>

ΠΑΡΑΤΗΡΗΣΗ :

Για οποιοδήποτε Διάνυσμα Α (Θέσεως, Ταχύτητας , Ροπής , Επιταχύνσεως ,κλπ.) που είναι συνάρτηση του χρόνου t ισχύει η σχέση (Α) .:

$$\left. \frac{d\vec{A}}{dt} \right|_{\Omega} = \left. \frac{d\vec{A}}{dt} \right|_{O} + \vec{\omega} \times \vec{A}$$ **(A)** **(Ω Αδρανειακό Σύστημα,Αδρανειακός παρατηρητής),(Ο <u>ΜΗ</u> Αδρανειακό Σύστημα ,ΜΗ Αδρανειακός παρατηρητής) .**

ΒΑΣΙΚΟ : Γιά ΑΔΡΑΝΕΙΑΚΟ παρατηρητή τα Μοναδιαία Διανύσματα Μεταβάλλονται Συναρτήσει του Χρόνου .
Επομένως πρέπει να Υπολογίζουμε τις Παραγώγους των Μοναδιαίων Διανυσμάτων Συναρτήσει του Χρόνου .

> *with(plots) :*

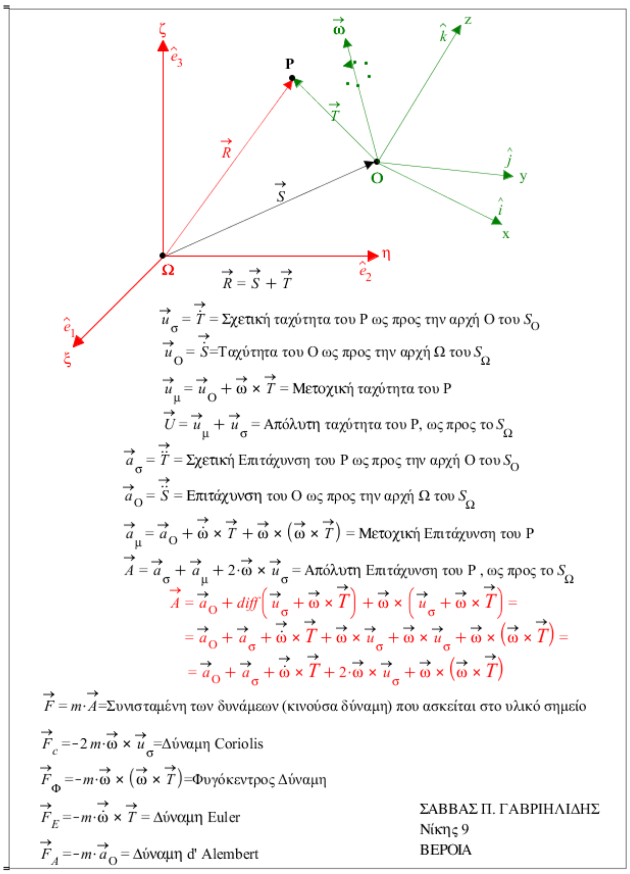
> *with(Physics[Vectors])*

[&x, `+`, `.`, ChangeBasis, ChangeCoordinates, Component, Curl, DirectionalDiff, Divergence, Gradient, Identify, Laplacian,∇, Norm, ParametrizeCurve, ParametrizeSurface, ParametrizeVolume, Setup, diff, int] **(1)**

> *Setup(mathematicalnotation = true)*

[mathematicalnotation = true] **(2)**

>



Καρτεσιανές Συντεταγμένες .

$\hat{i} = \cos(\phi(t)) \cdot \hat{\rho} - \sin(\phi(t)) \cdot \hat{\phi}$

$\hat{j} = \sin(\phi(t)) \cdot \hat{\rho} + \cos(\phi(t)) \cdot \hat{\phi}$

$\hat{\kappa} = \hat{\kappa}$

$\dot{\hat{i}} = \vec{\omega} \times \hat{i}$

$\dot{\hat{j}} = \vec{\omega} \times \hat{j}$

$\dot{\hat{k}} = \vec{\omega} \times \hat{k}$

Κυλινδρικές Συντεταγμένες .

$\hat{\rho} = \cos(\phi(t)) \cdot \hat{i} + \sin(\phi(t)) \cdot \hat{j}$

$\hat{\phi} = -\sin(\phi(t)) \cdot \hat{i} + \cos(\phi(t)) \cdot \hat{j}$

$\hat{\kappa} = \hat{\kappa}$

Με παραγώγιση ως προς t ,γιά Αδρανειακό Παρατηρητή ⇒

$\dot{\hat{\rho}} = \dot{\phi} \cdot \hat{\phi}$

$\dot{\hat{\phi}} = -\dot{\phi} \cdot \hat{\rho}$

ΣΥΜΒΟΛΙΣΜΟΙ :
GP : Γεωγραφικό Πλάτος
GM: Γεωγραφικό μήκος
B: ΒΕΡΟΙΑ
GR: ΓΚΡΗΝΟΥΪΤΣ
M: ΜΟΣΧΑ
S: ΣΙΔΝΕΫ
LS: ΛΟΣ ΑΝΤΖΕΛΕΣ
KT: ΚΕΪΠ ΤΑΟΥΝ

ΚΛΙΣΗ ΑΞΟΝΑ ΠΕΡΙΣΤΡΟΦΗΣ της ΓΗΣ ως προς την κατακόρυφο : 23, 439247 ° = 0.40909 *rad*.
Μέση ακτίνα ΓΗΣ :6371 km .
Αφήλιο :152.098.232 km , Περιήλιο :147.098.290 km , Εκκεντρότητα :0.01671123 .
Μεγάλος Ημιάξονας Ελλειπτικής : α=149.598.261 km **(5)**,⇒c=2.499.971 km **(1)**
Μικρός Ημιάξονας Ελλειπτικής :b=149.577.371 km **(4)**.

ΒΕΡΟΙΑ : (GP_B=$40^0$31'11.9" N , GM_B = $22^0$12'7"E): $40 + \frac{31}{60} + \frac{11.9}{3600} = 40.51997223$ $evalf\left(22 + \frac{12}{60} + \frac{7}{3600}\right) = 22.20194444$

ΓΚΡΗΝΟΥΪΤΣ : (GP_{GR}=$51^0$28'59.3" N , GM_{GR} = 0): $51 + \frac{28}{60} + \frac{59.3}{3600} = 51.48313889$ $0 = 0$

ΜΟΣΧΑ : (GP_M=$55^0$45'14" N , GM_M = $37^0$37'13.1"E) $evalf\left(55 + \frac{45}{60} + \frac{14}{3600}\right) = 55.75388889$ $37 + \frac{37}{60} + \frac{13.1}{3600} = 37.62030556$

ΣΙΔΝΕΫ : (GP_S=$33^0$51'23.9" S , GM_S = $151^0$12'56.1"E): $-33 - \frac{51}{60} - \frac{23.9}{3600} = -33.85663889$ $151 + \frac{12}{60} + \frac{56.1}{3600} = 151.2155833$

ΛΟΣ ΑΝΤΖΕΛΕΣ : (GP_{LS}=$34^0$06'48.0" N , GM_{LS} = $118^0$19'46.9"W): $evalf\left(34 + \frac{06}{60} + \frac{48}{3600}\right) = 34.11333333$ $-\left(118 + \frac{19}{60} + \frac{46.9}{3600}\right) = -118.3296945$

ΚΕΪΠ ΤΑΟΥΝ : (GP_{KT}=$33^0$54'16.6" S , GM_{KT} = $18^0$24'36.6"E): $evalf\left(-\left(33 + \frac{6}{60} + \frac{48}{3600}\right)\right) = -33.11333333$ $18 + \frac{24}{60} + \frac{36.6}{3600} = 18.41016667$

ΘΕΜΑ :

Βλήμα ***P*** εκτοξεύεται από τόπο **Γεωγραφικού Πλάτους ($\frac{Pi}{2} - \theta$)** και ύψος **H**

έτσι ώστε η αρχική του ταχύτητα u_0 να σχηματίζει **γωνία α** με το οριζόντιο επίπεδο($\hat{\theta}\, B\, \hat{\phi}$) και β με το κατακόρυφο επίπεδο($\hat{\theta}\, B\, \hat{r}$)

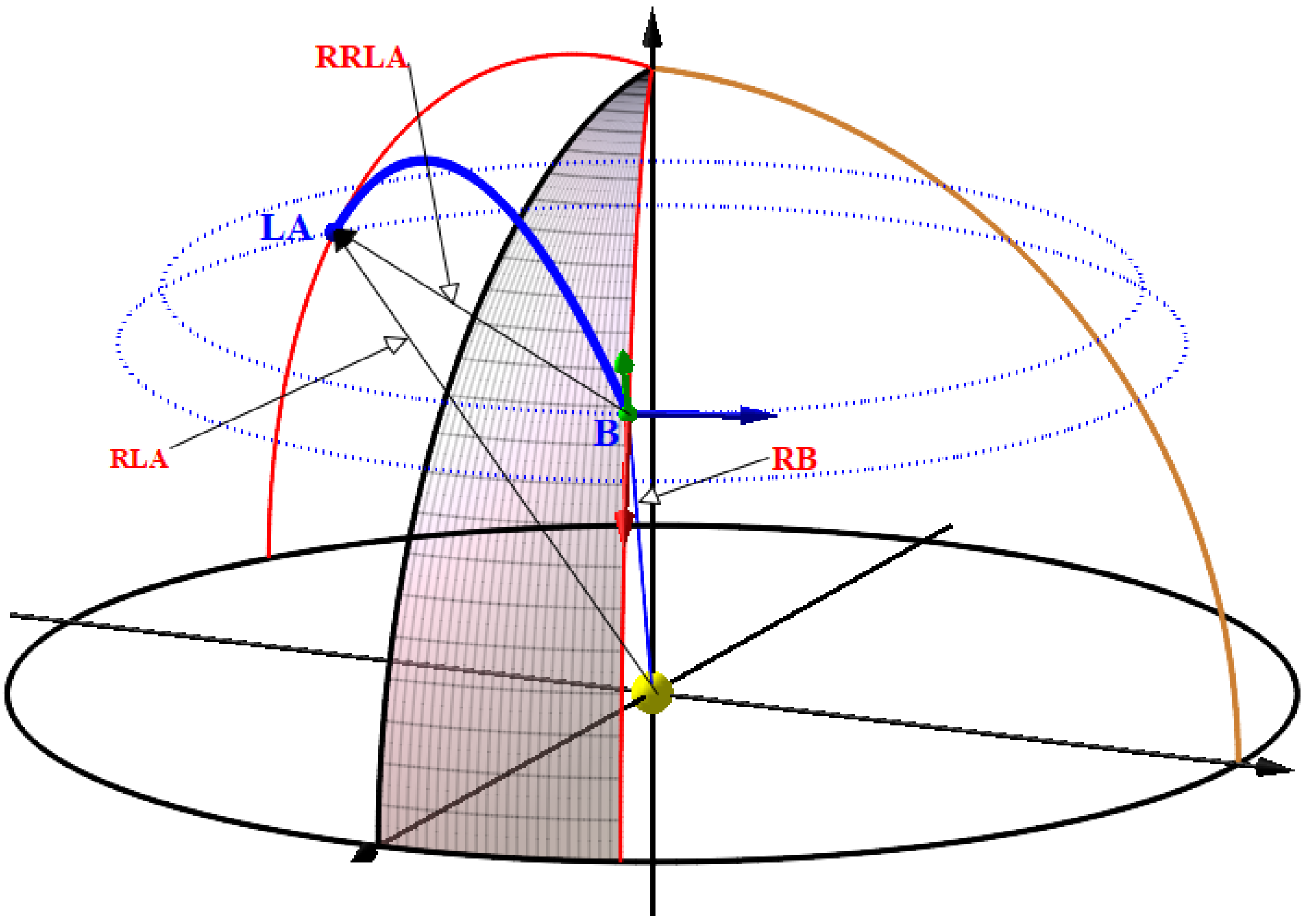
και να διευθύνεται πρός **ΒΔ** .

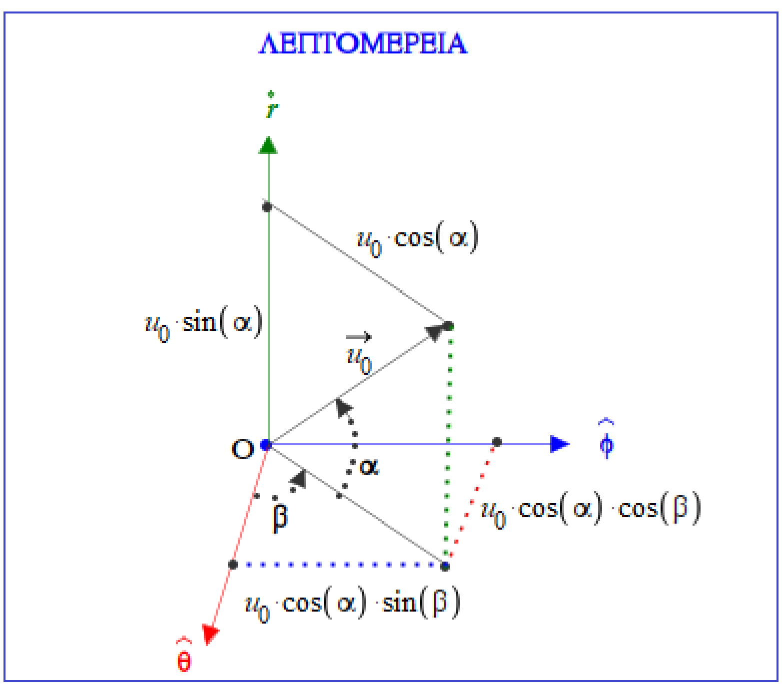
Να βρεθεί η θέση πρόσκρουσης στην Επιφάνεια της ΓΗΣ .

Παραδοχές :

Η τιμή g της επιτάχυνσης της βαρύτητας διατηρείται σταθερή .

ΜΕ ΣΤΟΧΟ ΤΟ LOS-ANGELES
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ





Προσοχή !!! .
Γιά στόχευση ΒΑ , ΓΩΝΙΑ β > $\frac{Pi}{2}$.

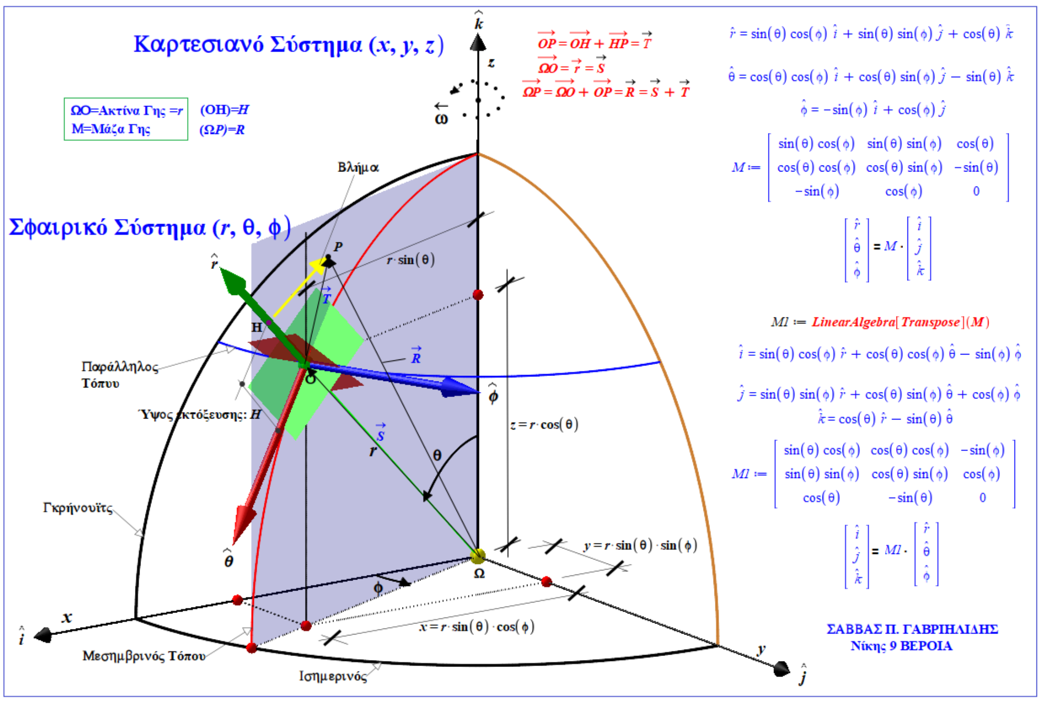
Γιά στόχευση προς Ανατολάς , ΓΩΝΙΑ β=$\frac{Pi}{2}$.

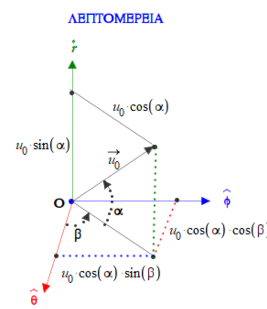
Γιά στόχευση προς Βορρά , ΓΩΝΙΑ β=Pi .

Γιά στόχευση προς Νότο , ΓΩΝΙΑ β=0 .

Γιά στόχευση ΒΔ , ΓΩΝΙΑ β > Pi .

Γιά στόχευση προς Δυσμάς , ΓΩΝΙΑ β=$\frac{3 \cdot Pi}{2}$.





$\overrightarrow{OP} = \overrightarrow{OH} + \overrightarrow{HP} = \vec{T}$

$\overrightarrow{\Omega O} = \vec{r} = \vec{S}$

$\overrightarrow{\Omega P} = \overrightarrow{\Omega O} + \overrightarrow{OP} = \vec{R} = \vec{S} + \vec{T}$

Συμβολισμοί :Ακτίνα Γης : ΩΟ= Q , Θ = θ, Φ = φ

(OH)=*H*

(ΩP)=*R*

M=Μάζα Γης

Γωνιακή ταχύτητα περιστροφής της ΓΗΣ περί τον άξονά της : $\omega = 7.292 \cdot 10^{-5} rad \cdot \sec^{-1}$

$$evalf\left(\frac{2 \cdot Pi}{\left(23 + \frac{56}{60} + \frac{4}{60 \cdot 60}\right) \cdot 60 \cdot 60}\right) = 7.292 \times 10^{-5}$$

>

>

Εξισώσεις της Κίνησης του Υλικου σημείου P

(Τα μήκη σε μέτρα .!!!)

> $H := 10000$

$$H := 10000 \quad (3)$$

> $g[0] := 9.80$

$$g_0 := 9.80 \quad (4)$$

> $\alpha := \frac{\text{Pi}}{4}$

$$\alpha := \frac{\pi}{4} \quad (5)$$

> ω_ := Q·(ω·cos(Θ)_r − ω·sin(Θ)_θ)

$$\vec{\omega} := Q\left(\omega\cos(\Theta)\,\hat{r} - \omega\sin(\Theta)\,\hat{\theta}\right) \quad (6)$$

> H_ := H·_r

$$\vec{H} := 10000\,\hat{r} \quad (7)$$

> S_ := Q·_r

$$\vec{S} := Q\,\hat{r} \quad (8)$$

> T_ := H_ + θ(t)·_θ + ϕ(t)·_ϕ + r(t)·_r

$$\vec{T} := \hat{r}\,(10000 + r(t)) + \theta(t)\,\hat{\theta} + \phi(t)\,\hat{\phi} \quad (9)$$

> R_ := S_ + T_

$$\vec{R} := \hat{r}\,(Q + 10000 + r(t)) + \theta(t)\,\hat{\theta} + \phi(t)\,\hat{\phi} \quad (10)$$

> u_[O] := diff(R_, t)

$$\vec{u}_{O} := \hat{r}\left(\frac{d}{dt}\,r(t)\right) + \left(\frac{d}{dt}\,\theta(t)\right)\hat{\theta} + \left(\frac{d}{dt}\,\phi(t)\right)\hat{\phi} \quad (11)$$

> u_[Ω] := u_[O] + ω_ × R_

$$\vec{u}_{\Omega} := \hat{r}\left(\frac{d}{dt}\,r(t) - \sin(\Theta)\,Q\,\omega\,\phi(t)\right) + \hat{\theta}\left(\frac{d}{dt}\,\theta(t) - \cos(\Theta)\,Q\,\omega\,\phi(t)\right) + \hat{\phi}\left(\frac{d}{dt}\,\phi(t) + Q\,\omega\left(\sin(\Theta)\,r(t) + \cos(\Theta)\,\theta(t) + \sin(\Theta)\,Q + 10000\sin(\Theta)\right)\right) \quad (12)$$

> a_[Ω] := diff(u_[Ω], t) + ω_ × u_[Ω]

$$\vec{a}_{\Omega} := \hat{r}\left(\frac{d^2}{dt^2}\,r(t) - \sin(\Theta)\,Q\,\omega\left(\frac{d}{dt}\,\phi(t)\right) - \sin(\Theta)\,Q\,\omega\left(\frac{d}{dt}\,\phi(t) + Q\,\omega\left(\sin(\Theta)\,r(t) + \cos(\Theta)\,\theta(t) + \sin(\Theta)\,Q + 10000\sin(\Theta)\right)\right)\right) + \hat{\theta}\left(\frac{d^2}{dt^2}\,\theta(t) - \cos(\Theta)\,Q\,\omega\left(\frac{d}{dt}\right.\right. \quad (13)$$

$$\phi(t)\Big) - \cos(\Theta)\, Q\,\omega\left(\frac{d}{dt}\phi(t) + Q\,\omega\,(\sin(\Theta)\, r(t) + \cos(\Theta)\,\theta(t) + \sin(\Theta)\, Q + 10000\sin(\Theta))\right)\Big) + \hat{\phi}\left(\frac{d^2}{dt^2}\phi(t) + Q\,\omega\left(\sin(\Theta)\left(\frac{d}{dt} r(t)\right) + \cos(\Theta)\left(\frac{d}{dt}\theta(t)\right)\right) - Q\,\omega\left(\phi(t)\, Q\,\omega - \cos(\Theta)\left(\frac{d}{dt}\theta(t)\right) - \sin(\Theta)\left(\frac{d}{dt} r(t)\right)\right)\right)$$

>

4. ΒΟΛΗ από Ανατολή προς Δύση .

ΕΚΤΡΟΠΗ ΠΡΟΣ ΒΟΡΡΑ και κάτω από το Οριζόντιο Επίπεδο Εκτόξευσης.

$u4[O] := -u \cdot \_\phi$ $\qquad u4_O := -u\,\hat{\phi}$

$Coriolis4 := simplify(-2\cdot\omega\_ \times u4[O])$ $\qquad Coriolis4 := -2\,Q\,\omega\,u\,(\sin(\Theta)\,\hat{r} + \cos(\Theta)\,\hat{\theta})$

>

> $Coriolis := -2\cdot\omega\_ \times u\_[O]$

$$Coriolis := 2\sin(\Theta)\left(\frac{d}{dt}\phi(t)\right) Q\,\hat{r}\,\omega + 2\cos(\Theta)\left(\frac{d}{dt}\phi(t)\right) Q\,\hat{\theta}\,\omega - 2\,\hat{\phi}\, Q\,\omega\left(\sin(\Theta)\left(\frac{d}{dt} r(t)\right) + \cos(\Theta)\left(\frac{d}{dt}\theta(t)\right)\right) \quad \textbf{(14)}$$

> $Component(Coriolis, 1)$

$$2\sin(\Theta)\, Q\,\omega\left(\frac{d}{dt}\phi(t)\right) \quad \textbf{(15)}$$

> $Component(Coriolis, 2)$

$$2\cos(\Theta)\, Q\,\omega\left(\frac{d}{dt}\phi(t)\right) \quad \textbf{(16)}$$

> $Component(Coriolis, 3)$

$$-2\sin(\Theta)\left(\frac{d}{dt} r(t)\right) Q\,\omega - 2\cos(\Theta)\left(\frac{d}{dt}\theta(t)\right) Q\,\omega \quad \textbf{(17)}$$

> $Fygokentros := -\omega\_ \times (\omega\_ \times R\_)$

$$Fygokentros := \sin(\Theta)\, Q^2\,\omega^2\,(\sin(\Theta)\, r(t) + \cos(\Theta)\,\theta(t) + \sin(\Theta)\, Q + 10000\sin(\Theta))\,\hat{r} + \cos(\Theta)\, Q^2\,\omega^2\,(\sin(\Theta)\, r(t) + \cos(\Theta)\,\theta(t) + \sin(\Theta)\, Q + 10000\sin(\Theta))\,\hat{\theta} + \hat{\phi}\, Q^2\,\omega^2\,\phi(t) \quad \textbf{(18)}$$

>

Μπορούμε να γράψουμε :

Είναι : $-G\cdot M\cdot\frac{\vec{R}}{\left(Norm(\vec{R})\right)^3} = -g\cdot\hat{r}$ Η επιτάχυνση της Βαρύτητας της ΓΗΣ στο σημείο P . (Χωρίς απλοποιήσεις)

ΕΑΝ Θεωρηθεί ότι η κίνηση του υλικό σημείου P γίνεται πλησίον της επιφάνειας της ΓΗΣ := $-g[0]\cdot\hat{r} = -G\cdot M\cdot\frac{\vec{S}}{S^3}$, όπου $g[0]=9.81\cdot\frac{m}{s^2}$

> $-\frac{G\cdot M\cdot R\_}{(Norm(R\_))^3} = a\_[\Omega]$

$$-\frac{GM\left(\hat{r}\,(Q+10000+r(t))+\theta(t)\,\hat{\theta}+\phi(t)\,\hat{\phi}\right)}{\left((Q+10000+r(t))^2+\theta(t)^2+\phi(t)^2\right)^{3/2}} = \hat{r}\left(\frac{d^2}{dt^2}r(t)-\sin(\Theta)\,Q\,\omega\left(\frac{d}{dt}\phi(t)\right)-\sin(\Theta)\,Q\,\omega\left(\frac{d}{dt}\phi(t)+Q\,\omega\left(\sin(\Theta)\,r(t)+\cos(\Theta)\,\theta(t)+\sin(\Theta)\,Q+10000\sin(\Theta)\right)\right)\right)+\hat{\theta}\left(\frac{d^2}{dt^2}\theta(t)-\cos(\Theta)\,Q\,\omega\left(\frac{d}{dt}\phi(t)\right)-\cos(\Theta)\,Q\,\omega\left(\frac{d}{dt}\phi(t)+Q\,\omega\left(\sin(\Theta)\,r(t)+\cos(\Theta)\,\theta(t)+\sin(\Theta)\,Q+10000\sin(\Theta)\right)\right)\right)+\hat{\phi}\left(\frac{d^2}{dt^2}\phi(t)+Q\,\omega\left(\sin(\Theta)\left(\frac{d}{dt}r(t)\right)+\cos(\Theta)\left(\frac{d}{dt}\theta(t)\right)\right)-Q\,\omega\left(\phi(t)\,Q\,\omega-\cos(\Theta)\left(\frac{d}{dt}\theta(t)\right)-\sin(\Theta)\left(\frac{d}{dt}r(t)\right)\right)\right) \quad (19)$$

> $-g[0]\cdot\_r = a\_[\Omega]$

$$-9.80\,\hat{r} = \hat{r}\left(\frac{d^2}{dt^2}r(t)-\sin(\Theta)\,Q\,\omega\left(\frac{d}{dt}\phi(t)\right)-\sin(\Theta)\,Q\,\omega\left(\frac{d}{dt}\phi(t)+Q\,\omega\left(\sin(\Theta)\,r(t)+\cos(\Theta)\,\theta(t)+\sin(\Theta)\,Q+10000\sin(\Theta)\right)\right)\right)+\hat{\theta}\left(\frac{d^2}{dt^2}\theta(t)-\cos(\Theta)\,Q\,\omega\left(\frac{d}{dt}\phi(t)\right)-\cos(\Theta)\,Q\,\omega\left(\frac{d}{dt}\phi(t)+Q\,\omega\left(\sin(\Theta)\,r(t)+\cos(\Theta)\,\theta(t)+\sin(\Theta)\,Q+10000\sin(\Theta)\right)\right)\right)+\hat{\phi}\left(\frac{d^2}{dt^2}\phi(t)+Q\,\omega\left(\sin(\Theta)\left(\frac{d}{dt}r(t)\right)+\cos(\Theta)\left(\frac{d}{dt}\theta(t)\right)\right)-Q\,\omega\left(\phi(t)\,Q\,\omega-\cos(\Theta)\left(\frac{d}{dt}\theta(t)\right)-\sin(\Theta)\left(\frac{d}{dt}r(t)\right)\right)\right) \quad (20)$$

> $lhs((\mathbf{20})) - rhs((\mathbf{20})) = 0$

$$\left(-1.\,\frac{d^2}{dt^2}\phi(t)-1.\,Q\,\omega\left(\sin(\Theta)\left(\frac{d}{dt}r(t)\right)+\cos(\Theta)\left(\frac{d}{dt}\theta(t)\right)\right)+Q\,\omega\left(\phi(t)\,Q\,\omega-1.\,\sin(\Theta)\left(\frac{d}{dt}r(t)\right)-1.\,\cos(\Theta)\left(\frac{d}{dt}\theta(t)\right)\right)\right)\hat{\phi}+\hat{r}\left(-9.80-1.\,\frac{d^2}{dt^2}r(t)\right. \quad (21)$$

$$+\sin(\Theta)\,Q\,\omega\left(\frac{d}{dt}\,\phi(t)\right)+\sin(\Theta)\,Q\,\omega\left(\frac{d}{dt}\,\phi(t)+Q\,\omega\,(\sin(\Theta)\,r(t)+\sin(\Theta)\,Q\right.$$
$$\left.\left.+\cos(\Theta)\,\theta(t)+10000.\sin(\Theta)\,)\right)\right)+\hat{\theta}\left(-1.\,\frac{d^2}{dt^2}\,\theta(t)+\cos(\Theta)\,Q\,\omega\left(\frac{d}{dt}\,\phi(t)\right)\right.$$
$$+\cos(\Theta)\,Q\,\omega\left(\frac{d}{dt}\,\phi(t)+Q\,\omega\,(\sin(\Theta)\,r(t)+\sin(\Theta)\,Q+\cos(\Theta)\,\theta(t)\right.$$
$$\left.\left.+10000.\sin(\Theta)\,)\right)\right)=0$$

> $Eq := seq(Component(lhs(\mathbf{(21)}), n) = 0, n = 1..3)$

$$Eq := -9.80-1.\,\frac{d^2}{dt^2}\,r(t)+2\sin(\Theta)\,Q\,\omega\left(\frac{d}{dt}\,\phi(t)\right)+\sin(\Theta)^2\,Q^2\,\omega^2\,r(t)+\sin(\Theta)^2\,Q^3\,\omega^2 \quad \textbf{(22)}$$
$$+\sin(\Theta)\,Q^2\,\omega^2\cos(\Theta)\,\theta(t)+10000.\sin(\Theta)^2\,Q^2\,\omega^2=0,\ -1.\,\frac{d^2}{dt^2}\,\theta(t)$$
$$+2\cos(\Theta)\,Q\,\omega\left(\frac{d}{dt}\,\phi(t)\right)+\cos(\Theta)\,Q^2\,\omega^2\sin(\Theta)\,r(t)+\cos(\Theta)\,Q^3\,\omega^2\sin(\Theta)$$
$$+\cos(\Theta)^2\,Q^2\,\omega^2\,\theta(t)+10000.\cos(\Theta)\,Q^2\,\omega^2\sin(\Theta)=0,\ -1.\,\frac{d^2}{dt^2}\,\phi(t)-2.\sin(\Theta)\,\left(\frac{d}{dt}\right.$$
$$\left.r(t)\right)Q\,\omega-2.\cos(\Theta)\,\left(\frac{d}{dt}\,\theta(t)\right)Q\,\omega+Q^2\,\omega^2\,\phi(t)=0$$

>

ΓΕΝΙΚΕΣ ΕΞΙΣΩΣΕΙΣ ΚΙΝΗΣΗΣ ΤΟΥ Ρ..

> $isolate\left(Eq[1], \frac{d^2}{dt^2}\,r(t)\right)$

$$\frac{d^2}{dt^2}\,r(t)=-9.80+2.\sin(\Theta)\,Q\,\omega\left(\frac{d}{dt}\,\phi(t)\right)+1.\sin(\Theta)^2\,Q^2\,\omega^2\,r(t)+1.\sin(\Theta)^2\,Q^3\,\omega^2 \quad \textbf{(23)}$$
$$+1.\sin(\Theta)\,Q^2\,\omega^2\cos(\Theta)\,\theta(t)+10000.\sin(\Theta)^2\,Q^2\,\omega^2$$

> $isolate\left(Eq[2], \frac{d^2}{dt^2}\,\theta(t)\right)$

$$\frac{d^2}{dt^2}\,\theta(t)=2.\cos(\Theta)\,Q\,\omega\left(\frac{d}{dt}\,\phi(t)\right)+1.\cos(\Theta)\,Q^2\,\omega^2\sin(\Theta)\,r(t) \quad \textbf{(24)}$$
$$+1.\cos(\Theta)\,Q^3\,\omega^2\sin(\Theta)+1.\cos(\Theta)^2\,Q^2\,\omega^2\,\theta(t)+10000.\cos(\Theta)\,Q^2\,\omega^2\sin(\Theta)$$

> $isolate\left(Eq[3], \frac{d^2}{dt^2}\,\phi(t)\right)$

$$\frac{d^2}{dt^2}\,\phi(t)=-2.\sin(\Theta)\,\left(\frac{d}{dt}\,r(t)\right)Q\,\omega-2.\cos(\Theta)\,\left(\frac{d}{dt}\,\theta(t)\right)Q\,\omega+1.\,Q^2\,\omega^2\,\phi(t) \quad \textbf{(25)}$$

> $\boldsymbol{ics := r(0)=H, \theta(0)=0, \phi(0)=0, D(r)(0)=u[0]\cdot\sin(\alpha), D(\theta)(0)=u[0]}$

$\cdot\cos(\alpha)\cdot\cos(\beta), D(\phi)(0) = u[0]\cdot\cos(\alpha)\cdot\sin(\beta):$

Απλοποιήσεις !!!:

>

Μπορούμε να γράψουμε :

ΕΑΝ Θεωρηθεί ότι η κίνηση του υλικό σημείου P γίνεται πλησίον της επιφάνειας της ΓΗΣ :⇒ $-g[0]\cdot\hat{r} = -G\cdot M\cdot\frac{\vec{S}}{S^3}$, όπου $g[0]=9.81\cdot\frac{m}{s^2}$

> $-g[0]\cdot\_r = simplify(a\_[\Omega] + Fygokentros + Coriolis)$

$$-9.80\,\hat{r} = \left(\frac{d^2}{dt^2} r(t)\right)\hat{r} + \left(\frac{d^2}{dt^2}\theta(t)\right)\hat{\theta} + \left(\frac{d^2}{dt^2}\phi(t)\right)\hat{\phi} \quad \textbf{(26)}$$

>

ΤΕΛΙΚΕΣ ΕΞΙΣΩΣΕΙΣ ΚΙΝΗΣΗΣ ΤΟΥ P

>

> $lhs((\textbf{26})) - rhs((\textbf{26})) = 0$

$$-1.\left(\frac{d^2}{dt^2}\phi(t)\right)\hat{\phi} + \hat{r}\left(-9.80 - 1.\frac{d^2}{dt^2}r(t)\right) - 1.\left(\frac{d^2}{dt^2}\theta(t)\right)\hat{\theta} = 0 \quad \textbf{(27)}$$

> $Eq2 := seq(Component(lhs((\textbf{27})), n) = 0, n = 1..3)$

$$Eq2 := -9.80 - 1.\frac{d^2}{dt^2}r(t) = 0,\ -1.\frac{d^2}{dt^2}\theta(t) = 0,\ -1.\frac{d^2}{dt^2}\phi(t) = 0 \quad \textbf{(28)}$$

> $isolate\left(Eq2[1], \frac{d^2}{dt^2}r(t)\right)$

$$\frac{d^2}{dt^2}r(t) = -9.800000000 \quad \textbf{(29)}$$

> $isolate\left(Eq2[2], \frac{d^2}{dt^2}\theta(t)\right)$

$$\frac{d^2}{dt^2}\theta(t) = -0. \quad \textbf{(30)}$$

> $isolate\left(Eq2[3], \frac{d^2}{dt^2}\phi(t)\right)$

$$\frac{d^2}{dt^2}\phi(t) = -0. \quad \textbf{(31)}$$

>

Άρα έχουμε πρός επίλυση το Σύστημα :

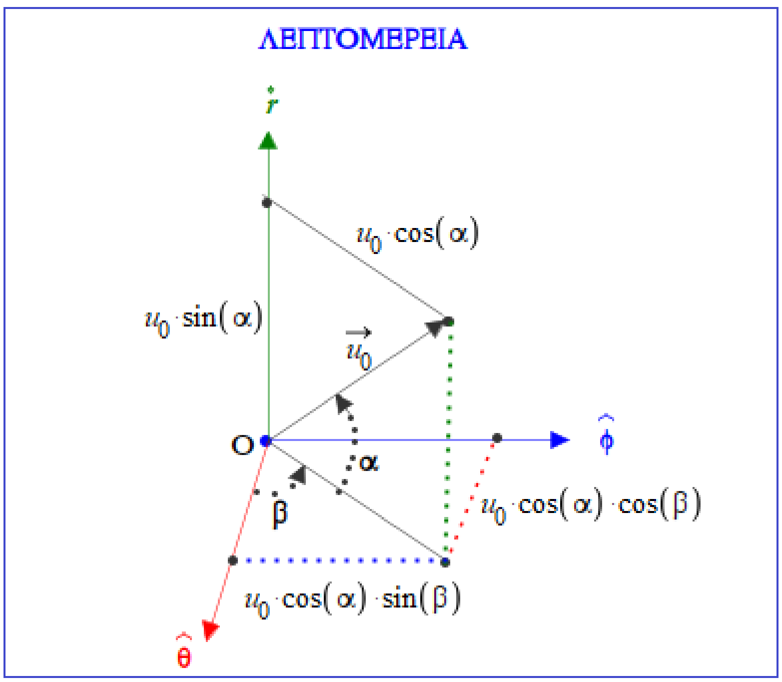
> $sys := \mathbf{(29), (30), (31)}$

$$sys := \frac{d^2}{dt^2}\, r(t) = -9.800000000,\ \frac{d^2}{dt^2}\, \theta(t) = -0.,\ \frac{d^2}{dt^2}\, \phi(t) = -0. \qquad \mathbf{(32)}$$

> $\boldsymbol{ics := r(0) = H, \theta(0) = 0, \phi(0) = 0, D(r)(0) = u[0]\cdot\sin(\alpha), D(\theta)(0) = u[0]\cdot\cos(\alpha)\cdot\cos(\beta), D(\phi)(0) = u[0]\cdot\cos(\alpha)\cdot\sin(\beta)}$

$$ics := r(0) = 10000, \theta(0) = 0, \phi(0) = 0, D(r)(0) = \frac{u_0\sqrt{2}}{2}, D(\theta)(0) = \frac{u_0\sqrt{2}\cos(\beta)}{2}, D(\phi)(0) = \frac{u_0\sqrt{2}\sin(\beta)}{2} \qquad \mathbf{(33)}$$

>



Προσοχή !!! .
Γιά στόχευση ΒΑ , ΓΩΝΙΑ $\beta > \frac{Pi}{2}$.

Γιά στόχευση προς Ανατολάς , ΓΩΝΙΑ $\beta = \frac{Pi}{2}$.

Γιά στόχευση προς Βορρά , ΓΩΝΙΑ β=Pi .

Γιά στόχευση προς Νότο , ΓΩΝΙΑ β=0 .

Γιά στόχευση ΒΔ , ΓΩΝΙΑ β > Pi .

Γιά στόχευση προς Δυσμάς , ΓΩΝΙΑ $\beta = \frac{3\cdot Pi}{2}$.

>

> $\boldsymbol{GP[B] := 0.7072607424 :}$

> $GM[B] := 0.3875212718:$

> $SOL := dsolve(\{sys, ics\})$

$$SOL := \left\{\phi(t) = \frac{t\,u_0\sqrt{2}\,\sin(\beta)}{2},\ r(t) = -\frac{49\,t^2}{10} + \frac{u_0\sqrt{2}\,t}{2} + 10000,\ \theta(t) = \frac{t\,u_0\sqrt{2}\,\cos(\beta)}{2}\right\} \quad (34)$$

> $SOL1 := subs([\Theta = \theta, Q = r], SOL)$

$$SOL1 := \left\{\phi(t) = \frac{t\,u_0\sqrt{2}\,\sin(\beta)}{2},\ r(t) = -\frac{49\,t^2}{10} + \frac{u_0\sqrt{2}\,t}{2} + 10000,\ \theta(t) = \frac{t\,u_0\sqrt{2}\,\cos(\beta)}{2}\right\} \quad (35)$$

> $evalf\left(subs\left(\theta = \left(\frac{Pi}{2} - GP[B]\right), SOL1[1]\right)\right)$

$$\phi(t) = 0.7071067810\,t\,u_0\sin(\beta) \quad (36)$$

> $SOL1[2]$

$$r(t) = -\frac{49\,t^2}{10} + \frac{u_0\sqrt{2}\,t}{2} + 10000 \quad (37)$$

> $SOL1[3]$

$$\theta(t) = \frac{t\,u_0\sqrt{2}\,\cos(\beta)}{2} \quad (38)$$

>

>

Καρτεσιανές Συντεταγμένες ΤΗΣ ΘΕΣΗΣ ΕΚΤΟΞΕΥΣΗΣ ως προς το ΑΔΡΑΝΕΙΑΚΟ ΣΥΣΤΗΜΑ Ωxyz :

> $LX := evalf\left(subs\left(\left\{r = 6371 \cdot 10^3, \theta = \left(\frac{Pi}{2} - GP[B]\right), \phi = GM[B]\right\}, r \cdot \sin(\theta) \cdot \cos(\phi)\right)\right)$

$$LX := 4.483774648 \times 10^6 \quad (39)$$

> $LY := evalf\left(subs\left(\left\{r = 6371 \cdot 10^3, \theta = \left(\frac{Pi}{2} - GP[B]\right), \phi = GM[B]\right\}, r \cdot \sin(\theta) \cdot \sin(\phi)\right)\right)$

$$LY := 1.830098846 \times 10^6 \quad (40)$$

> $LZ := evalf\left(subs\left(\left\{r = 6371 \cdot 10^3, \theta = \left(\frac{Pi}{2} - GP[B]\right), \phi = GM[B]\right\}, r \cdot \cos(\theta)\right)\right)$

$$LZ := 4.139582603 \times 10^6 \quad (41)$$

>

ΑΛΛΑΓΗ ΣΥΣΤΗΜΑΤΟΣ ΣΥΝΤΕΤΑΓΜΕΝΩΝ :

ΤΡΟΧΙΑΣ ΒΛΗΜΑΤΟΣ:

>

$$\phi(t) = 0.7071067810\, t\, u_0 \sin(\beta) \qquad \textbf{(36)}$$

$$r(t) = -\frac{49\, t^2}{10} + \frac{u_0 \sqrt{2}\, t}{2} + 10000 \qquad \textbf{(37)}$$

$$\theta(t) = \frac{t\, u_0 \sqrt{2} \cos(\beta)}{2} \qquad \textbf{(38)}$$

>

> $K[\theta] := rhs(\textbf{(38)})$:

> $K[\phi] := rhs(\textbf{(36)})$:

> $K[r] := rhs(\textbf{(37)})$:

> $K\_ := K[r] \cdot \_r + K[\theta] \cdot \_\theta + K[\phi] \cdot \_\phi$:

>

> $K[x] := Component\left(eval\left(subs\left(\left\{ \omega = 7.292 \cdot 10^{-5}, r = 6371 \cdot 10^{3}, \theta = \left(\frac{Pi}{2} - GP[B] \right), \phi = GM[B] \right\}, ChangeBasis(K\_, 1) \right) \right), 1 \right)$

$$K_x := -3.448516053\, t^2 + 0.4976467521\, u_0\, t + 7037.787863 + 0.4253769224\, t\, u_0 \cos(\beta) - 0.2672118704\, t\, u_0 \sin(\beta) \qquad \textbf{(42)}$$

> $K[y] := Component\left(eval\left(subs\left(\left\{ \omega = 7.292 \cdot 10^{-5}, r = 6371 \cdot 10^{3}, \theta = \left(\frac{Pi}{2} - GP[B] \right), \phi = GM[B] \right\}, ChangeBasis(K\_, 1) \right) \right), 2 \right)$

$$K_y := -1.407547378\, t^2 + 0.2031196522\, u_0\, t + 2872.545670 + 0.1736219762\, t\, u_0 \cos(\beta) + 0.6546738242\, t\, u_0 \sin(\beta) \qquad \textbf{(43)}$$

> $K[z] := Component\left(eval\left(subs\left(\left\{ \omega = 7.292 \cdot 10^{-5}, r = 6371 \cdot 10^{3}, \theta = \left(\frac{Pi}{2} - GP[B] \right), \phi = GM[B] \right\}, ChangeBasis(K\_, 1) \right) \right), 3 \right)$

$$K_z := -3.183794499\, t^2 + 0.4594454448\, u_0\, t + 6497.539794 - 0.5375033795\, t\, u_0 \cos(\beta) \qquad \textbf{(44)}$$

>

>

>

Επομένως η ΤΡΟΧΙΑ ΒΛΗΜΑΤΟΣ έχει Καρτεσιανές Συντεταγμένες ως προς το ΑΔΡΑΝΕΙΑΚΟ ΣΥΣΤΗΜΑ Ωxyz:

>

> $LX1 := K[x] + LX$

$$LX1 := -3.448516053\, t^2 + 0.4976467521\, u_0\, t + 4.490812436 \times 10^6 + 0.4253769224\, t\, u_0 \cos(\beta) - 0.2672118704\, t\, u_0 \sin(\beta) \qquad \mathbf{(45)}$$

> $LY1 := K[y] + LY$

$$LY1 := -1.407547378\, t^2 + 0.2031196522\, u_0\, t + 1.832971392 \times 10^6 + 0.1736219762\, t\, u_0 \cos(\beta) + 0.6546738242\, t\, u_0 \sin(\beta) \qquad \mathbf{(46)}$$

> $LZ1 := K[z] + LZ$

$$LZ1 := -3.183794499\, t^2 + 0.4594454448\, u_0\, t + 4.146080143 \times 10^6 - 0.5375033795\, t\, u_0 \cos(\beta) \qquad \mathbf{(47)}$$

>

>

ΕΦΑΡΜΟΓΗ

>

>

ΣΥΝΤΕΤΑΓΜΕΝΕΣ ΘΕΣΗΣ ΕΚΤΟΞΕΥΣΗΣ :

ΒΕΡΟΙΑ :

Γεωγραφικό Πλάτος : $\left(\frac{\pi}{2} - \theta\right) = 40^\circ 31' 23''$ **N** (Βορράς)

Γεωγραφικό Μήκος : $\phi = 22^\circ 12' 12''$ **E** (Ανατολή)

$X_B := 4483.774648$

$Y_B := 1830.098846$

$Z_B := 4139.582603$

$GP := evalf\left(convert\left(40 + \frac{31}{60} + \frac{23}{60 \cdot 60}, units, deg, rad\right)\right) rad$ $\qquad GP := 0.7072607424\, rad$

$GP1 := evalf\left(convert\left(40 + \frac{31}{60} + \frac{23}{60 \cdot 60}, units, deg, rad\right)\right)$ $\qquad GP1 := 0.7072607424$

$GM := evalf\left(convert\left(22 + \frac{12}{60} + \frac{12}{60 \cdot 60}, units, deg, rad\right)\right) rad$ $\qquad GM := 0.3875212718\, rad$

$GM1 := evalf\left(convert\left(22 + \frac{12}{60} + \frac{12}{60 \cdot 60}, units, deg, rad\right)\right)$ $\qquad GM1 := 0.3875212718$

ΣΚΟΠΙΑ : (0.7329911619rad N , 0.3739891520rad E) :

$GP[SK] := 0.7329911619:$

$GM[SK] := 0.3739891520:$

ΣΥΝΤΕΤΑΓΜΕΝΕΣ ΣΤΟΧΟΥ :

ΜΟΣΧΑ :

Γεωγραφικό Πλάτος : *GP[M]*= $\left(\frac{Pi}{2} - \theta\right) = 55^0.75388889$ N (Βορράς)

Γεωγραφικό Μήκος : *GM[M]*= $\phi = 37^0.62030556$ E (Ανατολή)

$X_M := 2839.799027$

$Y_M := 2188.543388$

$Z_M := 5266.446650$

$GP[M] := evalf\left(55 + \frac{45}{60} + \frac{14}{3600}\right) degree$ $\quad GP_M := 55.75388889\ degree$

$GP[M1] := evalf\left(convert\left(55 + \frac{45}{60} + \frac{14}{3600}, units, deg, rad\right)\right) rad$ $\quad GP_{M1} := 0.9730889322\ rad$

$GP[M2] := evalf\left(convert\left(55 + \frac{45}{60} + \frac{14}{3600}, units, deg, rad\right)\right)$ $\quad GP_{M2} := 0.9730889322$

$GM[M] := evalf\left(37 + \frac{37}{60} + \frac{13.1}{3600}\right) degree$ $\quad GM_M := 37.62030556\ degree$

$GP[M1] := convert\left(37 + \frac{37}{60} + \frac{13.1}{3600}, units, deg, rad\right) rad$ $\quad GP_{M1} := 0.6565981979\ rad$

$GM[M2] := convert\left(37 + \frac{37}{60} + \frac{13.1}{3600}, units, deg, rad\right)$ $\quad GM_{M2} := 0.6565981979$

ΣΥΝΤΕΤΑΓΜΕΝΕΣ ΣΤΟΧΟΥ :

ΛΟΣ ΑΝΤΖΕΛΕΣ

Γεωγραφικό Πλάτος : *GP[LA]* : $\left(\frac{Pi}{2} - \theta\right) = 34^0.11333333$ N (Βορράς)

Γεωγραφικό Μήκος : *GM[LA]* : $\phi = -118^0.3296945$ *W* (Δύση)

$X_{LA} := -2503.099182$

$Y_{LA} := -4642.993387$

$Z_{LA} := 3573.058618$

$GP[LA] := evalf\left(34 + \frac{06}{60} + \frac{48}{60 \cdot 60}\right) degree$ $\quad GP_{LA} := 34.11333333\ degree$

$GP[LA1] := evalf\left(convert\left(34 + \frac{06}{60} + \frac{48}{60 \cdot 60}, units, deg, rad\right)\right) rad$ $\quad GP_{LA1} := 0.5953899855\ rad$

$GP[LA2] := evalf\left(convert\left(34 + \frac{06}{60} + \frac{48}{60 \cdot 60}, units, deg, rad\right)\right)$ $\quad GP_{LA2} := 0.5953899855$

$GM[LA] := evalf\left(-\left(118 + \frac{19}{60} + \frac{46.9}{60 \cdot 60}\right)\right) degree$ $\quad GM_{LA} := -118.3296945\ degree$

$GM[LA1] := eval\left(convert\left(-\left(118 + \frac{19}{60} + \frac{46.9}{60 \cdot 60}\right), units, deg, rad\right)\right) rad$ $\quad GM_{LA1} := -2.065242772\ rad$

$GM[LA2] := eval\left(convert\left(-\left(118 + \frac{19}{60} + \frac{46.9}{60 \cdot 60}\right), units, deg, rad\right)\right)$ $\quad GM_{LA2} := -2.065242772$

ΣΥΝΤΕΤΑΓΜΕΝΕΣ ΣΤΟΧΟΥ :

ΚΩΝΣΤΑΝΤΙΝΟΥΠΟΛΗ :

Γεωγραφικό Πλάτος : $\left(\frac{\pi}{2} - \theta\right) = 41^0\, 0'\, 44''$ N · (Βορράς)

Γεωγραφικό Μήκος : $\phi = 28^0\, 58'\, 34''$ E · (Ανατολή)

$X_P := 4205.585805$

$Y_P := 2328.902562$

$Z_P := 4180.777665$

$GP[P] := evalf\left(41 + \frac{0}{60} + \frac{44}{60 \cdot 60}\right) degree$ $\quad GP_P := 41.01222222\ degree$

$GP[P1] := evalf\left(convert\left(41 + \frac{0}{60} + \frac{44}{60 \cdot 60}, units, deg, rad\right)\right) rad$ $\quad GP_{P1} := 0.7157983114\ rad$

$GP[P2] := evalf\left(convert\left(41 + \frac{0}{60} + \frac{44}{60 \cdot 60}, units, deg, rad\right)\right)$ $\quad GP_{P2} := 0.7157983114$

$GM[P] := evalf\left(28 + \frac{58}{60} + \frac{34}{60 \cdot 60}\right) degree$ $\quad GM_P := 28.97611111\ degree$

$GM[P1] := evalf\left(convert\left(28 + \frac{58}{60} + \frac{34}{60 \cdot 60}, units, deg, rad\right)\right) rad$ $\quad GM_{P1} := 0.5057285435\ rad$

$GM[P2] := evalf\left(convert\left(28 + \frac{58}{60} + \frac{34}{60 \cdot 60}, units, deg, rad\right)\right)$ $\quad GM_{P2} := 0.5057285435$

MILANO :

$evalf\left(45 + \frac{27}{60} + \frac{29}{60 \cdot 60}\right) degree$ $\quad 45.45805556\ degree$

$evalf\left(9 + \frac{11}{60} + \frac{17}{60 \cdot 60}\right) degree$ $\quad 9.188055556\ degree$

$evalf\left(45 + \frac{27}{60} + \frac{29}{60 \cdot 60}\right)$ $\quad 45.45805556$

$evalf\left(9 + \frac{11}{60} + \frac{17}{60 \cdot 60}\right)$ $\quad 9.188055556$

$GP[MIL] := convert((3), units, deg, rad)$ $\quad GP_{MIL} := 0.7933927411$

$GM[MIL] := convert((4), units, deg, rad$ $\quad GM_{MIL} := 0.1603618213$

ΑΡΧΙΚΗ ΤΑΧΥΤΗΤΑ ΕΚΤΟΞΕΥΣΗΣ $u_0 :=$ **???**

ΓΩΝΙΑ ΕΚΤΟΞΕΥΣΗΣ ΩΣ ΠΡΟΣ ΤΟ ΟΡΙΖΟΝΤΙΟ ΕΠΙΠΕΔΟ : $\alpha :=$ **???**

ΓΩΝΙΑ ΕΚΤΟΞΕΥΣΗΣ ΩΣ ΠΡΟΣ ΤΟ ΚΑΤΑΚΟΡΥΦΟ ΕΠΙΠΕΔΟ : $\beta := ???$

ΕΠΙΤΑΧΥΝΣΗ ΒΑΡΥΤΗΤΑΣ ΣΤΑΘΕΡΗ : $g = 9.80 \frac{m}{s^2}$

ΓΩΝΙΑΚΗ ΤΑΧΥΤΗΤΑ ΠΕΡΙΣΤΡΟΦΗΣ ΤΗΣ ΓΗΣ : $\Omega = 7.292 \cdot 10^{-5} \frac{rad}{s}$

ΑΚΤΙΝΑ ΓΗΣ : 6371 km

ΑΡΧΙΚΗ ΤΑΧΥΤΗΤΑ ΕΚΤΟΞΕΥΣΗΣ u_0: = ???

ΓΩΝΙΑ ΕΚΤΟΞΕΥΣΗΣ ΩΣ ΠΡΟΣ ΤΟ ΟΡΙΖΟΝΤΙΟ ΕΠΙΠΕΔΟ : $\alpha := ???$

ΓΩΝΙΑ ΕΚΤΟΞΕΥΣΗΣ ΩΣ ΠΡΟΣ ΤΟ ΚΑΤΑΚΟΡΥΦΟ ΕΠΙΠΕΔΟ : $\beta := ???$

ΕΠΙΤΑΧΥΝΣΗ ΒΑΡΥΤΗΤΑΣ ΣΤΑΘΕΡΗ : $g = 9.80 \frac{m}{s^2}$

ΓΩΝΙΑΚΗ ΤΑΧΥΤΗΤΑ ΠΕΡΙΣΤΡΟΦΗΣ ΤΗΣ ΓΗΣ : $\Omega = 7.292 \cdot 10^{-5} \frac{rad}{s}$

ΑΚΤΙΝΑ ΓΗΣ : 6371 km

>

>

BEROIA

> $GP[B] := 0.7072607424$:

> $GM[B] := 0.3875212718$:

>

>

>

MOSXA

> $GP[M] := 0.9730889322$:

> $GM[M] := 0.6565981979$:

>

ΣΚΟΠΙΑ : (0.7329911619rad N , 0.3739891520rad E):

> $GP[SK] := 0.7329911619$:

> $GM[SK] := 0.3739891520$:

>

MILANO

> $GP[MIL] := 0.7933927411$:

> $GM[MIL] := 0.1603618213$:

>

POLH

>

> $GP[P] := 0.7157983114$:

> $GM[P] := 0.5057285435 :$

>

>

LOS ANTZELES

> $GP[LA] := 0.5953899855 :$

> $GM[LA] := -2.065242772 :$

>

>

ΣΥΝΤΕΤΑΓΜΕΝΕΣ ΣΤΟΧΟΥ ΩΣ ΠΡΟΣ ΤΟ ΑΔΡΑΝΕΙΑΚΟ ΣΥΣΤΗΜΑ Ωxyz ΚΑΤΑ ΤΗΝ ΣΤΙΓΜΗ ΤΗΣ ΕΚΤΟΞΕΥΣΗΣ :

>

> $X1[LA] := evalf\left(subs\left(\left[\theta = \left(\frac{\text{Pi}}{2} - GP[LA]\right), \phi = (GM[LA]), r = 6371 \cdot 10^3\right], r \cdot \sin(\theta) \cdot \cos(\phi)\right)\right)$

$$X1_{LA} := -2.503099182 \times 10^6 \quad (48)$$

> $Y1[LA] := evalf\left(subs\left(\left[\theta = \left(\frac{\text{Pi}}{2} - GP[LA]\right), \phi = (GM[LA]), r = 6371 \cdot 10^3\right], r \cdot \sin(\theta) \cdot \sin(\phi)\right)\right)$

$$Y1_{LA} := -4.642993387 \times 10^6 \quad (49)$$

> $Z1[LA] := evalf\left(subs\left(\left[\theta = \left(\frac{\text{Pi}}{2} - GP[LA]\right), \phi = (GM[LA]), r = 6371 \cdot 10^3\right], r \cdot \cos(\theta)\right)\right)$

$$Z1_{LA} := 3.573058618 \times 10^6 \quad (50)$$

>

ΣΥΝΤΕΤΑΓΜΕΝΕΣ ΠΕΡΙΣΤΡΕΦΟΜΕΝΟΥ ΣΤΟΧΟΥ ΩΣ ΠΡΟΣ ΤΟ ΑΔΡΑΝΕΙΑΚΟ ΣΥΣΤΗΜΑ Ωxyz

>

$$> \Delta\phi := subs\left(\omega = 7.292\cdot 10^{-5}, \omega\cdot t\right)$$

$$\Delta\phi := 0.00007292000000\, t \quad \textbf{(51)}$$

$$> X[LA] := evalf\left(subs\left(\left[\theta = \left(\frac{Pi}{2} - GP[LA]\right), \phi = \left(GM[LA] + \Delta\phi\right), r = 6371\cdot 10^{3}\right], r\cdot\sin(\theta)\cdot\cos(\phi)\right)\right)$$

$$X_{LA} := 5.274741047 \times 10^{6} \cos(-2.065242772 + 0.00007292000000\, t) \quad \textbf{(52)}$$

$$> Y[LA] := evalf\left(subs\left(\left[\theta = \left(\frac{Pi}{2} - GP[LA]\right), \phi = \left(GM[LA] + \Delta\phi\right), r = 6371\cdot 10^{3}\right], r\cdot\sin(\theta)\cdot\sin(\phi)\right)\right)$$

$$Y_{LA} := 5.274741047 \times 10^{6} \sin(-2.065242772 + 0.00007292000000\, t) \quad \textbf{(53)}$$

$$> Z[LA] := evalf\left(subs\left(\left[\theta = \left(\frac{Pi}{2} - GP[LA]\right), \phi = \left(GM[LAL] + \Delta\phi\right), r = 6371\cdot 10^{3}\right], r\cdot\cos(\theta)\right)\right)$$

$$Z_{LA} := 3.573058618 \times 10^{6} \quad \textbf{(54)}$$

\>

ΕΠΟΜΕΝΩΣ ΕΧΟΥΜΕ ΠΡΟΣ ΕΠΙΛΥΣΗ ΤΟ ΣΥΣΤΗΜΑ :

$$> X[LA] - LX1 = 0$$

$$5.274741047 \times 10^{6} \cos(-2.065242772 + 0.00007292000000\, t) + 3.448516053\, t^{2} - 0.4976467521\, u_0\, t - 4.490812436 \times 10^{6} - 0.4253769224\, t\, u_0 \cos(\beta) + 0.2672118704\, t\, u_0 \sin(\beta) = 0 \quad \textbf{(55)}$$

$$> Y[LA] - LY1 = 0$$

$$5.274741047 \times 10^{6} \sin(-2.065242772 + 0.00007292000000\, t) + 1.407547378\, t^{2} - 0.2031196522\, u_0\, t - 1.832971392 \times 10^{6} - 0.1736219762\, t\, u_0 \cos(\beta) - 0.6546738242\, t\, u_0 \sin(\beta) = 0 \quad \textbf{(56)}$$

$$> Z[LA] - LZ1 = 0$$

$$-573021.525 + 3.183794499\, t^{2} - 0.4594454448\, u_0\, t + 0.5375033795\, t\, u_0 \cos(\beta) = 0 \quad \textbf{(57)}$$

\>

$$\alpha = \frac{Pi}{4}$$

ΤΡΕΙΣ ΕΞΙΣΩΣΕΙΣ ΜΕ ΑΓΝΩΣΤΟΥΣ :

{ β , u[0] , t } !!!

> $fsolve(\{(55), (56), (57)\}, \{\beta = Pi..2 \cdot Pi, u[0] = 0..20000, t = 0..3000\})$

$$\{\beta = 3.785405907, t = 1640.294528, u_0 = 5479.320830\} \quad (58)$$

> $\Delta\phi 1 := subs(\{\omega = 7.292 \cdot 10^{-5}, t = 1640.294528\}, \omega \cdot t)$

$$\Delta\phi 1 := 0.1196102770 \quad (59)$$

>

$$\Delta\phi 1 := 0.1196102770$$

LOS ANGELES

ΧΩΡΙΣ ΦΥΓΟΚΕΝΤΡΟ ΚΑΙ CORIOLIS

H=10000m

$$\alpha = \frac{Pi}{4}, \{\beta = 3.785405907, t = 1640.294528, u_0 = 5479.320830\}$$

$$MHKOSTROXIAS := 11154.34240\, km$$

LOS ANGELES

H=10000 m

$$\alpha = \frac{Pi}{4}, \{\beta = 4.521239410, t = 2573.630349, u_0 = 14194.73594\}$$

LOS ANGELES - ΑΚΙΝΗΤΟ

H=10000 m $MHKOSTROXIAS := 11369.95219\, km$

$$\alpha = \frac{Pi}{4}, \{\beta = 3.700410538, t = 1658.532747, u_0 = 5392.273368\}$$

LOS ANGELES

H=1000

$\alpha = \frac{\text{Pi}}{4}$, $\{\beta = 4.521103945, t = 2572.607327, u_0 = 14191.01955\}$

LOS ANGELES

H=0 m

$\alpha = \frac{\text{Pi}}{4}$, $\{\beta = 4.521084394, t = 2572.899365, u_0 = 14192.90854\}$

>

>

ΣΥΝΤΕΤΑΓΜΕΝΕΣ ΤΡΟΧΙΑΣ ΒΛΗΜΑΤΟΣ ΩΣ ΠΡΟΣ ΤΟ ΑΔΡΑΜΕΙΑΚΟ ΣΥΣΤΗΜΑ Ωxyz ΣΥΝΑΡΤΗΣΕΙ ΤΟΥ ΧΡΟΝΟΥ .

$\Delta\phi 1 := 0.1196102770$

LOS ANGELES

ΧΩΡΙΣ ΦΥΓΟΚΕΝΤΡΟ ΚΑΙ CORIOLIS

H=10000m

$\alpha = \frac{\text{Pi}}{4}$, $\{\beta = 3.785405907, t = 1640.294528, u_0 = 5479.320830\}$

$MHKOSTROXIAS := 11154.34240\, km$

> $LLX1 := evalf\left(subs\left(\left\{\alpha = \frac{\text{Pi}}{4}, \beta = 3.785405907, u_0 = 5479.320830\right\}, LX1\right)\right)$

$$LLX1 := -3.448516053\, t^2 + 1741.430841\, t + 4.490812436 \times 10^6 \qquad (60)$$

> $LLY1 := evalf\left(subs\left(\left\{\alpha = \frac{Pi}{4}, \beta = 3.785405907, u_0 = 5479.320830\right\}, LY1\right)\right)$

$$LLY1 := -1.407547378\, t^2 - 1801.124880\, t + 1.832971392 \times 10^6 \quad (61)$$

> $LLZ1 := evalf\left(subs\left(\left\{\alpha = \frac{Pi}{4}, \beta = 3.785405907, u_0 = 5479.320830\right\}, LZ1\right)\right)$

$$LLZ1 := -3.183794499\, t^2 + 4873.020063\, t + 4.146080143 \times 10^6 \quad (62)$$

> $TROXIA := spacecurve([LLX1, LLY1, LLZ1], t = 0 ..1640.294528, color = blue, thickness = 3, linestyle = 1) :$

>

> $MHKOSTROXIAS := convert\left(int\left(\operatorname{sqrt}\left((diff(LLX1, t))^2 + (diff(LLY1, t))^2 + (diff(LLZ1, t))^2\right), t = 0 ..1640.294528\right), units, m, km\right)km$

$$MHKOSTROXIAS := 11154.34240\, km \quad (63)$$

>

ΑΠΕΙΚΟΝΙΣΗ .

> $SF := subs(R = 6371 \cdot 10^3, R \cdot \sin(\theta) \cdot \cos(\phi) \cdot \_i + R \cdot \sin(\theta) \cdot \sin(\phi) \cdot \_j + R \cdot \cos(\theta) \cdot \_k)$

$$SF := 6371000 \sin(\theta) \cos(\phi)\, \hat{i} + 6371000 \sin(\theta) \sin(\phi)\, \hat{j} + 6371000 \cos(\theta)\, \hat{k} \quad (64)$$

> $SFAIRA := plot3d\left([Component(SF, 1), Component(SF, 2), Component(SF, 3)], \theta = 0 .. \frac{Pi}{2}, \phi = 0 ..GM[B], transparency = 0.50, style = surface\right) :$

> $Component(subs(\phi = GM[B], SF), 1), Component(subs(\phi = GM[B], SF), 2), Component(subs(\phi = GM[B], SF), 3)$

$$6371000 \sin(\theta) \cos(0.3875212718), 6371000 \sin(\theta) \sin(0.3875212718), 6371000 \cos(\theta) \quad (65)$$

> $Student[VectorCalculus][TNBFrame](\langle(65)\rangle, \theta)[1]$

$$\begin{bmatrix} 0.9258486015 \cos(\theta) \\ 0.3778946513 \cos(\theta) \\ -1.000000000 \sin(\theta) \end{bmatrix} \quad (66)$$

> $T\theta\_ := eval\left(subs\left(\theta = \left(\frac{Pi}{2} - GP[B]\right), ((\mathbf{66})[1]\cdot\_i + (\mathbf{66})[2]\cdot\_j + (\mathbf{66})[3]\cdot\_k)\right)\right)$

$$\overrightarrow{T\theta} := 0.6015738131\ \hat{i} + 0.2455385535\ \hat{j} - 0.7601445693\ \hat{k} \qquad \mathbf{(67)}$$

> $Norm((\mathbf{67}))$

$$1.000000000 \qquad \mathbf{(68)}$$

> $Component\left(subs\left(\theta = \left(\frac{Pi}{2} - GP[B]\right), SF\right), 1\right), Component\left(subs\left(\theta = \left(\frac{Pi}{2} - GP[B]\right), SF\right), 2\right), Component\left(subs\left(\theta = \left(\frac{Pi}{2} - GP[B]\right), SF\right), 3\right)$

$$6371000 \sin(0.8635355846) \cos(\phi), 6371000 \sin(0.8635355846) \sin(\phi), 6371000 \cos(0.8635355846) \qquad \mathbf{(69)}$$

> $\mathbf{Student[VectorCalculus][TNBFrame]}(\langle(\mathbf{69})\rangle, \phi)[1]$

$$\begin{bmatrix} -0.9999999999 \sin(\phi) \\ 0.9999999999 \cos(\phi) \\ 0. \end{bmatrix} \qquad \mathbf{(70)}$$

> $T\phi\_ := eval(subs(\phi = GM[B], ((\mathbf{70})[1]\cdot\_i + (\mathbf{70})[2]\cdot\_j + (\mathbf{70})[3]\cdot\_k)))$

$$\overrightarrow{T\phi} := -0.3778946513\ \hat{i} + 0.9258486012\ \hat{j} \qquad \mathbf{(71)}$$

> $Norm((\mathbf{71}))$

$$0.9999999999 \qquad \mathbf{(72)}$$

> $NBEROIA := (\mathbf{67}) \times (\mathbf{71})$

$$NBEROIA := 0.7037787862\ \hat{i} + 0.2872545670\ \hat{j} + 0.6497539795\ \hat{k} \qquad \mathbf{(73)}$$

> $Norm((\mathbf{73}))$

$$1.000000000 \qquad \mathbf{(74)}$$

>

> $T\theta := arrow(\langle LX, LY, LZ\rangle, \langle 1.5\cdot10^6\cdot Component(T\theta\_, 1), 1.5\cdot10^6\cdot Component(T\theta\_, 2), 1.5\cdot10^6\cdot Component(T\theta\_, 3)\rangle, width = 70000, head\_width = 200000, head\_length = 400000, color = red)$:

> $T\phi := arrow(\langle LX, LY, LZ\rangle, \langle 1.5\cdot10^6\cdot Component(T\phi\_, 1), 1.5\cdot10^6\cdot Component(T\phi\_, 2), 1.5\cdot10^6\cdot Component(T\phi\_, 3)\rangle, width = 70000, head\_width = 200000, head\_length = 400000, \mathbf{color} = blue)$:

> $NB := arrow(\langle LX, LY, LZ\rangle, \langle 1.5\cdot10^6\cdot Component(NBEROIA, 1), 1.5\cdot10^6\cdot Component(NBEROIA, 2), 1.5\cdot10^6\cdot Component(NBEROIA, 3)\rangle, width = 70000, head\_width = 200000, head\_length = 400000, \mathbf{color} = green)$:

>

> $BEROIA := pointplot3d([LX, LY, LZ], color = green, symbol = solidsphere, symbolsize = 7)$:

> $ONOMA1 := textplot3d([LX, LY - 250000, LZ - 200000, \text{"B"}], font = [arial, bold, 14], color = black)$:

> $GRNMES := spacecurve\Big([Component(subs(\phi = 0, SF), 1), Component(subs(\phi = 0, SF), 2),$

$Component(subs(\phi = 0, SF), 3)], \theta = 0..\frac{Pi}{2}, color = black, thickness = 3\Big):$

$> MES90 := spacecurve\Big(\Big[Component\Big(subs\Big(\phi = \frac{Pi}{2}, SF\Big), 1\Big), Component\Big(subs\Big(\phi = \frac{Pi}{2}, SF\Big), 2\Big), Component\Big(subs\Big(\phi = \frac{Pi}{2}, SF\Big), 3\Big)\Big], \theta = 0..\frac{Pi}{2}, color = gold, thickness = 3\Big):$

$> ISHM := spacecurve\Big(\Big[Component\Big(subs\Big(\theta = \frac{Pi}{2}, SF\Big), 1\Big), Component\Big(subs\Big(\theta = \frac{Pi}{2}, SF\Big), 2\Big), Component\Big(subs\Big(\theta = \frac{Pi}{2}, SF\Big), 3\Big)\Big], \phi = 0..2\cdot Pi, color = black, thickness = 3\Big):$

$> BMES := spacecurve\Big([Component(subs(\phi = GM[B], SF), 1), Component(subs(\phi = GM[B], SF), 2), Component(subs(\phi = GM[B], SF), 3)], \theta = 0..\frac{Pi}{2}, color = red, thickness = 2\Big):$

$> BPAR := spacecurve\Big(\Big[Component\Big(subs\Big(\theta = \Big(\frac{Pi}{2} - GP[B]\Big), SF\Big), 1\Big), Component\Big(subs\Big(\theta = \Big(\frac{Pi}{2} - GP[B]\Big), SF\Big), 2\Big), Component\Big(subs\Big(\theta = \Big(\frac{Pi}{2} - GP[B]\Big), SF\Big), 3\Big)\Big], \phi = 0..2\cdot Pi, color = blue, thickness = 2, linestyle = 2\Big):$

$> AKTINA := spacecurve([0 + \lambda\cdot(LX - 0), 0 + \lambda\cdot(LY - 0), 0 + \lambda\cdot(LZ - 0)], \lambda = 0..1, color = blue, thickness = 1, linestyle = solid):$

>

$> LOSANGELES := pointplot3d(subs(t = 1640.294528, [X[LA], Y[LA], Z[LA]]), color = red, symbol = solidsphere, symbolsize = 7):$

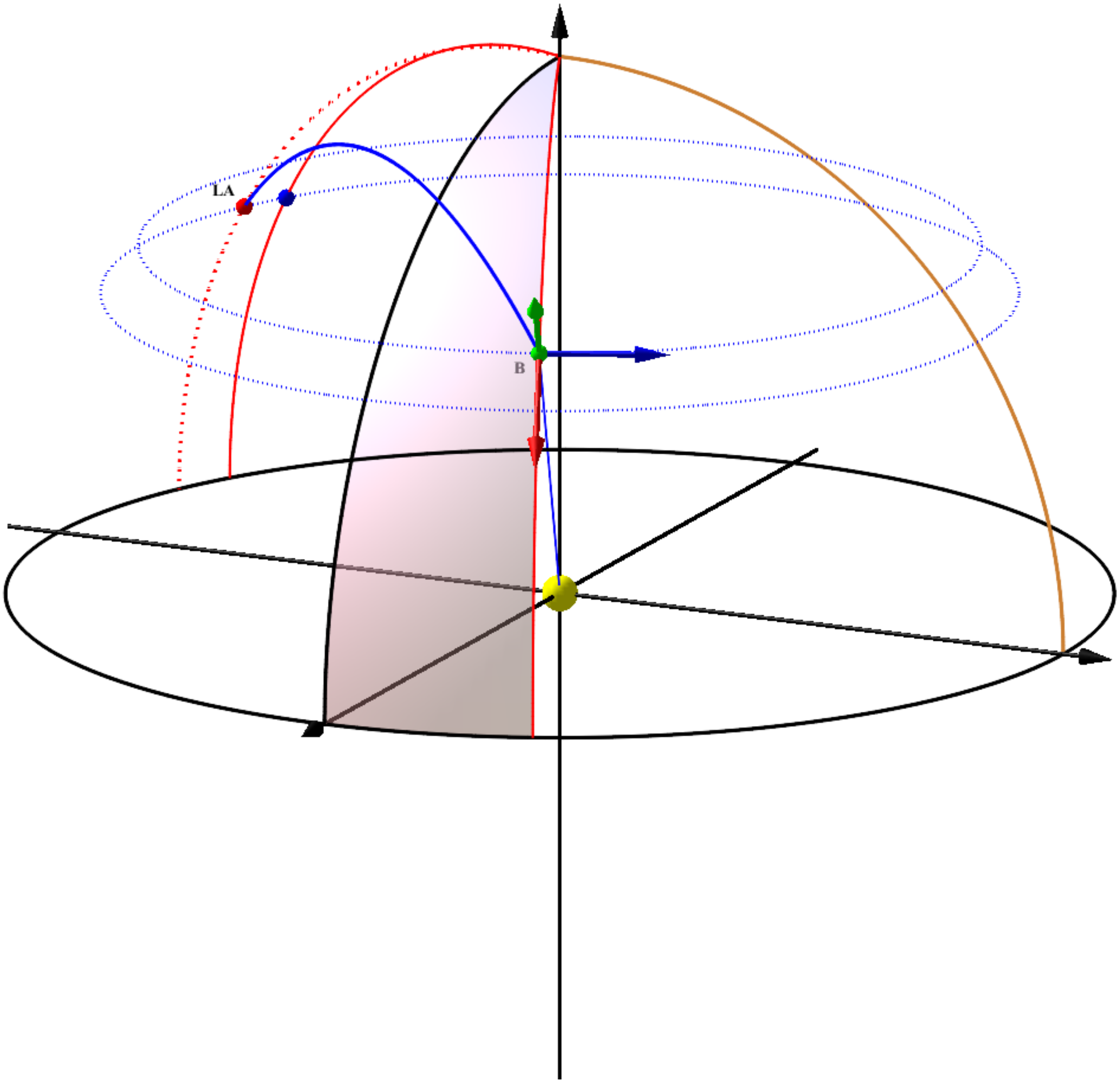
>

$> LAMES := spacecurve\Big([Component(subs(\phi = GM[LA] + \Delta\phi 1, SF), 1), Component(subs(\phi = GM[LA] + \Delta\phi 1, SF), 2), Component(subs(\phi = GM[LA + \Delta\phi 1], SF), 3)], \theta = 0..\frac{Pi}{2}, color = red, thickness = 3, linestyle = 2\Big):$

$> LAPAR := spacecurve\Big(\Big[Component\Big(subs\Big(\theta = \Big(\frac{Pi}{2} - GP[LA]\Big), SF\Big), 1\Big), Component\Big(subs\Big(\theta = \Big(\frac{Pi}{2} - GP[LA]\Big), SF\Big), 2\Big),$

```
Component(subs(θ = (Pi/2 - GP[LA]), SF), 3)], φ = 0 ..2·Pi,
    color = blue, thickness = 2, linestyle = 2) :
> ONOMA := textplot3d([X1[LA], Y1[LA] - 800000, Z1[LA] + 10000, "LA"], font = [arial,
    bold, 14], color = black) :
>
> LOSANGELES1 := pointplot3d([X1[LA], Y1[LA], Z1[LA]], color
    = blue, symbol = solidsphere, symbolsize = 7) :
>
> LAMES1 := spacecurve([Component(subs(φ = GM[LA], SF), 1),
    Component(subs(φ = GM[LA], SF), 2), Component(subs(φ
    = GM[LA], SF), 3)], θ = 0 .. Pi/2, color = red, thickness = 2) :
>
> A := pointplot3d([0, 0, 0], color = yellow, symbol = solidsphere, symbolsize = 15, axes = none) :
> Ax := arrow(⟨-1.1·6371·10^3, 0, 0⟩, ⟨2.2·6371·10^3, 0, 0⟩, width = 50000, head_width
    = 200000, head_length = 400000, color = black) :
> Ay := arrow(⟨0, -1.1·6371·10^3, 0⟩, ⟨0, 2.2·6371·10^3, 0⟩, width = 50000, head_width
    = 200000, head_length = 400000, color = black) :
> Az := arrow(⟨0, 0, -1.1·6371·10^3⟩, ⟨0, 0, 2.2·6371·10^3⟩, width = 50000, head_width
    = 200000, head_length = 400000, color = black) :
> SYNOLO := display(SFAIRA, BEROIA, AKTINA, Tθ, Tφ, NB, GRNMES, ISHM, MES90,
    BMES, BPAR, LOSANGELES1, LAMES1) :
> SYNOLO1 := display(A, Ax, Ay, Az, BEROIA, ONOMA, ONOMA1,
    LOSANGELES, LAMES, LAPAR, TROXIA) :
> display(A, Ax, Ay, Az, SFAIRA, BEROIA, AKTINA, Tθ, Tφ, NB, GRNMES, ISHM, MES90,
    BMES, BPAR, LOSANGELES, LAMES, LAPAR, LOSANGELES1, LAMES1,
    ONOMA, ONOMA1, TROXIA, scaling = constrained, axes = none, orientation
    = [25, 75, 0], labels = [x, y, z], labelfont = [arial, bold, 14], title
    = "ΜΕ ΣΤΟΧΟ ΤΟ LOS-ANGELES\nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ", titlefont = [arial, bold,
    14])
```

ΜΕ ΣΤΟΧΟ ΤΟ LOS-ANGELES
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ



>

> $BLHMA := animate(pointplot3d, [[LLX1, LLY1, LLZ1], symbol = solidcircle, symbolsize = 8, color = red,], t = 0..1640.294528, frames = 100) :$

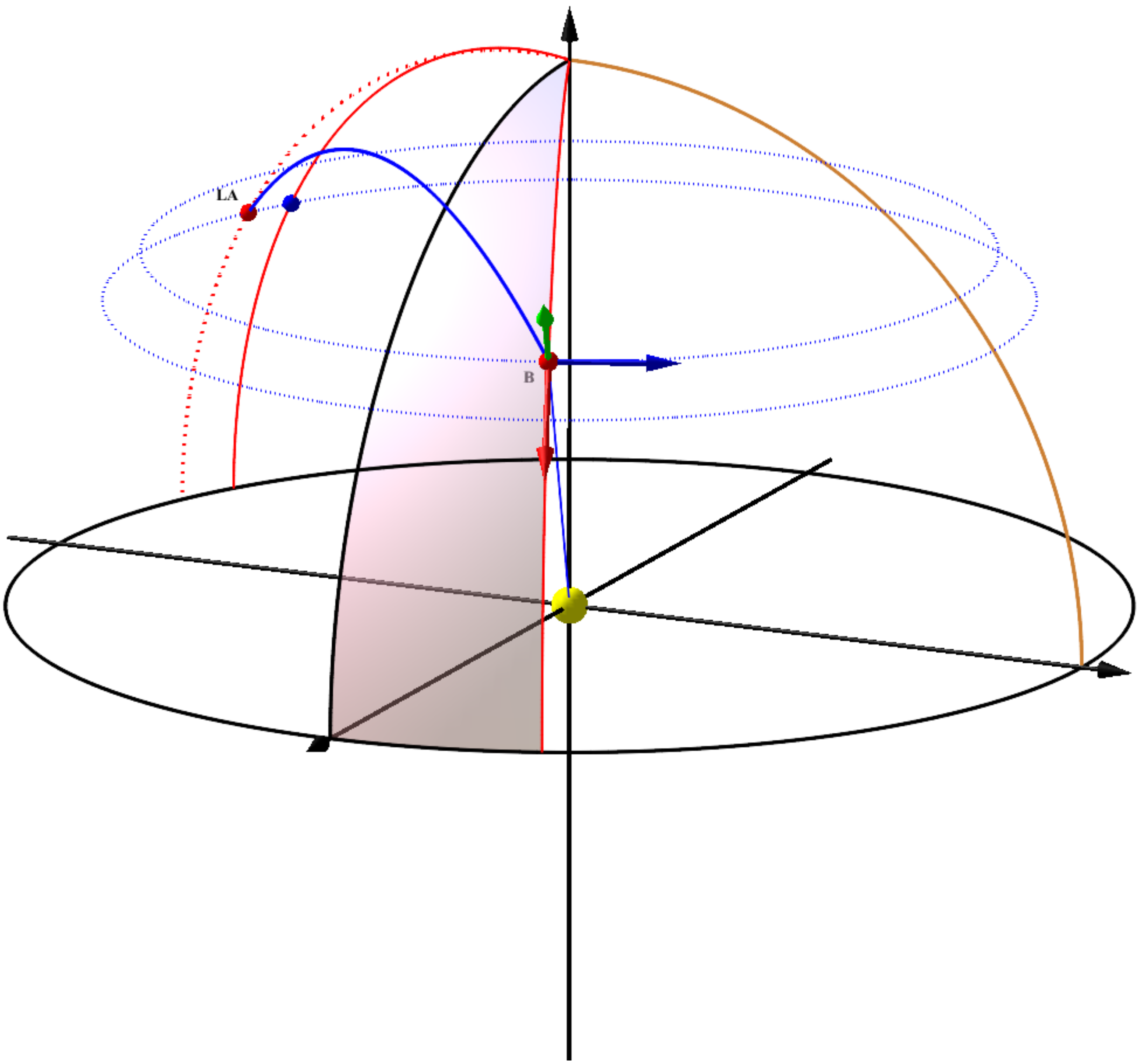
>

> $ANIM := animate(plottools[rotate], [SYNOLO, \Delta\phi 1 \cdot k, [[0, 0, 0], [0, 0, 1]]], k = 0..1, frames = 100, scaling = constrained,$

***orientation* = [25, 75, 0], *axes* = *none*) :**

> *display*(*SYNOLO1*, *ANIM*, *BLHMA*, ***scaling*** = *constrained*, *axes* = *none*, *orientation* = [25, 75, 0], *labels* = [*x*, *y*, *z*], *labelfont* = [*arial*, *bold*, 14], *title* = "ANIMATE\nΜΕ ΣΤΟΧΟ ΤΟ LOS-ANGELES\nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ", *titlefont* = [*arial*, *bold*, 14])

ANIMATE
ΜΕ ΣΤΟΧΟ ΤΟ LOS-ANGELES
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ



>