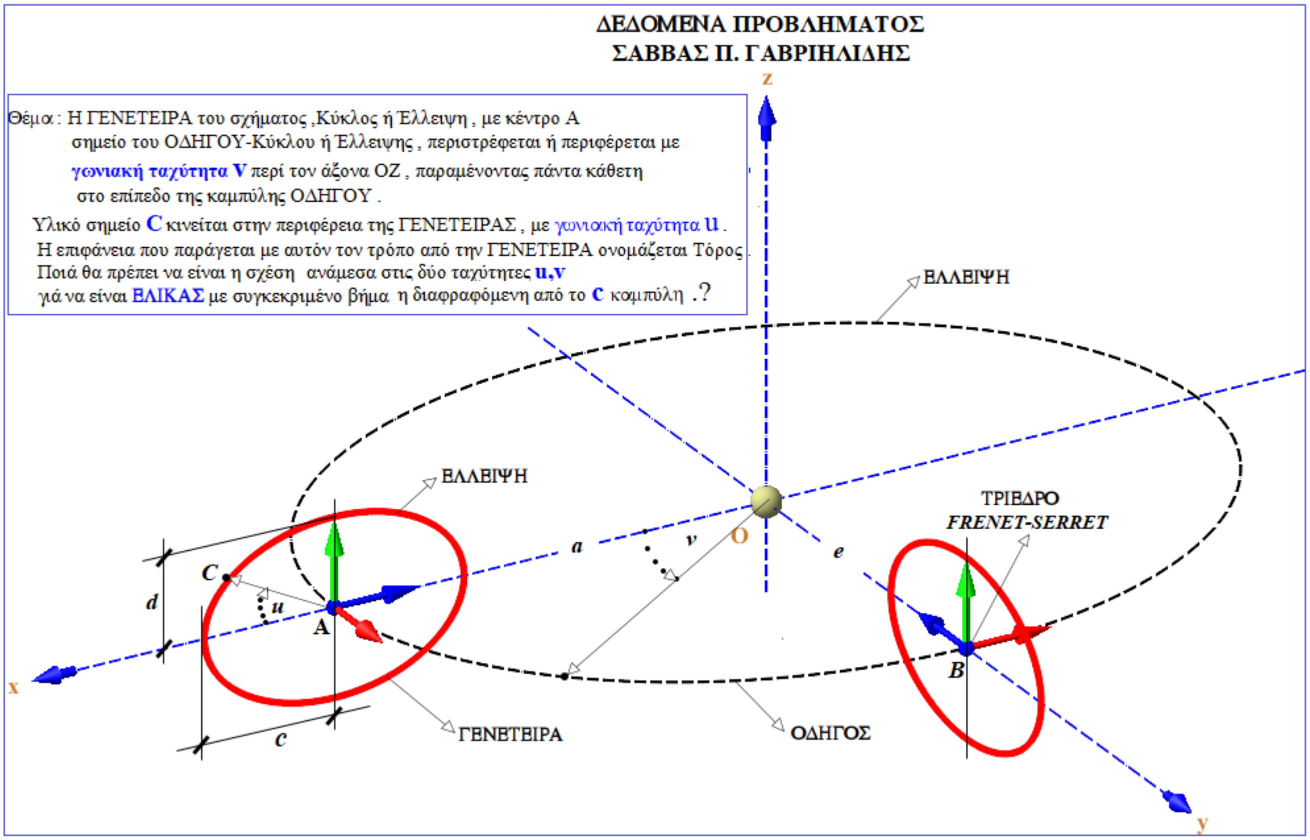
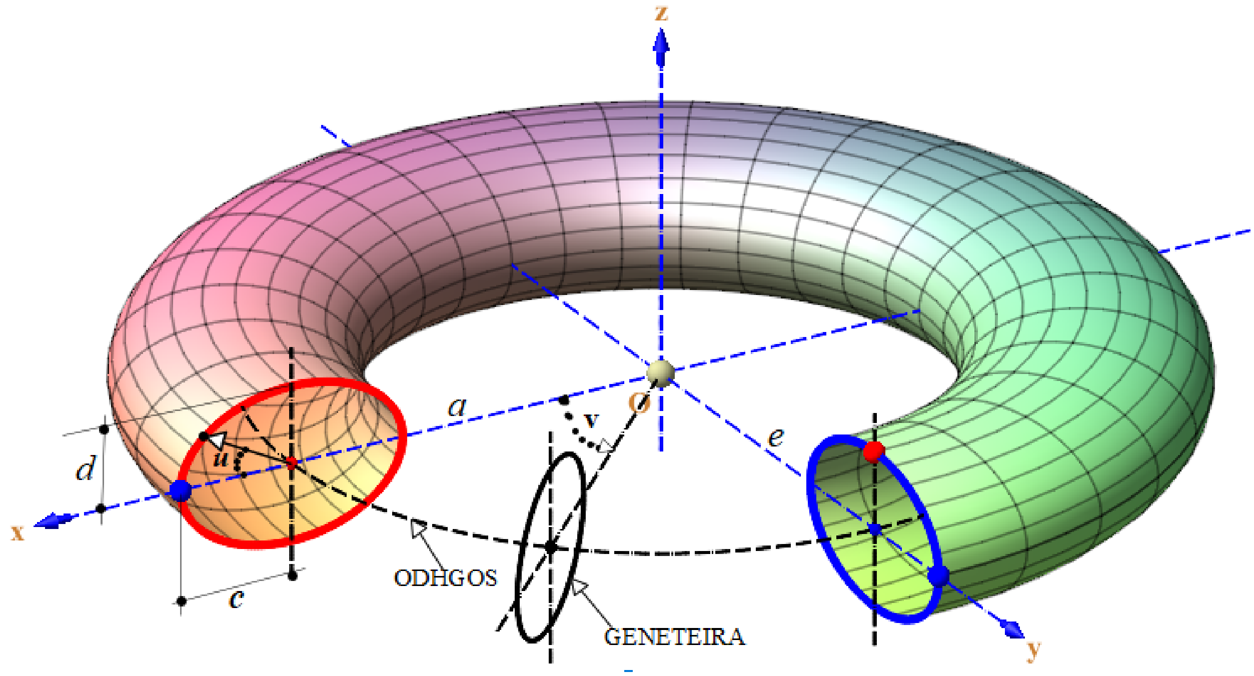
```
>
> with(plots) :
> with(Physics[Vectors]) :
> Setup(mathematicalnotation = true) :
>
```



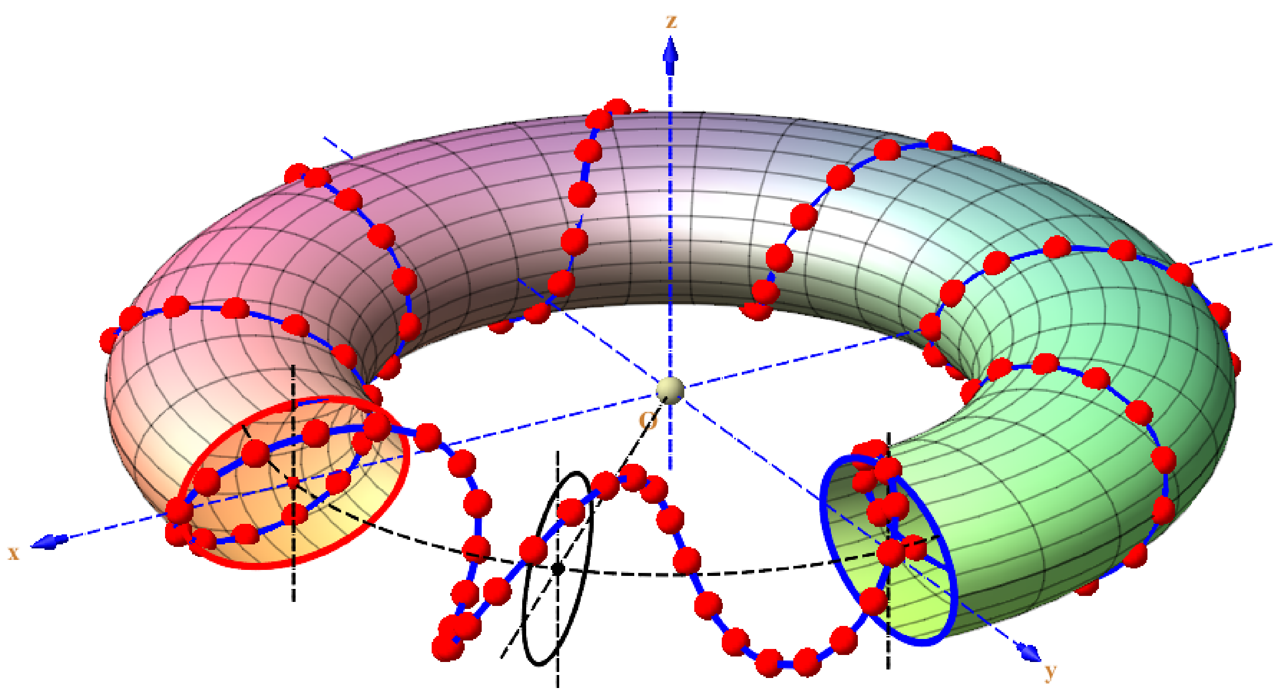

```
>
```

ΠΕΡΙΣΤΡΟΦΗ ΕΠΙΠΕΔΗΣ ΚΑΜΠΥΛΗΣ ΓΥΡΩ ΑΠΟ ΚΥΡΙΟ ΑΞΟΝΑ ΤΟΥ ΕΠΙΠΕΔΟΥ ΤΗΣ .

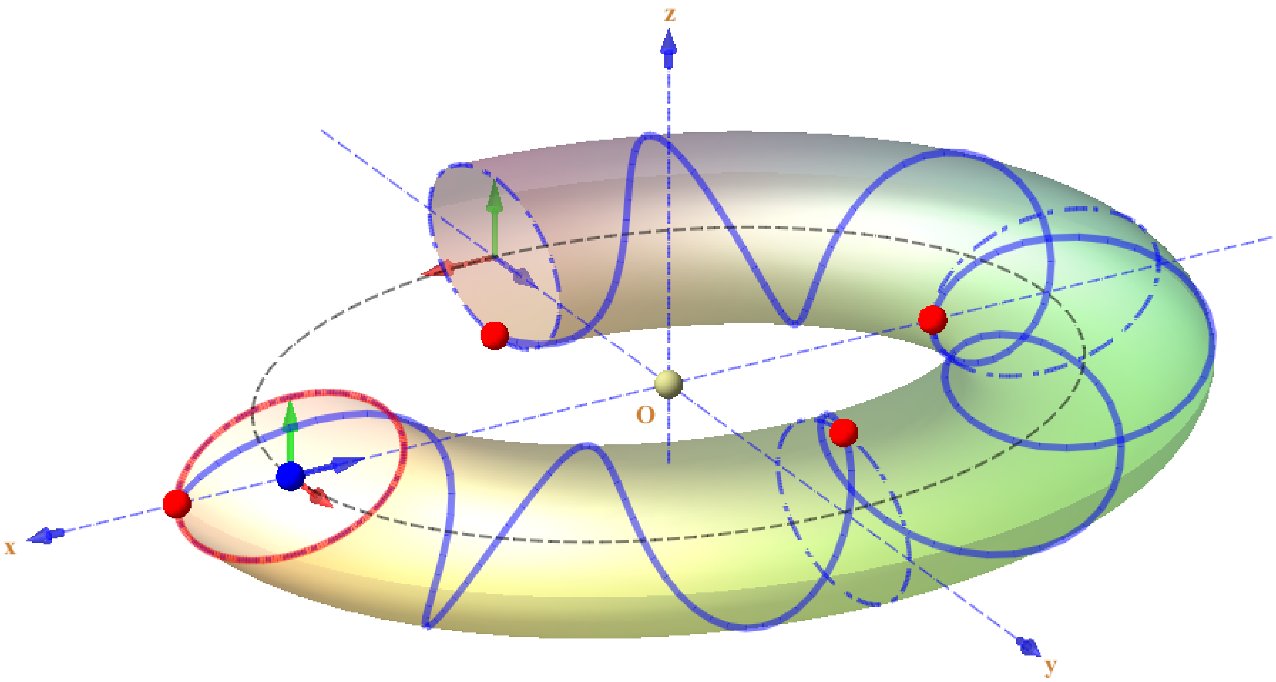
ΤΟΡΟΣ
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ

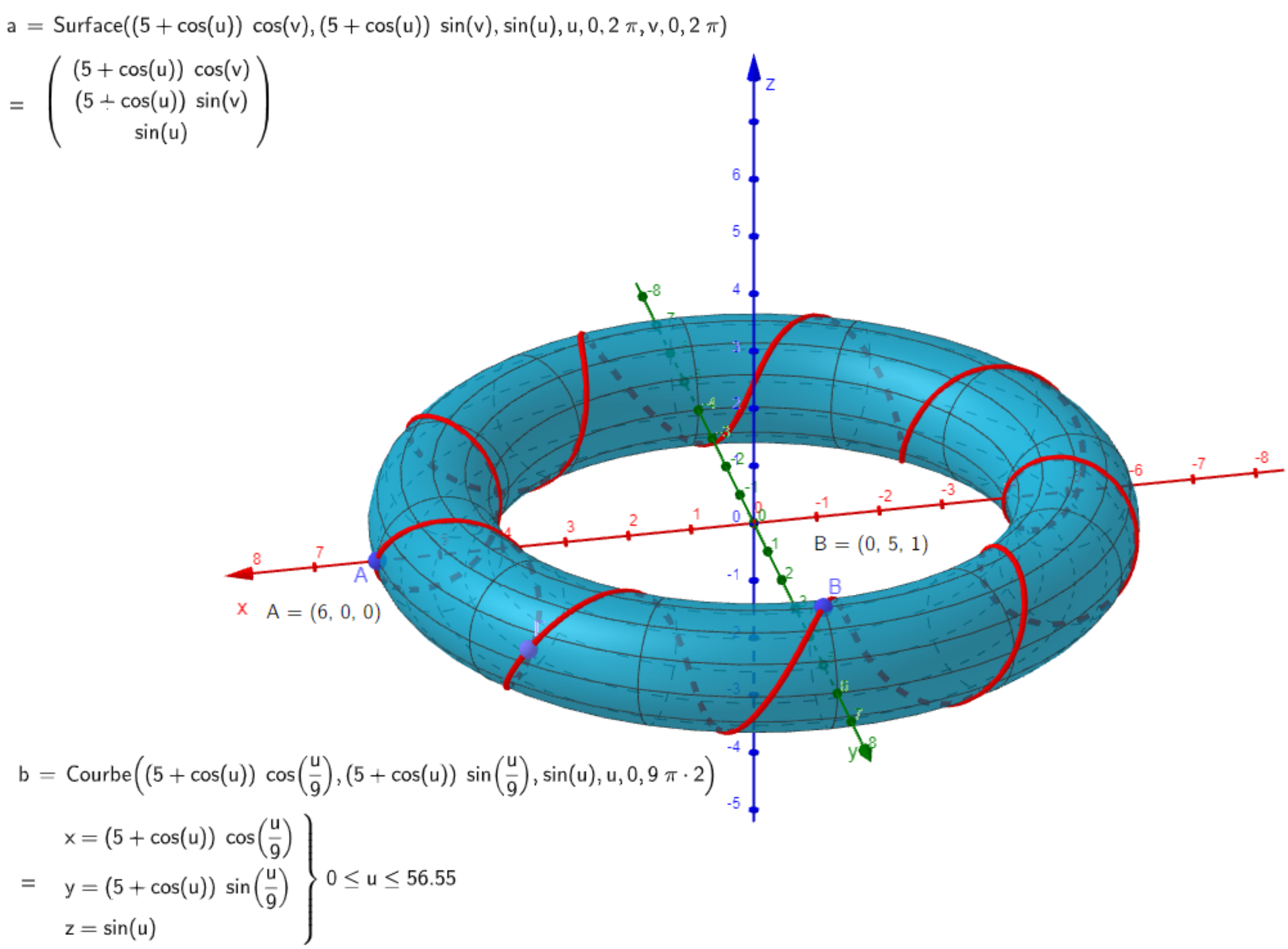


ΤΟΡΟΣ με ΠΕΡΙΕΛΙΞΗ
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ



ANIMATION
ΤΟΡΟΣ ΜΕ ΠΕΡΙΕΛΙΞΗ
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ





>

> $A := 7:$

> $B := 7:$

> $H := 2:$

> $OO := pointplot3d([0, 0, 0], symbol = solidcircle, symbolsize = 10):$

>

> $axX := spacecurve([x, 0, 0], x = -(A+1)..(A+1), linestyle = 3, thickness = 1, color = blue):$

> $axY := spacecurve([0, y, 0], y = -(B+1)..(B+1), linestyle = 3, thickness = 1, color = blue):$

> $axZ := spacecurve\left([0, 0, z], z = -\frac{H}{2}..H + 2, linestyle = 3, thickness = 2, color = blue\right):$

>

> $ARaxX := arrow([(A+1), 0, 0], [0.5, 0, 0], width = 0.1, head\_length = 0.3, shape = cylindrical\_arrow, color = blue):$

> $ARaxY := arrow([0, (B+1), 0], [0, 0.5, 0], width = 0.1, head\_length = 0.3, shape = cylindrical\_arrow, color = blue):$

> $ARaxZ := arrow([0, 0, (H+2)], [0, 0, 0.5], width = 0.1, head\_length = 0.3, shape = cylindrical\_arrow, color = blue):$

>

> $tX := textplot3d([(A + 1.7), 0.0, 0, "x"], color = gold, font = [arial, bold, 14])$:

> $tY := textplot3d([0, (B + 1.7), 0, "y"], color = gold, font = [arial, bold, 14])$:

> $tZ := textplot3d([0, 0, (H + 2.7), "z"], color = gold, font = [arial, bold, 14])$:

> $tO := textplot3d([0 + 0.3, 0, -0.3, "O"], color = gold, font = [arial, bold, 14])$:

> $AXONES := display(OO, axX, axY, axZ, ARaxX, ARaxY, ARaxZ, tX, tY, tZ, tO, scaling = constrained, axes = none, orientation = [60, 65, 0], lightmodel = light4)$:

>

Συντεταγμένες του κέντρου της καμπύλης-Γενέτειρας . (ΚΥΚΛΟΣ ή ΕΛΛΕΙΨΗ)

> $a := 5$:

> $b := 0$:

Ημιάξονες της καμπύλης-Γενέτειρας . (ΚΥΚΛΟΣ ή ΕΛΛΕΙΨΗ)

> $c := 1.5$:

> $d := 1$:

>

Ημιάξονες της καμπύλης-ΟΔΗΓΟΥ . (ΚΥΚΛΟΣ ή ΕΛΛΕΙΨΗ)

> $a := 5$:

> $e := 4$:

>

Καμπύλη στο xoz : $[x, 0, z]$

Διανυσματικές παραμετρικές εξισώσεις καμπύλης (Γενέτειρα)

> $r\_ := (a + c\cdot\cos(u))\cdot\_i + 0\cdot\_j + (b + d\cdot\sin(u))\cdot\_k$

$$\vec{r} := (5. + 1.5\cos(u))\,\hat{i} + \sin(u)\,\hat{k} \qquad (1)$$

> $simplify(Norm(diff(r\_, u)))$

$$\sqrt{-1.25\cos(u)^2 + 2.25} \qquad (2)$$

> $C := [Component(r\_, 1), Component(r\_, 2), Component(r\_, 3)]$

$$C := [5. + 1.5\cos(u), 0, \sin(u)] \qquad (3)$$

> $P := spacecurve(C, u = 0 ..2\cdot\mathrm{Pi}, color = green, thickness = 5, linestyle = 1)$:

>

1. ΠΕΡΙΣΤΡΟΦΗ ΓΥΡΩ ΑΠΟ ΤΟΝ ΑΞΟΝΑ ΟΖ .

ΟΔΗΓΟΣ ΚΥΚΛΟΣ ΑΚΤΙΝΑΣ *a*.

Παραμετρικές εξισώσεις διαγραφόμενης Επιφάνειας .

ΤΟΡΟΣ

>

> $SURF := [C[1]\cdot\cos(v), C[1]\cdot\sin(v), C[3]]$

$$SURF := [(5. + 1.5\cos(u))\cos(v), (5. + 1.5\cos(u))\sin(v), \sin(u)] \quad \textbf{(4)}$$

> $SURFACE := plot3d\left(SURF, u = 0..2\cdot\text{Pi}, v = \frac{\text{Pi}}{2}..2\cdot\ \text{Pi}, style = surfacewireframe\right):$

> $simplify(subs(v = 2\cdot\text{Pi}, SURF))$

$$[5. + 1.5\cos(u), 0, \sin(u)] \quad \textbf{(5)}$$

> $GENETEIRA1 := spacecurve(\textbf{(5)}, u = 0..2\cdot\text{Pi}, color = red, thickness = 5, linestyle = 1):$

> $simplify\left(subs\left(v = \frac{\text{Pi}}{2}, SURF\right)\right)$

$$[0, 5. + 1.5\cos(u), \sin(u)] \quad \textbf{(6)}$$

> $GENETEIRA2 := spacecurve(\textbf{(6)}, u = 0..2\cdot\text{Pi}, color = blue, thickness = 5, linestyle = 1):$

> $simplify\left(subs\left(v = \frac{\text{Pi}}{4}, SURF\right)\right)$

$$[3.535533905 + 1.060660172\cos(u), 3.535533905 + 1.060660172\cos(u), \sin(u)] \quad \textbf{(7)}$$

> $GENETEIRA3 := spacecurve(\textbf{(7)}, u = 0..2\cdot\text{Pi}, color = black, thickness = 3, linestyle = 1):$

>

> $K1 := pointplot3d([a, 0, 0], color = red, symbol = solidcircle, symbolsize = 5):$

> $K2 := pointplot3d([0, a, 0], color = blue, symbol = solidcircle, symbolsize = 5):$

> $K3 := pointplot3d\left(\left[a\cdot\cos\left(\frac{\text{Pi}}{4}\right), a\cdot\sin\left(\frac{\text{Pi}}{4}\right), 0\right], color = black, symbol = solidcircle, symbolsize = 5\right):$

> $linK1 := spacecurve([a, 0, t\cdot d], t = -1.5..1.5, color = black, thickness = 2, linestyle = 3):$

> $linK2 := spacecurve([0, a, t\cdot d], t = -1.5..1.5, color = black, thickness = 2, linestyle = 3):$

> $linK3 := spacecurve\left(\left[a\cdot\cos\left(\frac{\text{Pi}}{4}\right), a\cdot\sin\left(\frac{\text{Pi}}{4}\right), t\cdot d\right], t = -1.5..1.5, color = black, thickness = 2, linestyle = 3\right):$

> $linOK3 := spacecurve\left(\left[t\cdot a\cdot\cos\left(\frac{\text{Pi}}{4}\right), t\cdot a\cdot\sin\left(\frac{\text{Pi}}{4}\right), 0\right], t = 0..1.5, color = black, thickness = 2, linestyle = 3\right):$

> $ODHGOS := spacecurve([a\cdot\cos(v), a\cdot\sin(v), 0], v = 0..2\cdot\text{Pi}, color = black, thickness = 2, linestyle = 3):$

> $P1 := pointplot3d([a + c, 0, 0], color = blue, symbol = solidcircle, symbolsize = 8):$

> $P2 := pointplot3d\left(\left[(a + c)\cdot\cos\left(\frac{\text{Pi}}{2}\right), (a + c)\cdot\sin\left(\frac{\text{Pi}}{2}\right), 0\right], color = blue, symbol = solidcircle, symbolsize = 8\right):$

> $P3 := pointplot3d\left(\left[a\cdot\cos\left(\frac{\text{Pi}}{2}\right), a\cdot\sin\left(\frac{\text{Pi}}{2}\right), d\right], color = red, symbol = solidcircle, symbolsize\right.$

$= 8\Big)$:

> simplify(expand((5 + 1.5·cos(u))^2))

$$2.25\ (\cos(u) + 3.333333333)^2 \quad \textbf{(8)}$$

> SURF := [C[1]·cos(v), C[1]·sin(v), C[3]]

$$SURF := [(5. + 1.5\cos(u))\cos(v), (5. + 1.5\cos(u))\sin(v), \sin(u)] \quad \textbf{(9)}$$

> simplify(subs([u = 0, v = 0], SURF))

$$[6.5, 0., 0] \quad \textbf{(10)}$$

> $simplify\left(subs\left(\left[u = 0, v = \frac{\text{Pi}}{4}\right], SURF\right)\right)$

$$[4.596194076, 4.596194076, 0] \quad \textbf{(11)}$$

> $simplify\left(subs\left(\left[u = \frac{\text{Pi}}{2}, v = \frac{\text{Pi}}{4}\right], SURF\right)\right)$

$$[3.535533905, 3.535533905, 1] \quad \textbf{(12)}$$

>

Αριθμός Περιελίξεων : *n*

Βήμα έλικας : $S := \dfrac{(ΜΗΚΟΣΟΔΗΓΟΥ)}{n}$

> n := 9 :

> $ELIKA := subs\left(v = \frac{u}{n}, SURF\right)$

$$ELIKA := \left[(5. + 1.5\cos(u))\cos\left(\frac{u}{9}\right), (5. + 1.5\cos(u))\sin\left(\frac{u}{9}\right), \sin(u)\right] \quad \textbf{(13)}$$

ΜΗΚΟΣ ΕΛΙΚΑΣ : sEL .

> sEL := int(sqrt((diff(ELIKA[1], u))^2 + (diff(ELIKA[2], u))^2 + (diff(ELIKA[3], u))^2), u = 0 ..n·2·Pi, numeric)

$$sEL := 78.35739334 \quad \textbf{(14)}$$

Στην Επιφάνεια εκ Περιστροφής :

$$\vec{r}(u,v) = [f(u)\cdot\cos(v), f(u)\cdot\sin(v), g(u)]$$

Καμπύλη στο x-z συντεταγμένο επίπεδο : $\vec{r}(u) = f(u)\cdot\vec{i} + 0\cdot\vec{j} + g(u)\cdot\vec{k}$.
Περιστροφή γύρω από τον άξονα OZ .

$$E = \frac{\partial}{\partial u}\vec{r}.\frac{\partial}{\partial u}\vec{r} = \left[\frac{d}{du}f(u)\cdot\cos(v), \frac{d}{du}f(u)\cdot\sin(v), \frac{d}{du}g(u)\right].\left[\frac{d}{du}f(u)\cdot\cos(v), \frac{d}{du}f(u)\cdot\sin(v), \frac{d}{du}g(u)\right] = \left(\frac{d}{du}f(u)\cdot\cos(v)\right)^2 + \left(\frac{d}{du}f(u)\cdot\sin(v)\right)^2 + \left(\frac{d}{du}g(u)\right)^2 =$$

$$= \left(\frac{d}{du}f(u)\right)^2 + \left(\frac{d}{du}g(u)\right)^2$$

$$F = \frac{\partial}{\partial u}\vec{r}.\frac{\partial}{\partial v}\vec{r} = \left[\frac{d}{du}f(u)\cdot\cos(v), \frac{d}{du}f(u)\cdot\sin(v), \frac{d}{du}g(u)\right].[f(u)\cdot diff(\cos(v),v), f(u)\cdot diff(\sin(v),v), 0] = \left[\frac{d}{du}f(u)\cdot\cos(v), \frac{d}{du}f(u)\cdot\sin(v), \frac{d}{du}g(u)\right].[-f(u)\cdot\sin(v), f(u)\cdot\cos(v), 0] = 0$$

$$G = \frac{\partial}{\partial v}\vec{r}.\frac{\partial}{\partial v}\vec{r} = [f(u)\cdot diff(\cos(v),v), f(u)\cdot diff(\sin(v),v), 0].[f(u)\cdot diff(\cos(v),v), f(u)\cdot diff(\sin(v),v), 0] = (f(u))^2$$

Μήκος Καμπύλης $\vec{r} = \vec{r}(u(t), v(t))$ πάνω στην Επιφάνεια εκ Περιστροφής

$$s = \int_{t_1}^{t_2} \sqrt{\left(\left(\frac{d}{du}f(u)\right)^2 + \left(\frac{d}{du}g(u)\right)^2\right)\cdot\left(\frac{d}{dt}(u(t))\right)^2 + (f(u))^2\cdot\left(\frac{d}{dt}(v(t))\right)^2}\, dt$$

Μήκος Μεσημβρινού πάνω στην Επιφάνεια εκ Περιστροφής $(v = const)$

$$s = \int_{u_1}^{u_2} \sqrt{\left(\frac{d}{du}f(u)\cdot\cos(v)\right)^2 + \left(\frac{d}{du}f(u)\cdot\sin(v)\right)^2 + \left(\frac{d}{du}g(u)\right)^2}\, du = \int_{u_1}^{u_2}\sqrt{E}\, du$$

>

ΜΗΚΟΣ ΕΛΙΚΑΣ ΜΕ E,F,G .

> $SURF := [C[1]\cdot\cos(v), C[1]\cdot\sin(v), C[3]]$

$$SURF := [(5. + 1.5\cos(u))\cos(v), (5. + 1.5\cos(u))\sin(v), \sin(u)] \quad \textbf{(15)}$$

> $R\_ := SURF[1]\cdot\_i + SURF[2]\cdot\_j + SURF[3]\cdot\_k$

$$\vec{R} := (5. + 1.5\cos(u))\cos(v)\,\hat{i} + (5. + 1.5\cos(u))\sin(v)\,\hat{j} + \sin(u)\,\hat{k} \quad \textbf{(16)}$$

> $E := subs\left(\left[u = t, v = \frac{t}{9}\right], simplify(diff(R\_, u) \cdot diff(R\_, u))\right)$

$$E := -1.25\cos(t)^2 + 2.25 \quad \textbf{(17)}$$

> $F := subs\left(\left[u = t, v = \frac{t}{9}\right], diff(R\_, u) \cdot diff(R\_, v)\right)$

$$F := 0 \quad \textbf{(18)}$$

> $G := subs\left(\left[u = t, v = \frac{t}{9}\right], simplify(diff(R\_, v) \cdot diff(R\_, v))\right)$

$$G := 2.25\,(\cos(t) + 3.333333333)^2 \quad \textbf{(19)}$$

> $diff(t, t)$

$$1 \quad \textbf{(20)}$$

> $diff\left(\frac{t}{9}, t\right)$

$$\frac{1}{9} \quad \textbf{(21)}$$

> $diff(t, t)\cdot diff\left(\frac{t}{9}, t\right)$

$$\frac{1}{9} \qquad (22)$$

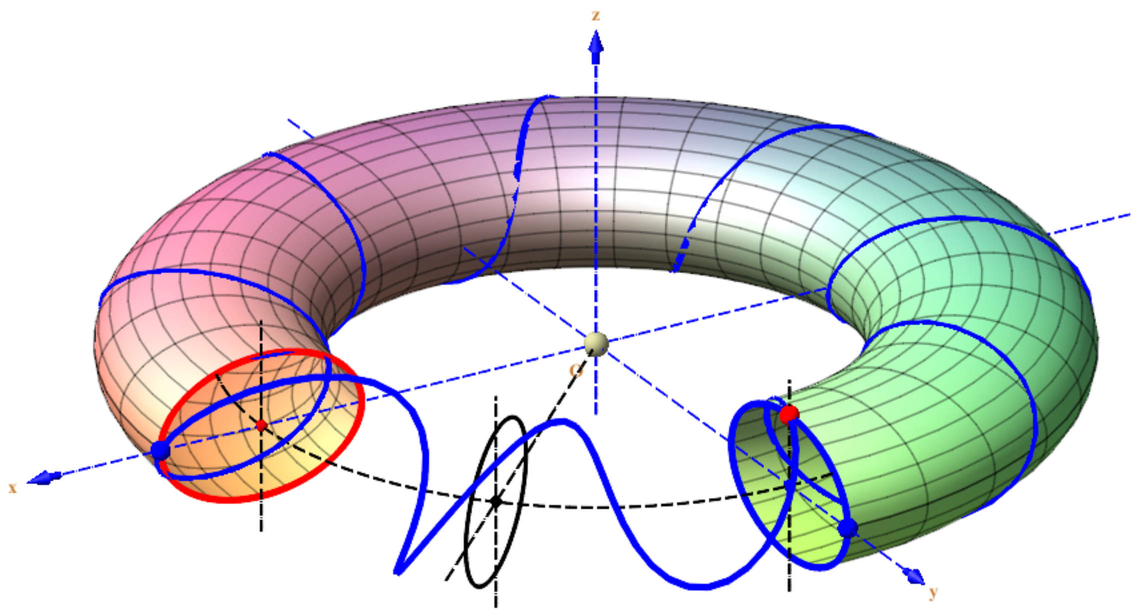
> $MHLOSELIKAS := int\left(\text{sqrt}\left(E\cdot(diff(t,t))^2 + 2\cdot F\cdot diff(t,t)\cdot diff\left(\frac{t}{9}, t\right) + G\cdot\left(diff\left(\frac{t}{9}, t\right)\right)^2\right), t = 0..9\cdot 2\cdot \text{Pi}, numeric\right)$

$$MHLOSELIKAS := 78.35739334 \qquad (23)$$

> $ELIKAPLOT := spacecurve(ELIKA, u = 0..n\cdot 2\cdot \text{Pi}, color = blue, thickness = 5, linestyle = 1):$

> *display*(*AXONES*, *SURFACE*, *GENETEIRA1*, *GENETEIRA2*, *GENETEIRA3*, *K1*, *K2*, *K3*, *linK1*, *linK2*, *linK3*, *linOK3*, *ODHGOS*, *P1*, *P2*, *P3*, *ELIKAPLOT*, *title* = "ΤΟΡΟΣ ΜΕ ΠΕΡΙΕΛΙΞΗ\nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ", *titlefont* = [*arial*, *bold*, 14])

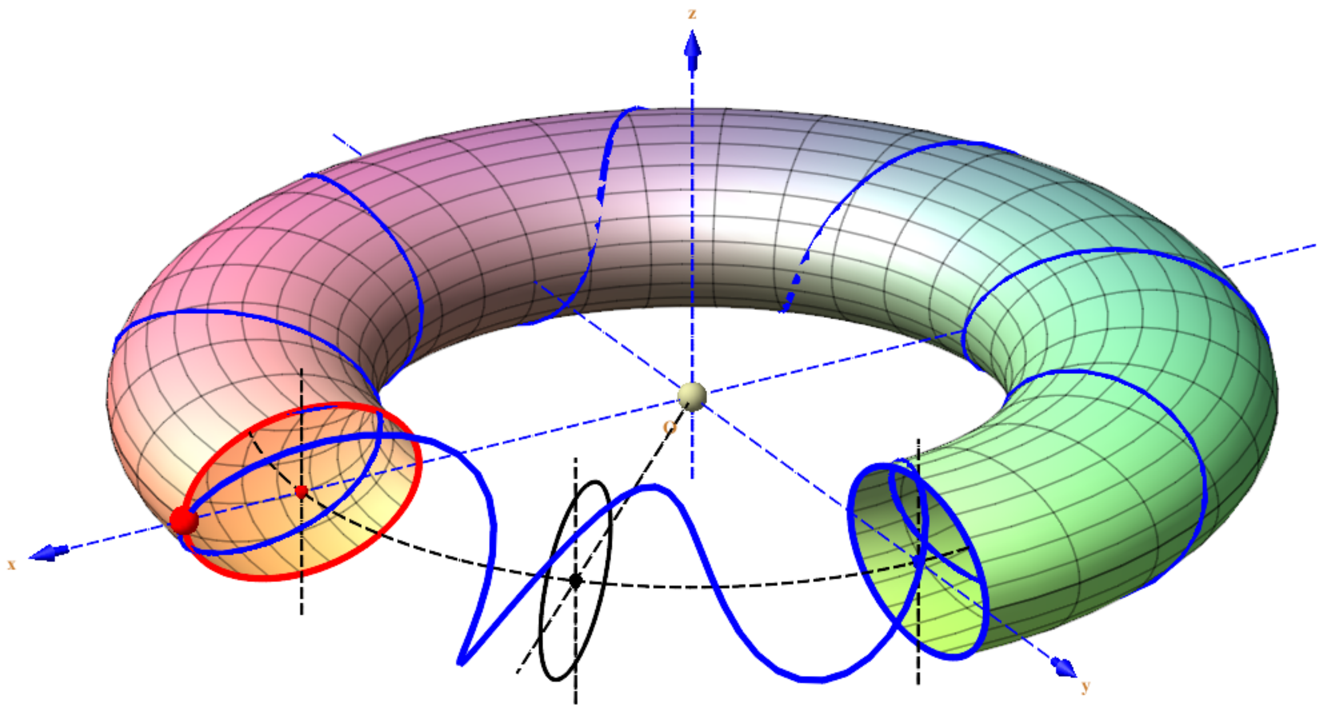
ΤΟΡΟΣ ΜΕ ΠΕΡΙΕΛΙΞΗ
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ



> *ANIM* := *animate*(*pointplot3d*, [*ELIKA*, *color* = *red*, *symbol* = *solidcircle*, *symbolsize* = 10], *u* = 0 ..*n*·2·Pi, *frames* = 21, *trace* = 20) :

> *display*(*AXONES*, *SURFACE*, *GENETEIRA1*, *GENETEIRA2*, *GENETEIRA3*, *linK1*, *linK2*, *linK3*, *linOK3*, *ODHGOS*, *K1*, *K2*, *K3*, *ELIKAPLOT*, *ANIM*, *title* = "ANIMATION\nΤΟΡΟΣ ΜΕ ΠΕΡΙΕΛΙΞΗ\nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ", *titlefont* = [*arial*, *bold*, 14])

ANIMATION
ΤΟΡΟΣ ΜΕ ΠΕΡΙΕΛΙΞΗ
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ



2. ΠΕΡΙΦΟΡΑ ΓΥΡΩ ΑΠΟ ΤΟΝ ΑΞΟΝΑ ΟΖ .

ΟΔΗΓΟΣ ΕΛΛΕΙΨΗ : [$a \cdot \cos(v)$, $e \cdot \sin(v)$, 0]

ΧΡΗΣΗ ΤΡΙΕΔΡΟΥ FRENET-SERRET

ΓΙΑ ΤΗΝ ΕΞΩΘΗΣΗ (EXTRUDE) ΤΗΣ ΚΑΜΠΥΛΗΣ-ΓΕΝΕΤΕΙΡΑΣ .

> $ODHGOS1 := a \cdot \cos(v), e \cdot \sin(v), 0$

$$ODHGOS1 := 5\cos(v), 4\sin(v), 0 \qquad (24)$$

> *Student[VectorCalculus][TNBFrame]*(⟨*ODHGOS1*⟩)

$$\left[\begin{bmatrix} -\frac{5\sin(v)}{\sqrt{-9\cos(v)^2+25}} \\ \frac{4\cos(v)}{\sqrt{-9\cos(v)^2+25}} \\ 0 \end{bmatrix}, \begin{bmatrix} -\frac{4\cos(v)}{\sqrt{\frac{1}{(3\cos(v)-5)^2(3\cos(v)+5)^2}}\left(-9\cos(v)^2+25\right)^{3/2}} \\ -\frac{5\sin(v)}{\sqrt{\frac{1}{(3\cos(v)-5)^2(3\cos(v)+5)^2}}\left(-9\cos(v)^2+25\right)^{3/2}} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{\left(9\cos(v)^2-25\right)\sqrt{\frac{1}{(3\cos(v)-5)^2(3\cos(v)+5)^2}}} \end{bmatrix}\right] \quad \textbf{(25)}$$

> $T\_ := \mathbf{(25)}[1][1]\cdot\_i + \mathbf{(25)}[1][2]\cdot\_j + \mathbf{(25)}[1][3]\cdot\_k$

$$\vec{T} := -\frac{5\sin(v)\,\hat{i}}{\sqrt{-9\cos(v)^2+25}} + \frac{4\cos(v)\,\hat{j}}{\sqrt{-9\cos(v)^2+25}} \quad \textbf{(26)}$$

> *simplify*(*Norm*(*T_*))

$$1 \quad \textbf{(27)}$$

> $TA\_ := ODHGOS1[1]\cdot\_i + ODHGOS1[2]\cdot\_j + ODHGOS1[3]\cdot\_k + T\_$

$$\overrightarrow{TA} := \hat{i}\left(5\cos(v) - \frac{5\sin(v)}{\sqrt{-9\cos(v)^2+25}}\right) + \hat{j}\left(4\sin(v) + \frac{4\cos(v)}{\sqrt{-9\cos(v)^2+25}}\right) \quad \textbf{(28)}$$

> *simplify*(*subs*($v=0$, *T_*))

$$\hat{j} \quad \textbf{(29)}$$

> *simplify*(*subs*($v=0$, *TA_*))

$$5\,\hat{i} + \hat{j} \quad \textbf{(30)}$$

> $N\_ := \mathbf{(25)}[2][1]\cdot\_i + \mathbf{(25)}[2][2]\cdot\_j + \mathbf{(25)}[2][3]\cdot\_k$

$$\vec{N} := -\frac{4\cos(v)\,\hat{i}}{\sqrt{\frac{1}{(3\cos(v)-5)^2(3\cos(v)+5)^2}}\left(-9\cos(v)^2+25\right)^{3/2}} \quad \textbf{(31)}$$

$$-\frac{5\sin(v)\,\hat{j}}{\sqrt{\frac{1}{(3\cos(v)-5)^2(3\cos(v)+5)^2}}\left(-9\cos(v)^2+25\right)^{3/2}}$$

> simplify(Norm(N_))

$$1 \quad (32)$$

> NA_ := ODHGOS1[1]·_i + ODHGOS1[2]·_j + ODHGOS1[3]·_k + N_

$$\overrightarrow{NA} := \hat{i}\left(5\cos(v)-\frac{4\cos(v)}{\sqrt{\frac{1}{(3\cos(v)-5)^2(3\cos(v)+5)^2}}\left(-9\cos(v)^2+25\right)^{3/2}}\right)+\hat{j}\left(4\sin(v)-\frac{5\sin(v)}{\sqrt{\frac{1}{(3\cos(v)-5)^2(3\cos(v)+5)^2}}\left(-9\cos(v)^2+25\right)^{3/2}}\right) \quad (33)$$

> simplify(subs(v=0, N_))

$$-\hat{i} \quad (34)$$

> simplify(subs(v=0, NA_))

$$4\,\hat{i} \quad (35)$$

> B_ := **(25)**[3][1]·_i + **(25)**[3][2]·_j + **(25)**[3][3]·_k

$$\vec{B} := -\frac{\hat{k}}{\left(9\cos(v)^2-25\right)\sqrt{\frac{1}{(3\cos(v)-5)^2(3\cos(v)+5)^2}}} \quad (36)$$

> simplify(Norm(B_))

$$1 \quad (37)$$

> BA_ := ODHGOS1[1]·_i + ODHGOS1[2]·_j + ODHGOS1[3]·_k + B_

$$\overrightarrow{BA} := 5\cos(v)\,\hat{i}+4\sin(v)\,\hat{j}-\frac{\hat{k}}{\left(9\cos(v)^2-25\right)\sqrt{\frac{1}{(3\cos(v)-5)^2(3\cos(v)+5)^2}}} \quad (38)$$

> simplify(Norm(B_))

$$1 \quad (39)$$

> simplify(subs(v=0, B_))

$$\hat{k} \quad (40)$$

> simplify(subs(v=0, BA_))

$$5\,\hat{i}+\hat{k} \quad (41)$$

> OD_ := ODHGOS1[1]·_i + ODHGOS1[2]·_j + ODHGOS1[3]·_k

$$\overrightarrow{OD} := 5\cos(v)\,\hat{i}+4\sin(v)\,\hat{j} \quad (42)$$

ΓΕΝΕΤΕΙΡΑ (c,d) ΣΤΟ ΣΥΣΤΗΜΑ TNB , ΣΤΟ ΕΠΙΠΕΔΟ NB (ΚΑΘΕΤΗ ΣΤΗΝ ΚΑΜΠΥΛΗ ΟΔΗΓΟ) .

> $rSURF\_ := OD\_ - N\_\cdot c\cdot \cos(u) + B\_\cdot d\cdot \sin(u)$

$$\overrightarrow{rSURF} := \left(5.\cos(v) + \frac{6.0\cos(u)\cos(v)}{\sqrt{\frac{1}{(3\cos(v)-5)^2(3\cos(v)+5)^2}}\left(-9\cos(v)^2+25\right)^{3/2}}\right)\hat{i} + \hat{j}\left(4.\sin(v) + \frac{7.5\cos(u)\sin(v)}{\sqrt{\frac{1}{(3\cos(v)-5)^2(3\cos(v)+5)^2}}\left(-9\cos(v)^2+25\right)^{3/2}}\right) - \frac{1.\hat{k}\sin(u)}{\left(9.\cos(v)^2-25.\right)\sqrt{\frac{1}{(3\cos(v)-5)^2(3\cos(v)+5)^2}}} \quad (43)$$

> $X := Component(rSURF\_, 1)$

$$X := 5.\cos(v) + \frac{6.0\cos(u)\cos(v)}{\sqrt{\frac{1}{(3\cos(v)-5)^2(3\cos(v)+5)^2}}\left(-9\cos(v)^2+25\right)^{3/2}} \quad (44)$$

> $Y := Component(rSURF\_, 2)$

$$Y := 4.\sin(v) + \frac{7.5\cos(u)\sin(v)}{\sqrt{\frac{1}{(3\cos(v)-5)^2(3\cos(v)+5)^2}}\left(-9\cos(v)^2+25\right)^{3/2}} \quad (45)$$

> $Z := Component(rSURF\_, 3)$

$$Z := -\frac{1.\sin(u)}{\left(9.\cos(v)^2-25.\right)\sqrt{\frac{1}{(3\cos(v)-5)^2(3\cos(v)+5)^2}}} \quad (46)$$

> $evalf(subs(v=0, [X, Y, Z]))$

$$[5. + 1.500000000\cos(u), 0., 1.000000000\sin(u)] \quad (47)$$

> $ELIKA1 := subs\left(v = \frac{u}{n}, [X, Y, Z]\right)$

$$ELIKA1 := \left[5.\cos\left(\frac{u}{9}\right) + \frac{6.0\cos(u)\cos\left(\frac{u}{9}\right)}{\sqrt{\frac{1}{\left(3\cos\left(\frac{u}{9}\right)-5\right)^2\left(3\cos\left(\frac{u}{9}\right)+5\right)^2}}\left(-9\cos\left(\frac{u}{9}\right)^2+25\right)^{3/2}}, 4.\sin\left(\frac{u}{9}\right) \right. \quad (48)$$

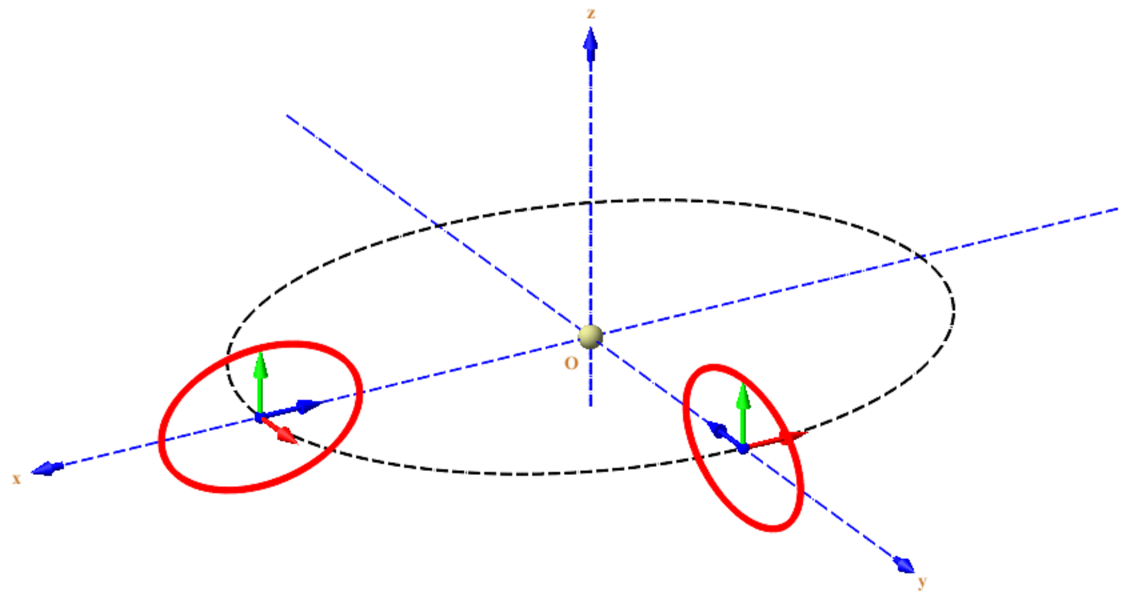
$$+\frac{7.5\cos(u)\sin\left(\frac{u}{9}\right)}{\sqrt{\frac{1}{\left(3\cos\left(\frac{u}{9}\right)-5\right)^2\left(3\cos\left(\frac{u}{9}\right)+5\right)^2}}\left(-9\cos\left(\frac{u}{9}\right)^2+25\right)^{3/2}},$$

$$-\left.\frac{1.\sin(u)}{\left(9.\cos\left(\frac{u}{9}\right)^2-25.\right)\sqrt{\frac{1}{\left(3\cos\left(\frac{u}{9}\right)-5\right)^2\left(3\cos\left(\frac{u}{9}\right)+5\right)^2}}}\right]$$

```
> PLELIKA1 := spacecurve(ELIKA1, u = 0 ..n·2· Pi, color = blue, thickness = 5, linestyle = 1) :
> ANIMELIKA1 := animate(pointplot3d, [ELIKA1, color = red, symbol = solidcircle, symbolsize
      = 10], u = 0 ..n·2·Pi, frames = 21, trace = 4) :
> ANIMELIKA2 := animate(spacecurve, [ELIKA1, u = 0 ..L, color = blue, thickness = 5, linestyle
      = 1], L = 0 ..n·2·Pi, frames = 21, trace = 00) :
> GENETEIRA := spacecurve(evalf(subs(v = 0, [X, Y, Z]) ), u = 0 ..2·Pi , color = red, thickness
      = 5, linestyle = 1) :
> TORUS := plot3d([X, Y, Z], u = 0 ..2·Pi, v = 0 ..2·Pi) :
> ANIMTORUS := animate(plot3d, [[X, Y, Z], u = 0 ..2·Pi, v = 0 ..M, style = surface], M = 0 ..2·Pi,
      frames = 21, trace = 0, transparency = 0.55) :
> ANIMGEN := animate(spacecurve, [[X, Y, Z], u = 0 ..2·Pi, color = blue, thickness = 3, linestyle
      = 4], v = 0 ..2·Pi, frames = 21, trace = 4) :
> CENTRE := pointplot3d([a, b, 0], color = blue, symbol = solidcircle, symbolsize = 5) :
> ODHGOS1A := spacecurve([a·cos(v), e·sin(v), 0], v = 0 ..2·Pi, color = black, thickness = 2,
      linestyle = 3) :
>
> plotTA := arrow(subs(v = 0, [ODHGOS1[1], ODHGOS1[2], ODHGOS1[3]]), subs(v = 0,
      [Component(T_, 1), Component(T_, 2), Component(T_, 3)]), width = 0.07, head_width
      = 0.2, head_length = 0.4, length = 1, color = red) :
> plotNA := arrow(subs(v = 0, [ODHGOS1[1], ODHGOS1[2], ODHGOS1[3]]), subs(v = 0,
      [Component(N_, 1), Component(N_, 2), Component(N_, 3)]), width = 0.07, head_width
      = 0.2, head_length = 0.4, length = 1, color = blue) :
> plotBA := arrow(subs(v = 0, [ODHGOS1[1], ODHGOS1[2], ODHGOS1[3]]), subs(v = 0,
      [Component(B_, 1), Component(B_, 2), Component(B_, 3)]), width = 0.07, head_width
      = 0.2, head_length = 0.4, length = 1, color = green) :
>
> plotTA1 := arrow(subs(v = Pi/2, [ODHGOS1[1], ODHGOS1[2], ODHGOS1[3]]), subs(v
      = Pi/2, [Component(T_, 1), Component(T_, 2), Component(T_, 3)]), width = 0.07,
      head_width = 0.2, head_length = 0.4, length = 1, color = red) :
```

```
> plotNA1 := arrow(subs(v = Pi/2, [ODHGOS1[1], ODHGOS1[2], ODHGOS1[3]]), subs(v
      = Pi/2, [Component(N_, 1), Component(N_, 2), Component(N_, 3)]), width = 0.07,
      head_width = 0.2, head_length = 0.4, length = 1, color = blue) :

> plotBA1 := arrow(subs(v = Pi/2, [ODHGOS1[1], ODHGOS1[2], ODHGOS1[3]]), subs(v
      = Pi/2, [Component(B_, 1), Component(B_, 2), Component(B_, 3)]), width = 0.07,
      head_width = 0.2, head_length = 0.4, length = 1, color = green) :

> BGENETEIRA := spacecurve(evalf(subs(v = Pi/2, [X, Y, Z])), u = 0 ..2·Pi , color = red,
      thickness = 5, linestyle = 1) :

> CENTREGENB := pointplot3d(subs(v = Pi/2, [ODHGOS1]), color = blue, symbol
      = solidcircle, symbolsize = 5) :

> display(AXONES, ODHGOS1A, GENETEIRA, BGENETEIRA, plotTA, plotNA, plotBA, plotTA1,
      plotNA1, plotBA1, CENTRE, CENTREGENB, title
      = "ΔΕΔΟΜΕΝΑ ΠΡΟΒΛΗΜΑΤΟΣ\nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ", titlefont = [arial, bold,
      14])
```

ΔΕΔΟΜΕΝΑ ΠΡΟΒΛΗΜΑΤΟΣ
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ

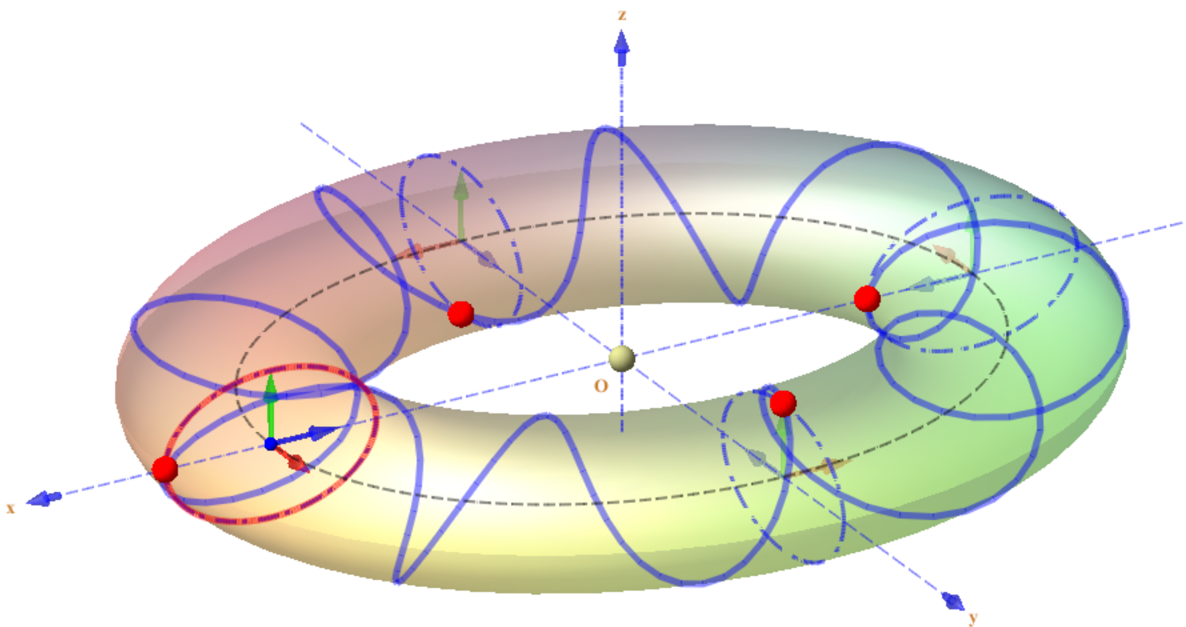


> *animTA* ≔ *animate*(*arrow*, [[*ODHGOS1*[1], *ODHGOS1*[2], *ODHGOS1*[3]], [*Component*(*T_*, 1), *Component*(*T_*, 2), *Component*(*T_*, 3)], *width* = 0.07, *head_width* = 0.2, *head_length* = 0.4, *length* = 1, *color* = *red*], *v* = 0 ..2·Pi, *frames* = 21, *trace* = 4) :

> *animNA* ≔ *animate*(*arrow*, [[*ODHGOS1*[1], *ODHGOS1*[2], *ODHGOS1*[3]],

```
[Component(N_, 1), Component(N_, 2), Component(N_, 3)], width = 0.07, head_width
 = 0.2, head_length = 0.4, length = 1, color = blue], v = 0 ..2·Pi, frames = 21, trace = 4) :
> animBA := animate(arrow, [[ODHGOS1[1], ODHGOS1[2], ODHGOS1[3]],
    [Component(B_, 1), Component(B_, 2), Component(B_, 3)], width = 0.07, head_width
    = 0.2, head_length = 0.4, length = 1, color = green], v = 0 ..2·Pi, frames = 21, trace = 4) :
> animFRENET := Student[VectorCalculus][TNBFrame](⟨ODHGOS1⟩, output = animation,
    range = 0 ..2·Pi, frames = 21) :
> display(animTA, animNA, animBA, AXONES, GENETEIRA, plotTA, plotNA, plotBA,
    ANIMELIKA1, ANIMELIKA2, ANIMTORUS, ANIMGEN, CENTRE, ODHGOS1A, title
    = "ANIMATION\nΤΟΡΟΣ ΜΕ ΠΕΡΙΕΛΙΞΗ\nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ", titlefont
    = [arial, bold, 14], scaling = constrained)
```

ANIMATION
ΤΟΡΟΣ ΜΕ ΠΕΡΙΕΛΙΞΗ
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ



ΜΗΚΟΣ ΕΛΙΚΑΣ : sEL1 .

> $ELIKA1 := subs\left(v = \frac{u}{n}, [X, Y, Z]\right)$

$$ELIKA1 := \left[5.\cos\left(\frac{u}{9}\right) + \frac{6.0\cos(u)\cos\left(\frac{u}{9}\right)}{\sqrt{\frac{1}{\left(3\cos\left(\frac{u}{9}\right)-5\right)^2\left(3\cos\left(\frac{u}{9}\right)+5\right)^2}}\left(-9\cos\left(\frac{u}{9}\right)^2+25\right)^{3/2}}, 4.\sin\left(\frac{u}{9}\right) + \frac{7.5\cos(u)\sin\left(\frac{u}{9}\right)}{\sqrt{\frac{1}{\left(3\cos\left(\frac{u}{9}\right)-5\right)^2\left(3\cos\left(\frac{u}{9}\right)+5\right)^2}}\left(-9\cos\left(\frac{u}{9}\right)^2+25\right)^{3/2}}, -\frac{1.\sin(u)}{\left(9.\cos\left(\frac{u}{9}\right)^2-25.\right)\sqrt{\frac{1}{\left(3\cos\left(\frac{u}{9}\right)-5\right)^2\left(3\cos\left(\frac{u}{9}\right)+5\right)^2}}}\right] \quad \textbf{(49)}$$

> $sEL1 := int\left(\text{sqrt}\left((diff(ELIKA1[1], u))^2 + (diff(ELIKA1[2], u))^2 + (diff(ELIKA1[3], u))^2\right), u = 0..n\cdot 2\cdot \text{Pi}, numeric\right)$

$$sEL1 := 77.21444784 \quad \textbf{(50)}$$

ΜΗΚΟΣ ΕΛΙΚΑΣ ΜΕ E,F,G .

F≠0 !!!

> $SURF1 := [X, Y, Z]$

$$SURF1 := \left[5.\cos(v) + \frac{6.0\cos(u)\cos(v)}{\sqrt{\frac{1}{(3\cos(v)-5)^2(3\cos(v)+5)^2}}\left(-9\cos(v)^2+25\right)^{3/2}}, 4.\sin(v) + \frac{7.5\cos(u)\sin(v)}{\sqrt{\frac{1}{(3\cos(v)-5)^2(3\cos(v)+5)^2}}\left(-9\cos(v)^2+25\right)^{3/2}}, -\frac{1.\sin(u)}{\left(9.\cos(v)^2-25.\right)\sqrt{\frac{1}{(3\cos(v)-5)^2(3\cos(v)+5)^2}}}\right] \quad \textbf{(51)}$$

> $R1\_ := SURF1[1]\cdot\_i + SURF1[2]\cdot\_j + SURF1[3]\cdot\_k$

(52)

$$\overrightarrow{R1} := \left(5.\cos(v) + \frac{6.0\cos(u)\cos(v)}{\sqrt{\frac{1}{(3\cos(v)-5)^2(3\cos(v)+5)^2}}\left(-9\cos(v)^2+25\right)^{3/2}}\right)\hat{i} + \hat{j}\left(4.\sin(v) + \frac{7.5\cos(u)\sin(v)}{\sqrt{\frac{1}{(3\cos(v)-5)^2(3\cos(v)+5)^2}}\left(-9\cos(v)^2+25\right)^{3/2}}\right) - \frac{1.\hat{k}\sin(u)}{\left(9.\cos(v)^2-25.\right)\sqrt{\frac{1}{(3\cos(v)-5)^2(3\cos(v)+5)^2}}} \quad \textbf{(52)}$$

> $E1 := subs\left(\left[u=t, v=\frac{t}{9}\right], simplify(diff(R1\_, u) \cdot diff(R1\_, u))\right):$

> $F1 := subs\left(\left[u=t, v=\frac{t}{9}\right], diff(R1\_, u) \cdot diff(R1\_, v)\right):$

> $G1 := subs\left(\left[u=t, v=\frac{t}{9}\right], simplify(diff(R1\_, v) \cdot diff(R1\_, v))\right):$

> $diff(t, t)$

$$1 \quad \textbf{(53)}$$

> $diff\left(\frac{t}{9}, t\right)$

$$\frac{1}{9} \quad \textbf{(54)}$$

> $diff(t, t) \cdot diff\left(\frac{t}{9}, t\right)$

$$\frac{1}{9} \quad \textbf{(55)}$$

> $MHLOSELIKAS1 := int\left(\mathrm{sqrt}\left(E1 \cdot (diff(t,t))^2 + 2 \cdot F1 \cdot diff(t,t) \cdot diff\left(\frac{t}{9}, t\right) + G1 \cdot \left(diff\left(\frac{t}{9}, t\right)\right)^2\right), t = 0..9 \cdot 2 \cdot \mathrm{Pi}, numeric\right)$

$$MHLOSELIKAS1 := 77.21447259 \quad \textbf{(56)}$$

Όπως αναμενόνταν : MHKOSELIKAS1<MHKOSELIKAS !!!!!!!

>

ΠΕΡΙ ΓΕΩΔΑΙΣΙΑΚΩΝ .

Έστω M μια κανονική επιφάνεια του $\mathbb{R}^3$. Μια καμπύλη $\gamma :\to M$ (με παραμέτρηση κατά μήκος τόξου) στην M ονομάζεται γεωδαισιακή (geodesic) εάν η εφαπτομενική συνιστώσα της δεύτερης παραγώγου $\ddot{\gamma}(t)$ μηδενίζεται, δηλαδή ισχύει

$$\ddot{\gamma}(t)^{\tan} = 0 \quad \text{για κάθε } t \in I.$$

Ελέγχουμε εάν η παραμετροποίηση της Περιστρεφόμενης Καμπύλης γίνεται ως προς Φυσική παράμετρο (Καμπύλη Μοναδιαίας Ταχύτητας).
$Norm\left(\frac{d}{du}\vec{r}\right) = 1$.
Υπάρχει αναπαραμέτρηση που να ικανοποιεί την ως άνω απαίτηση .

Παράδειγμα : Έστω κύκλος στο συντεταγμένο επίπεδο xz ακτίνας ρ :
Μία παραμέτρηση είναι : $\vec{r} := \rho \cdot \cos(\phi) \cdot \vec{i} + \rho \cdot \sin(\phi) \cdot \vec{k} \Rightarrow Norm\left(\frac{d}{d\phi}\vec{r}\right) = \rho \neq 1$

Η αναπαραμέτρηση γιά να είναι $Norm\left(\frac{d}{d\phi}\vec{r}\right) = 1$ **:**

$$s = \int_0^{\phi} Norm\left(\frac{d}{d\phi}\vec{r}\right) d\phi = \rho \cdot \phi \Rightarrow \phi = \frac{s}{\rho} \Rightarrow \vec{r} := \rho \cdot \cos\left(\frac{s}{\rho}\right) \cdot \vec{i} + \rho \cdot \sin\left(\frac{s}{\rho}\right) \cdot \vec{k}, \; Norm\left(\frac{d}{ds}\vec{r}\right) = 1.$$

ΑΚΟΛΟΥΘΩΝΤΑΣ ΤΑ ΒΗΜΑΤΑ ΤΗΣ ΘΕΩΡΙΑΣ :

ΓΡΑΦΟΥΜΕ ΤΗΝ ΔΙΑΝΥΣΜΑΤΙΚΗ ΑΚΤΙΝΑ ΤΗΣ ΚΑΜΠΥΛΗΣ ΣΤΟ ΧΩΡΟ : $\overrightarrow{r(\theta)} := \langle x(\theta), y(\theta), z(\theta) \rangle$. **Έστω Χρόνος t.**

ΥΠΟΛΟΓΙΖΟΥΜΕ (ΠΡΩΤΑ) ΤΟ ΜΗΚΟΣ ΤΟΥ ΤΟΞΟΥ sOM : s:=int($\sqrt{(x'(\theta))^2 + (y'(\theta))^2 + (z'(\theta))^2}$, θ), (που είναι συνάρτηση της παραμέτρου θ.). **Έστω Χρόνος t.**

Εφαπτόμενο μοναδιαίο διάνυσμα $\overrightarrow{TM} = \left\langle \frac{dx}{ds}, \frac{dy}{ds}, \frac{dz}{ds} \right\rangle = \left\langle \left(\frac{ds}{dx}\right)^{-1}, \left(\frac{ds}{dy}\right)^{-1}, \left(\frac{ds}{dz}\right)^{-1} \right\rangle = < \left[\frac{ds}{d\theta} \cdot \left(\frac{d\theta}{dx}\right)\right]^{-1}, \left[\frac{ds}{d\theta} \cdot \left(\frac{d\theta}{dy}\right)\right]^{-1}, \left[\frac{ds}{d\theta} \cdot \left(\frac{d\theta}{dz}\right)\right]^{-1} > = < \left(\frac{ds}{d\theta}\right)^{-1} \cdot \left(\frac{dx}{d\theta}\right), \left(\frac{ds}{d\theta}\right)^{-1} \cdot \left(\frac{dy}{d\theta}\right), \left(\frac{ds}{d\theta}\right)^{-1} \cdot \left(\frac{dz}{d\theta}\right) >$ **στο σημείο M(x,y,z) . (Κανόνας Αλυσίδας στην παραγώγιση) .**

Ακτίνα καμπυλότητας R ,στο σημείο M(x,y,z) (Κανόνας Αλυσίδας στην παραγώγιση): $\frac{d}{ds}\vec{T} = \frac{1}{R} \cdot \vec{N} \Rightarrow \frac{d}{d\theta}\vec{T} \cdot \left(\frac{ds}{d\theta}\right)^{-1} = \frac{1}{R} \cdot \vec{N} \Rightarrow \left|\frac{d}{d\theta}\vec{T}\right| = \frac{\left|\frac{ds}{d\theta}\right| \cdot 1}{R} \Rightarrow R = \frac{\left|\frac{ds}{d\theta}\right|}{\left|\frac{d}{d\theta}\vec{T}\right|}$.

Καμπυλότητα K , στο Σημείο M(x,y,z) : K=1/R

Κάθετο Μοναδιαίο διάνυσμα $\overrightarrow{NM}$**, στο Σημείο M(x,y,z) :** $\overrightarrow{NM} = R \cdot \frac{d}{ds}\vec{T} = R \cdot \frac{d}{d\theta}\vec{T} \cdot \left(\frac{ds}{d\theta}\right)^{-1}$

Μοναδιαίο διάνυσμα : $\overrightarrow{BM} = \vec{B} = \vec{T} \times \vec{N}$

Ταχύτητα : $\vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{ds}\vec{r} \cdot \frac{d}{dt}s = \vec{T} \cdot \frac{d}{dt}s$

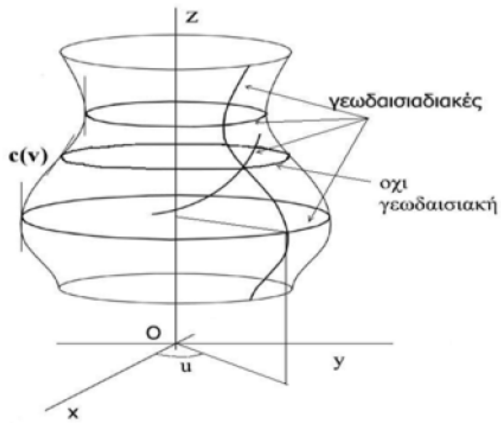
Επιτάχυνση : $\vec{a} = \frac{d}{dt}\vec{v} = \frac{d}{dt}\left(\vec{T} \cdot \frac{d}{dt}s\right) = \frac{d^2}{dt^2}s \cdot \vec{T} + \frac{d}{dt}s \cdot \frac{d}{dt}\vec{T} = \frac{d^2}{dt^2}s \cdot \vec{T} + \frac{d}{dt}s \cdot \left(\frac{d}{ds}\vec{T} \cdot \frac{d}{dt}s\right) = \frac{d^2}{dt^2}s \cdot \vec{T} + \left(\frac{d}{dt}s\right)^2 \cdot \frac{1}{R} \cdot \vec{N} = \frac{d^2}{dt^2}s \cdot \vec{T} + K\left(\frac{d}{dt}s\right)^2 \cdot \vec{N}$

Επιτάχυνση : $\vec{a} = \frac{d^2}{dt^2}s \cdot \vec{T} + K\left(\frac{d}{dt}s\right)^2 \cdot \vec{N} \rightarrow \vec{a} = a_T \cdot \vec{T} + a_N \cdot \vec{N}$ **όπου :** $a_T = \frac{d^2}{dt^2}s = \frac{d}{dt}|\vec{v}|$ **και** $a_N = K\left(\frac{d}{dt}s\right)^2 = K \cdot |\vec{v}|^2, \; a_N = \sqrt{|\vec{a}|^2 - a_T^2}$

Μια καμπύλη $\alpha : I \to M$ της επιφάνειας M ονομάζεται **γεωδαισιακή** αν η επιτάχυνσή της είναι κάθετη στην επιφάνεια σε κάθε σημείο της δηλαδή $\alpha''(t) \in T^{\perp}_{\alpha(t)}M$. Να δειχθεί ότι σε μια επιφάνεια εκ περιστροφής κάθε μεσημβρινός είναι γεωδαισιακή.

Αν μια καμπύλη είναι γεωδαισιακή,
τότε έχει σταθερό μέτρο ταχύτητας και γεωδαισιακή καμπυλότητα μηδέν.

ένας παράλληλος είναι γεωδαισιακή αν αντιστοιχεί σε σημείο της γενέτειρας που αυτή έχει εφαπτομένη παράλληλη προς τον άξονα περιστροφής.



Γεωδεσιακές επιφάνειας εκ περιστροφής

Εφαρμογή των Διαφορικών εξισώσεων Euler-Lagrange

$$\frac{d}{dt}\left(\frac{\partial}{\partial \dot{u}}L\right)-\frac{\partial}{\partial u}L=0:$$

$$\frac{d}{dt}\left(\frac{\partial}{\partial \dot{v}}L\right)-\frac{\partial}{\partial v}L=0:$$

ΔΙΑΦΟΡΙΚΕΣ ΕΞΙΣΩΣΕΙΣ ΓΕΩΔΑΙΣΙΑΚΩΝ :

$$\begin{cases} \dfrac{d}{dt}\left(E\cdot\dot{u}+F\cdot\dot{v}\right)=\dfrac{1}{2}\left(E_u\cdot\dot{u}^2+2F_u\cdot\dot{u}\cdot\dot{v}+G_u\,\dot{v}^2\right) \\ \dfrac{d}{dt}\left(F\cdot\dot{u}+G\cdot\dot{v}\right)=\dfrac{1}{2}\left(E_v\cdot\dot{u}^2+2F_v\cdot\dot{u}\cdot\dot{v}+G_v\,\dot{v}^2\right) \end{cases}$$

Η ΓΕΝΕΤΕΙΡΑ ΕΙΝΑΙ ΓΕΩΔΑΙΣΙΑΚΗ ?

Μια καμπύλη $\boldsymbol{\alpha}: I \to M$ της επιφάνειας M ονομάζεται **γεωδαισιακή** αν η επιτάχυνσή της είναι κάθετη στην επιφάνεια σε κάθε σημείο της δηλαδή $\boldsymbol{\alpha}''(t) \in T^{\perp}_{\boldsymbol{\alpha}(t)}M$. Να δειχθεί ότι σε μια επιφάνεια εκ περιστροφής κάθε μεσημβρινός είναι γεωδαισιακή.

"ΕΠΙΤΑΧΥΝΣΗ" ΜΕΣΗΜΒΡΙΝΩΝ . $(v = const)$

Το κάθετο διάνυσμα της Καμπύλης (Μεσημβρινού) σε κάθε σημείο της , είναι Συγγραμμικό με το κάθετο διάνυσμα της Επιφάνειας στο συγκεκριμένο σημείο .

ΑΠΟΔΕΙΞΗ

>

> $C := [Component(r\_, 1), Component(r\_, 2), Component(r\_, 3)]$

$$C := [5. + 1.5\cos(u), 0, \sin(u)] \quad (57)$$

> $C1 := simplify\left(subs\left(v = \frac{\text{Pi}}{2}, [C[1]\cdot\cos(v), C[1]\cdot\sin(v), C[3]]\right)\right)$

$$C1 := [0, 5. + 1.5\cos(u), \sin(u)] \quad (58)$$

> $FRENET := Student[VectorCalculus][TNBFrame](\langle C1\rangle):$

> $kathetosC1\_ := simplify\left(subs\left(u = \frac{\text{Pi}}{2}, FRENET[2][1]\cdot\_i + FRENET[2][2]\cdot\_j + FRENET[2][3]\cdot\_k\right)\right)$

$$\overrightarrow{kathetosC1} := -1.\ \hat{k} \quad (59)$$

> $simplify(Norm(kathetosC1\_), symbolic)$

$$1. \quad (60)$$

> $SURF := [C[1]\cdot\cos(v), C[1]\cdot\sin(v), C[3]]$

$$SURF := [(5. + 1.5\cos(u))\cos(v), (5. + 1.5\cos(u))\sin(v), \sin(u)] \quad (61)$$

> $R\_ := SURF[1]\cdot\_i + SURF[2]\cdot\_j + SURF[3]\cdot\_k$

$$\vec{R} := (5. + 1.5\cos(u))\cos(v)\ \hat{i} + (5. + 1.5\cos(u))\sin(v)\ \hat{j} + \sin(u)\ \hat{k} \quad (62)$$

> $NS\_ := simplify\left(subs\left(\left[u = \frac{\text{Pi}}{2}, v = \frac{\text{Pi}}{2}\right], diff(R\_, u) \times diff(R\_, v)\right)\right)$

$$\overrightarrow{NS} := -7.5\ \hat{k} \quad (63)$$

> $Norm(NS\_)$

$$7.500000000 \quad (64)$$

> $\frac{NS\_}{Norm(NS\_)}$

$$-1.000000000\ \hat{k} \quad (65)$$

> $\frac{NS\_}{Norm(NS\_)} \times kathetosC1\_$

$$0 \tag{66}$$

>

Η ΕΛΙΚΑ ΕΙΝΑΙ ΓΕΩΔΑΙΔΙΑΚΗ ??? ΟΧΙ .

> $Rel\_ := ELIKA[1] \cdot \_i + ELIKA[2] \cdot \_j + ELIKA[3] \cdot \_k$

$$\overrightarrow{Rel} := (5. + 1.5 \cos(u)) \cos\left(\frac{u}{9}\right) \hat{i} + (5. + 1.5 \cos(u)) \sin\left(\frac{u}{9}\right) \hat{j} + \sin(u) \hat{k} \tag{67}$$

> $epitaxEL := simplify\left(subs\left(u = \frac{\mathrm{Pi}}{9}, \frac{diff(Rel\_, u\$2)}{Norm(diff(Rel\_, u\$2))}\right)\right)$

$$epitaxEL := -0.9686814483\, \hat{i} - 0.1121069214\, \hat{j} - 0.2233848867\, \hat{k} \tag{68}$$

> $kathetosEPIF := simplify\left(subs\left(\left[u = \frac{\mathrm{Pi}}{9}, v = \frac{\frac{\mathrm{Pi}}{9}}{9}\right], \frac{NS\_}{simplify(Norm(NS\_))}\right)\right)$

$$kathetosEPIF := -1.\, \hat{k} \tag{69}$$

> **(68) × (69)**

$$0.1121069214\, \hat{i} - 0.9686814483\, \hat{j} \tag{70}$$

>