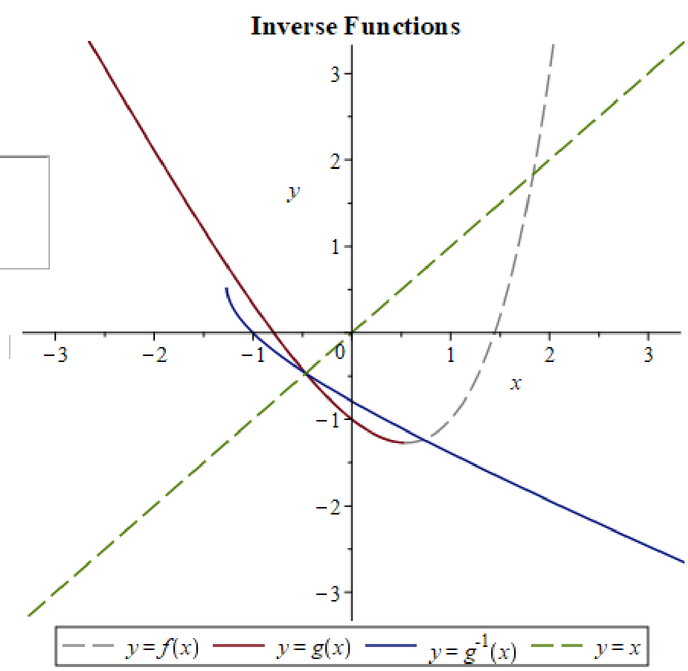
> $alias(W = \mathrm{LambertW})$

$$W \quad \textbf{(4)}$$

>

Example = User Defined

$f(x) =$ $3^x - 2\cdot x - 2$



Plot f(x) | Restrict Domain | Inverse

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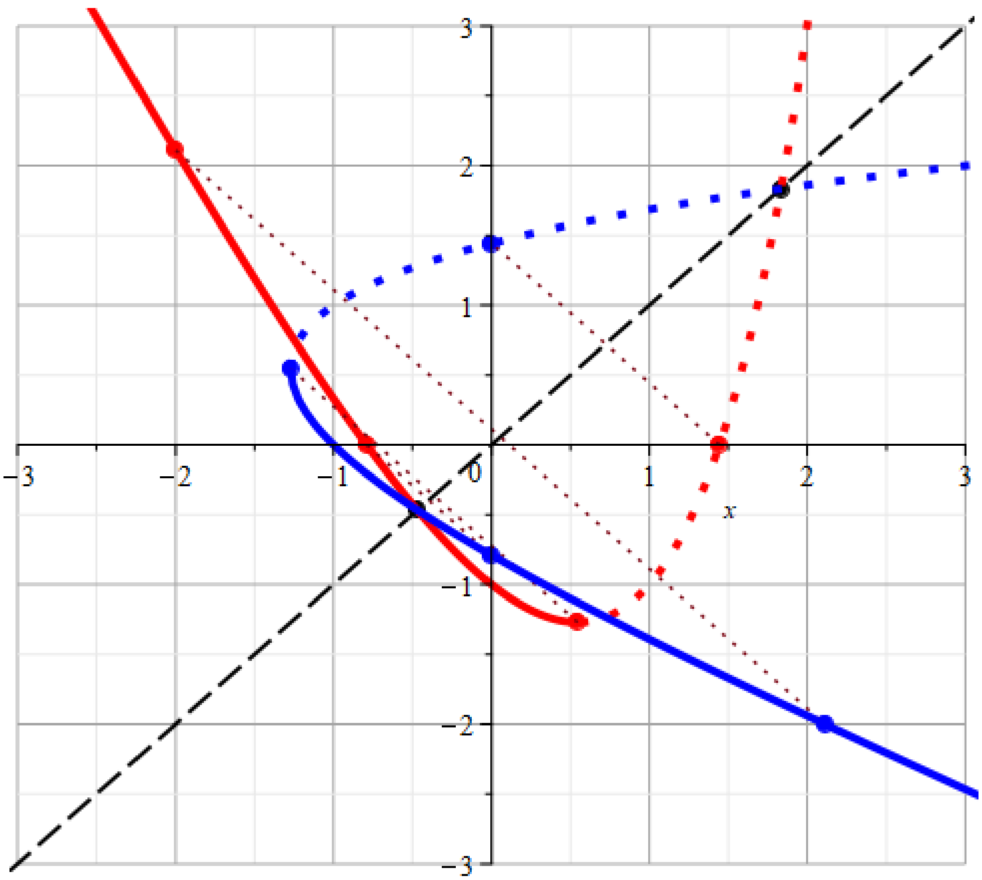

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>

>

ΝΑ ΜΕΛΕΤΗΘΕΙ Η ΣΥΝΑΡΤΗΣΗ :

$$f := x \rightarrow 3^{x} - 2\cdot x - 2$$

> $f := x \rightarrow 3^{x} - 2\cdot x - 2$

$$f := x \rightarrow \grave{}+\grave{}(\grave{}+\grave{}(3^{x}, -2\,x), -2) \quad (5)$$

> $g := x \rightarrow 3^{x} - 2\cdot x - 2$

$$g := x \rightarrow \grave{}+\grave{}(\grave{}+\grave{}(3^{x}, -2\,x), -2) \quad (6)$$

>

ΑΝΑΖΗΤΗΣΗ ΤΩΝ ΡΙΖΩΝ ΤΗΣ ΣΥΝΑΡΤΗΣΗΣ

$$f := x \rightarrow 3^{x} - 2\cdot x - 2$$

$$x = -\frac{W\left(-\frac{\ln(3)}{6}\right)}{\ln(3)} - 1 \Rightarrow x1 := -0.7901100111$$

$$x = -\frac{W\left(-1, -\frac{\ln(3)}{6}\right)}{\ln(3)} - 1 \Rightarrow x2 := 1.444561392$$

>

Γράφουμε :

> $3^x = 2\cdot(x+1)$:

> $(x+1)\cdot 3^{-x} = \frac{1}{2}$:

> $(x+1)\cdot 3^{-1}\cdot 3^{-x} = \frac{1}{2}\cdot\frac{1}{3}$:

> $-(x+1)\cdot 3^{-(x+1)} = -\frac{1}{6}$:

> $-(x+1)\cdot\exp\left(\ln\left(3^{-(x+1)}\right)\right) = -\frac{1}{6}$:

> $-(x+1)\cdot\exp(-(x+1)\cdot\ln(3)) = -\frac{1}{6}$:

> $-(x+1)\cdot\ln(3)\cdot\exp(-(x+1)\cdot\ln(3)) = -\frac{\ln(3)}{6}$:

> $-(x+1)\cdot\ln(3) = W\left(-\frac{\ln(3)}{6}\right)$

$$-(x+1)\ln(3) = W\left(-\frac{\ln(3)}{6}\right) \tag{7}$$

> *isolate*((7), x)

$$x = -\frac{W\left(-\frac{\ln(3)}{6}\right)}{\ln(3)} - 1 \tag{8}$$

> $x = -\frac{W\left(-1, -\frac{\ln(3)}{6}\right)}{\ln(3)} - 1$

$$x = -\frac{W\left(-1, -\frac{\ln(3)}{6}\right)}{\ln(3)} - 1 \tag{9}$$

>

MAPLE ΕΝΤΟΛΗ

> *solve*$(f(x), x)$ assuming $x :: real$

$$-\frac{W\left(-\frac{\ln(3)}{6}\right) + \ln(3)}{\ln(3)}, -\frac{W\left(-1, -\frac{\ln(3)}{6}\right) + \ln(3)}{\ln(3)} \tag{10}$$

> $op(\ map(\ simplify, [\mathbf{(10)}], 'constant'\ )\ )$

$$-\frac{W\left(-\frac{\ln(3)}{6}\right)}{\ln(3)}-1,\ -\frac{W\left(-1,\ -\frac{\ln(3)}{6}\right)}{\ln(3)}-1 \quad \mathbf{(11)}$$

>

>

Επομένως οι ΡΙΖΕΣ της Εξίσωσης είναι :

> $x1 := evalf(rhs(\mathbf{(8)}))$

$$x1 := -0.7901100112 \quad \mathbf{(12)}$$

> $x2 := evalf(rhs(\mathbf{(9)}))$

$$x2 := 1.444561392 \quad \mathbf{(13)}$$

>

>

ΚΡΙΣΙΜΑ ΣΗΜΕΙΑ -ΑΚΡΟΤΑΤΑ-ΔΙΑΣΤΗΜΑΤΑ 1-1 :

> $diff(f(x), x)$

$$3^x \ln(3) - 2 \quad \mathbf{(14)}$$

> $evalf(solve(\mathbf{(14)}, x))\text{assuming } x :: real$

$$0.5453237314 \quad \mathbf{(15)}$$

> $diff(f(x), x\$2)$

$$3^x \ln(3)^2 \quad \mathbf{(16)}$$

> $subs(x = \mathbf{(15)}, \mathbf{(16)})$

$$1.820478453 \ln(3)^2 \quad \mathbf{(17)}$$

>

ΣΗΜΕΙΟ MIN:

> $MIN := [\mathbf{(15)}, f(\mathbf{(15)})]$

$$MIN := [0.5453237314,\ -1.270169010] \quad \mathbf{(18)}$$

>

ΣΗΜΕΙΟ ΚΑΜΠΗΣ : Δεν έχουμε : Η δεύτερη παράγωγος πάντοτε >0 .Τα κοίλα της εξίσωσης προς τα πάνω .!

>

ΔΙΑΣΤΗΜΑΤΑ 1-1:

>

1-ΠΕΔΙΟ ΟΡΙΣΜΟΥΣΥΝΑΡΤΗΣΗΣ =ΠΕΔΙΟ ΤΙΜΩΝ ΑΝΤΙΣΤΡΟΦΗΣ ΣΥΝΑΡΤΗΣΗΣ

> $[-3\,..\,MIN[1]]$

$$[-3\,..0.5453237314] \quad \mathbf{(19)}$$

>

1- ΠΕΔΙΟ ΤΙΜΩΝ ΣΥΝΑΡΤΗΣΗΣ=ΠΕΔΙΟ ΟΡΙΣΜΟΥ ΑΝΤΙΣΤΡΟΦΗΣ ΣΥΝΑΡΤΗΣΗΣ

> $evalf([f(-3), MIN[2]])$

$$[4.037037037,\ -1.270169010] \quad \mathbf{(20)}$$

>

2-ΠΕΔΙΟ ΟΡΙΣΜΟΥ ΣΥΝΑΡΤΗΣΗΣ =ΠΕΔΙΟ ΤΙΜΩΝ ΑΝΤΙΣΤΡΟΦΗΣ ΣΥΝΑΡΤΗΣΗΣ

> $[MIN[1]..3]$

$$[0.5453237314..3] \tag{21}$$

>

2- ΠΕΔΙΟ ΤΙΜΩΝ ΣΥΝΑΡΤΗΣΗΣ=ΠΕΔΙΟ ΟΡΙΣΜΟΥ ΑΝΤΙΣΤΡΟΦΗΣ ΣΥΝΑΡΤΗΣΗΣ

> $[MIN[2]..f(3)]$

$$[-1.270169010..19] \tag{22}$$

>

ΑΝΤΙΣΤΡΟΦΗ ΣΥΝΑΡΤΗΣΗ :

> $\boldsymbol{x = 3^y - 2\cdot y - 2}$

$$x = 3^y - 2\,y - 2 \tag{23}$$

> $\boldsymbol{solve(rhs((23)), y)}$ assuming $y :: real$

$$-\frac{W\left(-\frac{\ln(3)}{6}\right) + \ln(3)}{\ln(3)},\ -\frac{W\left(-1, -\frac{\ln(3)}{6}\right) + \ln(3)}{\ln(3)} \tag{24}$$

>

Επομένως τα σημεία τομής με τον Άξονα y της ΑΝΤΙΣΤΡΟΦΗΣ ΣΥΝΑΡΤΗΣΗΣ είναι :

>

> $y1 := evalf((24)[1])$

$$y1 := -0.7901100111 \tag{25}$$

> $y2 := evalf((24)[2])$

$$y2 := 1.444561392 \tag{26}$$

>

>

>

ΕΥΘΕΙΕΣ ΚΑΘΕΤΕΣ ΣΤΗΝ ΔΙΧΟΤΟΜΟ y=x :

>

> $\boldsymbol{MIN}$

$$[0.5453237314, -1.270169010] \tag{27}$$

> $\boldsymbol{INVMIN := [MIN[2], MIN[1]]}$

$$INVMIN := [-1.270169010, 0.5453237314] \tag{28}$$

> $r1\_ := MIN[1]\cdot\_i + MIN[2]\cdot\_j$

$$\overrightarrow{r1} := 0.5453237314\,\hat{i} - 1.270169010\,\hat{j} \tag{29}$$

> $r2\_ := INVMIN[1]\cdot\_i + INVMIN[2]\cdot\_j$

$$\overrightarrow{r2} := -1.270169010\,\hat{i} + 0.5453237314\,\hat{j} \tag{30}$$

> $r\_ := r1\_ + t\cdot(r2\_ - r1\_)$

$$\vec{r} := \hat{i}\,(0.5453237314 - 1.815492741\,t) + \hat{j}\,(-1.270169010 + 1.815492741\,t) \tag{31}$$

> $x = Component((\mathbf{31}), 1)$

$$x = 0.5453237314 - 1.815492741\ t \tag{32}$$

> $y = Component((\mathbf{31}), 2)$

$$y = -1.270169010 + 1.815492741\ t \tag{33}$$

> $\boldsymbol{K1 := plot([rhs((32)), rhs((33)), t = 0..1], linestyle = 2):}$

>

>

> $\boldsymbol{RIZES1} := [x1, 0]$

$$RIZES1 := [-0.7901100112, 0] \tag{34}$$

> $\boldsymbol{INVRIZES1} := [0, y1]$

$$INVRIZES1 := [0, -0.7901100111] \tag{35}$$

> $r3\_ := RIZES1[1] \cdot \_i + RIZES1[2] \cdot \_j$

$$\overrightarrow{r3} := -0.7901100112\ \hat{i} \tag{36}$$

> $r4\_ := INVRIZES1[1] \cdot \_i + INVRIZES1[2] \cdot \_j$

$$\overrightarrow{r4} := -0.7901100111\ \hat{j} \tag{37}$$

> $R\_ := r3\_ + t \cdot (r4\_ - r3\_)$

$$\vec{R} := \hat{i}\ (-0.7901100112 + 0.7901100112\ t) - 0.7901100111\ t\ \hat{j} \tag{38}$$

> $x = Component((\mathbf{38}), 1)$

$$x = -0.7901100112 + 0.7901100112\ t \tag{39}$$

> $y = Component((\mathbf{38}), 2)$

$$y = -0.7901100111\ t \tag{40}$$

> $\boldsymbol{K2 := plot([rhs((39)), rhs((40)), t = 0..1], linestyle = 2):}$

>

>

> $\boldsymbol{RIZES2} := [x2, 0]$

$$RIZES2 := [1.444561392, 0] \tag{41}$$

> $\boldsymbol{INVRIZES2} := [0, y2]$

$$INVRIZES2 := [0, 1.444561392] \tag{42}$$

> $r5\_ := RIZES2[1] \cdot \_i + RIZES2[2] \cdot \_j$

$$\overrightarrow{r5} := 1.444561392\ \hat{i} \tag{43}$$

> $r6\_ := INVRIZES2[1] \cdot \_i + INVRIZES2[2] \cdot \_j$

$$\overrightarrow{r6} := 1.444561392\ \hat{j} \tag{44}$$

> $S\_ := r5\_ + t \cdot (r6\_ - r5\_)$

$$\vec{S} := \hat{i}\ (1.444561392 - 1.444561392\ t) + 1.444561392\ t\ \hat{j} \tag{45}$$

> $x = Component((\mathbf{45}), 1)$

$$x = 1.444561392 - 1.444561392\ t \tag{46}$$

> $y = Component((\mathbf{45}), 2)$

$$y = 1.444561392\ t \tag{47}$$

> $K3 := \boldsymbol{plot([rhs((46)), rhs((47)), t = 0..1], linestyle = 2)}:$

>

>

> $TEST := evalf([-2, f(-2)])$

$$TEST := [-2., 2.111111111] \quad (48)$$

> $INVTEST := [TEST[2], TEST[1]]$

$$INVTEST := [2.111111111, -2.] \quad (49)$$

> $r7\_ := TEST[1] \cdot \_i + TEST[2] \cdot \_j$

$$\overrightarrow{r7} := -2.\ \hat{i} + 2.111111111\ \hat{j} \quad (50)$$

> $r8\_ := INVTEST[1] \cdot \_i + INVTEST[2] \cdot \_j$

$$\overrightarrow{r8} := 2.111111111\ \hat{i} - 2.\ \hat{j} \quad (51)$$

> $T\_ := r7\_ + t \cdot (r8\_ - r7\_)$

$$\vec{T} := \hat{i}\ (-2. + 4.111111111\ t) + \hat{j}\ (2.111111111 - 4.111111111\ t) \quad (52)$$

> $x = Component(\mathbf{(52)}, 1)$

$$x = -2. + 4.111111111\ t \quad (53)$$

> $y = Component(\mathbf{(52)}, 2)$

$$y = 2.111111111 - 4.111111111\ t \quad (54)$$

> $K4 := \mathbf{plot}([\mathbf{rhs}(\mathbf{(53)}), \mathbf{rhs}(\mathbf{(54)}), t = 0..1], \mathbf{linestyle} = 2):$

>

>

ΤΟΜΗ ΤΩΝ ΣΥΝΑΡΤΗΣΕΩΝ ΜΕ ΤΗΝ ΔΙΧΟΤΟΜΟ y=x :

> $\boldsymbol{x = f(x)}$

$$x = 3^x - 2x - 2 \quad (55)$$

> $solve(\mathbf{(55)}, x)$ assuming $x :: real$

$$-\frac{2\ln(3) + 3\,W\left(-\frac{\ln(3)\,3^{1/3}}{9}\right)}{3\ln(3)},\ -\frac{2\ln(3) + 3\,W\left(-1, -\frac{\ln(3)\,3^{1/3}}{9}\right)}{3\ln(3)} \quad (56)$$

> $evalf(\mathbf{(56)}[1])$

$$-0.4671427213 \quad (57)$$

> $evalf(\mathbf{(56)}[2])$

$$1.834450392 \quad (58)$$

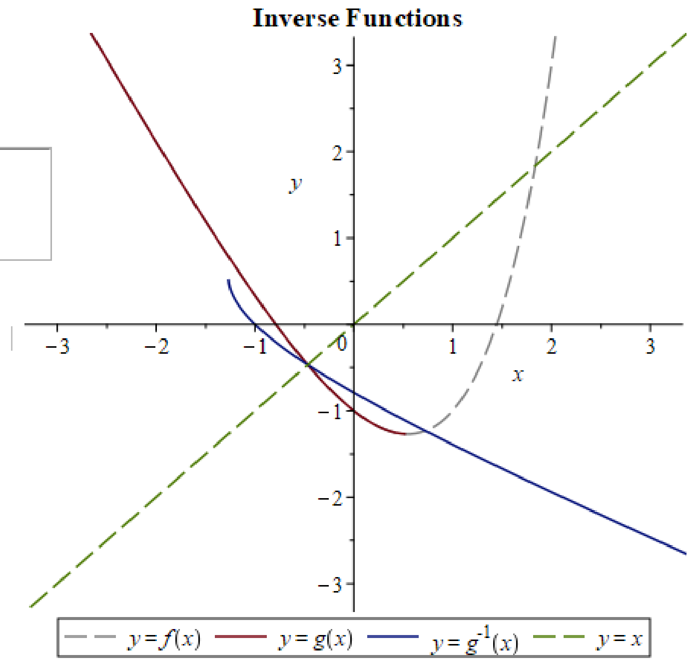
>

>

ΑΠΕΙΚΟΝΙΣΗ :

Example = User Defined

$f(x)$ = $3^x - 2 \cdot x - 2$



Plot f(x) Restrict Domain Inverse

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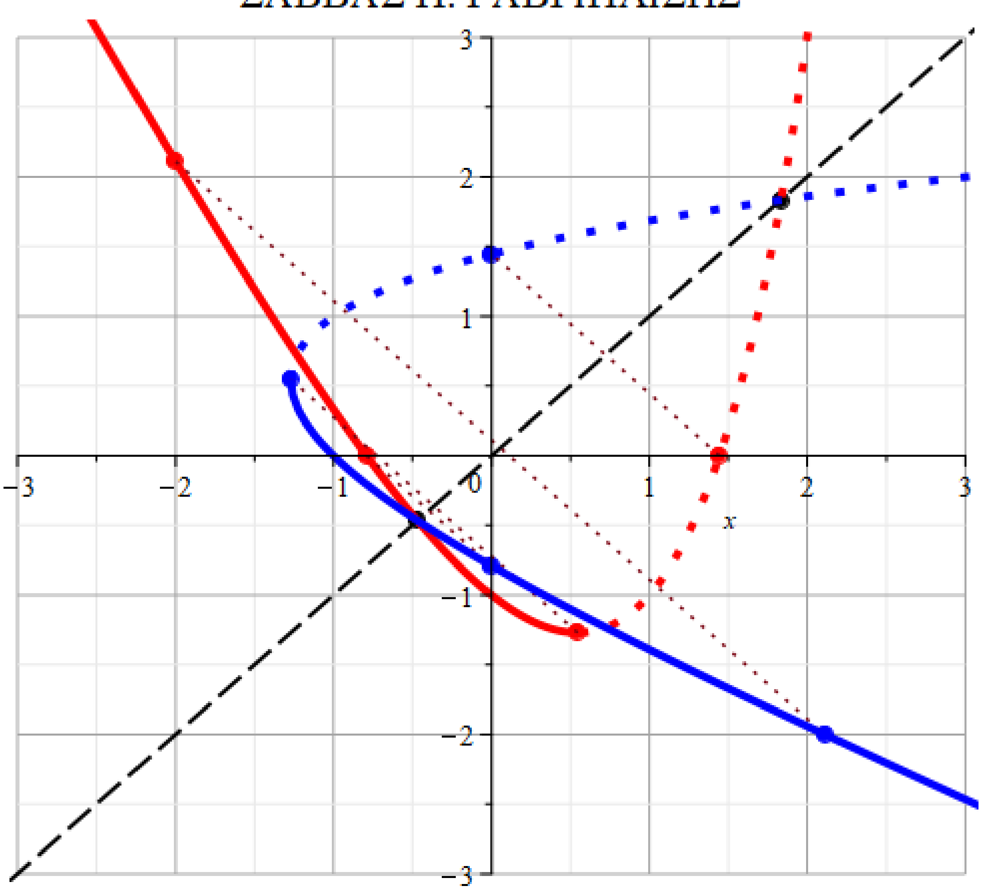

> $Plist := [MIN, INVMIN, [x1, 0], [x2, 0], [0, y1], [0, y2], TEST, INVTEST, [\mathbf{(57)}, f(\mathbf{(57)})], [\mathbf{(58)}, f(\mathbf{(58)})]]:$

> $Point := pointplot(Plist, symbol = solidcircle, symbolsize = 12, color = [red, blue, red, red, blue, blue, red, blue, black, black]):$

> $A := plot(f(x), x = -10..MIN[1], color = red, linestyle = 1, thickness = 4):$

> $B := plot(f(x), x = MIN[1]..10, color = red, linestyle = 2, thickness = 4):$

> $LIN := plot(x, x = -10..10, color = black, linestyle = 3, thickness = 2):$

>

> $INVERSE1 := implicitplot\left(x = 3^y - 2\,y - 2, x = f(-10)..f(MIN[1]), y = -10..MIN[1], color = blue, linestyle = 1, thickness = 4\right):$

> $INVERSE2 := implicitplot\left(x = 3^y - 2\,y - 2, x = f(MIN[1])..f(10), y = MIN[1]..10, color = blue, linestyle = 2, thickness = 4\right):$

>

> $display(Point, A, B, LIN, K1, K2, K3, K4, INVERSE1, INVERSE2, view = [-3..3, -3..3], gridlines, title$
= "Η κόκκινη συνάρτηση & η αντίστροφη μπλε\nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΣΗΣ" , $titlefont = [arial, bolt, 16])$

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