```
> with(plots) :
```

Εφαρμόζοντας την μέθοδο Newton-Raphson :

Θέμα : Να βρούμε , τις ρίζες της μιγαδικής συνάρτησης : $z^3 - 2$, το Γράφημα διαχωρισμού και την ροή της αναζήτησης των λύσεων :

```
with(plots) :
p := z^3 - 2
```

$$p := z^3 - 2$$

```
F := unapply(p, z)
```

$$F := z \mapsto z^3 - 2$$

```
RIZES := [evalf(solve(z^3 - 2, z))]
```

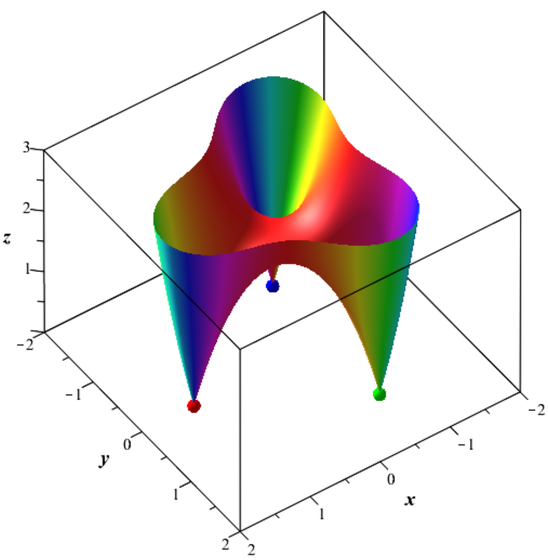
$$RIZES := [1.25992, -0.62996 + 1.09112\,I, -0.62996 - 1.09112\,I]$$

```
RIZES3D := [Re, Im, 0]~(RIZES)
```

$$RIZES3D := [[1.25992, 0., 0], [-0.62996, 1.09112, 0], [-0.62996, -1.09112, 0]]$$

```
A := pointplot3d(RIZES3D, symbol = solidcircle, symbolsize = 20, color = [red, green, blue]) :
B := complexplot3d(F, -2 - 2·I..2 + 2·I, style = surface, transparency = 0.00, grid = [1500, 1500]) :
display(A, B, orientation = [55, 45, 0], labels = [x, y, z], view = [default, default, 0 ..3], scaling = constrained, labelfont = [arial, bold, 14], title = "Συνάρτηση F& OI PIZEΣ THΣ", titlefont = [arial, bold, 14])
```

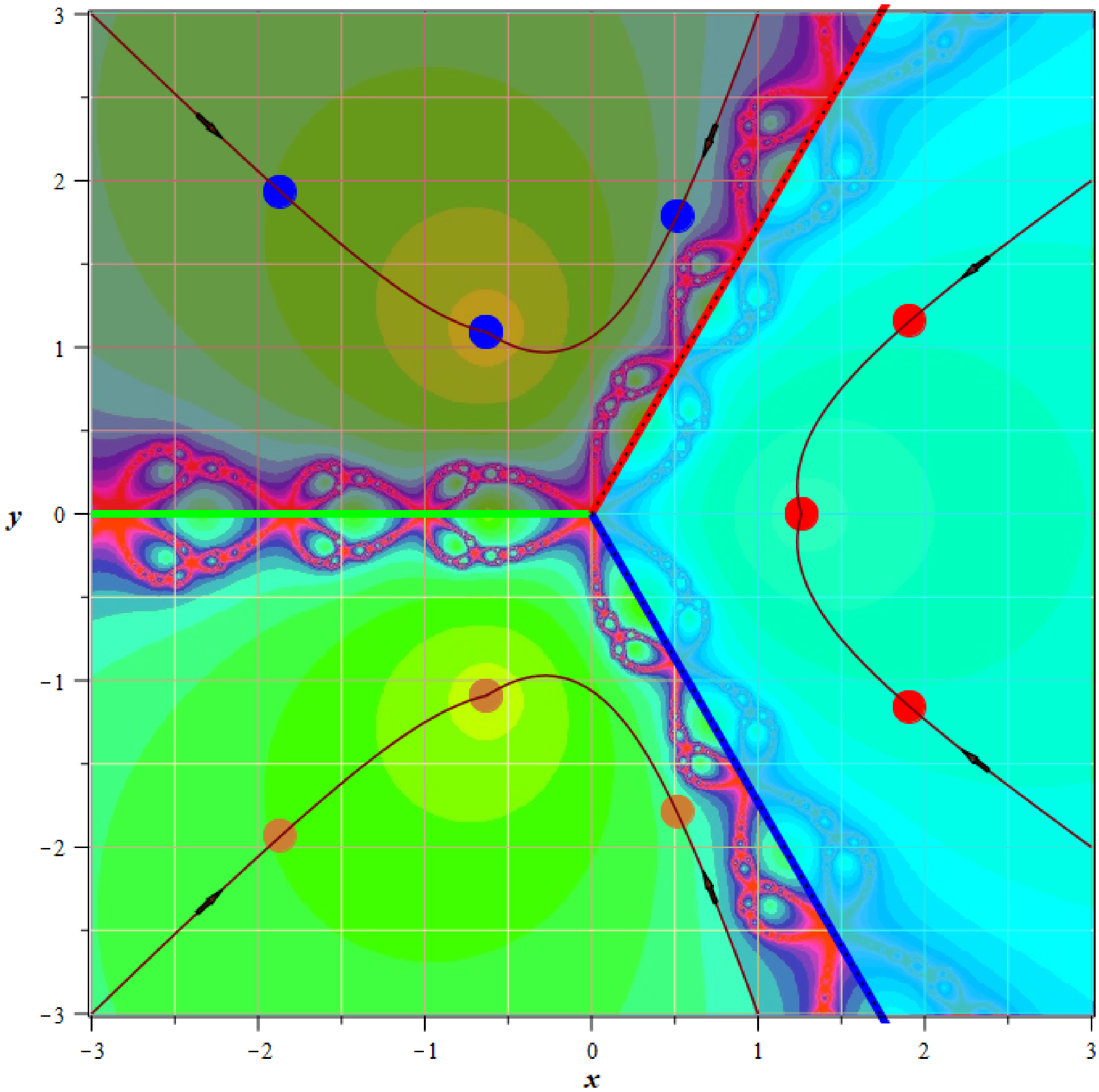
Συνάρτηση F& OI PIZEΣ THΣ

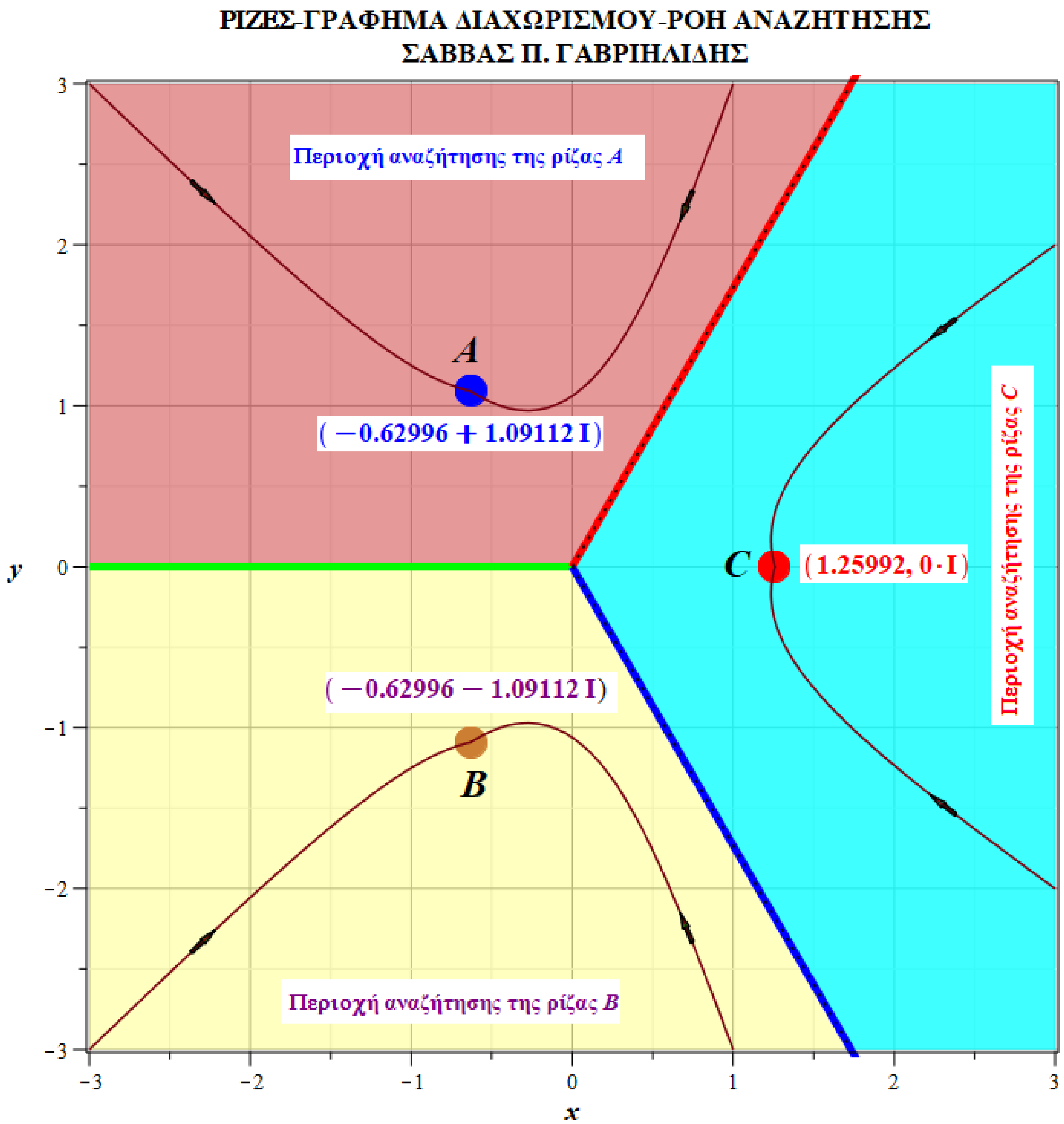


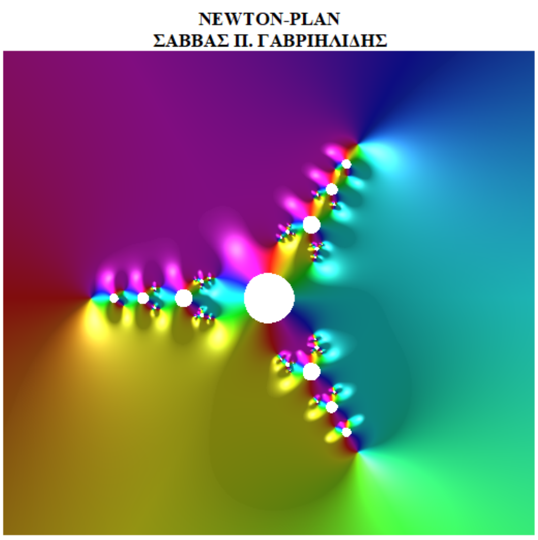
FRACTAL-NEWTON

ΡΙΖΕΣ-ΓΡΑΦΗΜΑ ΔΙΑΧΩΡΙΣΜΟΥ-ΡΟΗ ΑΝΑΖΗΤΗΣΗΣ

ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ







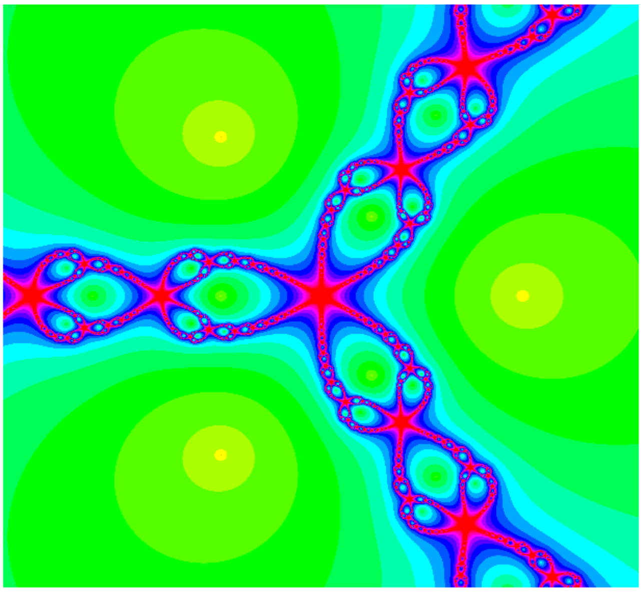
Πολλαπλασιάζουμε το βήμα με τον παράγοντα : $h := 0.1$.

$h := 0.1$

$F := unapply\left(z - h\cdot\frac{z^3-2}{3z^2}, z\right) \quad F := z \mapsto z - \frac{0.03333\,(z^3-2)}{z^2}$

$RIZES := evalf(solve(z^3-2, z)) \quad RIZES := 1.25992, -0.62996 + 1.09112\,I, -0.62996 - 1.09112\,I$

```
complexplot3d(F^(4),-1.3 - 1.3 I..1.3 + 1.3 I, view =- 4 ..4, grid = [500, 500], orientation = [-90, 0, 0], style = patchnogrid, transparency = 0.0, axes = none, title
    = "NEWTON-PLAN\nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ", titlefont = [arial, bold, 14]) :
```



NEWTON FRACTAL

$p := z^3 - 2 :$

$RIZES := 1.25992, -0.62996 + 1.09112\,I, -0.62996 - 1.09112\,I$

$p1 := diff(p, z) = 3\,z^2 :$

$\frac{p}{p1} = \frac{z^3 - 2}{3\,z^2} :$

$f := z \mapsto z - \frac{z^3 - 2}{3\,z^2} :$

$bl := -2 - 2\,I : ur := 2 + 2\,I :$

$Embed(Newton(3000, bl, ur, p, iterationlimit = 18, output = color)) :$

ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ

>

Πολλαπλασιάζουμε το βήμα με τον παράγοντα : $h := 0.1.$

> $h := 0.1$

$$h := 0.10000 \tag{1}$$

> $F := unapply\left(z - h \cdot \frac{z^3 - 2}{3\,z^2}, z\right)$

$$F := z \mapsto z - \frac{0.03333\,(z^3 - 2)}{z^2} \tag{2}$$

> $RIZES := evalf(solve(z^3 - 2, z))$

$$RIZES := 1.25992, -0.62996 + 1.09112\,I, -0.62996 - 1.09112\,I \tag{3}$$

>

> $y1 := \text{abs}\left(\frac{\text{Im}(RIZES[1])}{\text{Re}(RIZES[1])}\right) \cdot x$

$$y1 := 0. \tag{4}$$

> $y2 := \text{abs}\left(\frac{\text{Im}(RIZES[2])}{\text{Re}(RIZES[2])}\right) \cdot x$

$$y2 := 1.73205\,x \tag{5}$$

> $y3 := -\text{abs}\left(\frac{\text{Im}(RIZES[3])}{\text{Re}(RIZES[3])}\right) \cdot x$

$$y3 := -1.73205\,x \tag{6}$$

> $LIN := plot([[x, y1, x = -3..0], [x, y2, x = 0..3], [x, y3, x = 0..3]], color = [green, red, blue], thickness = 5, gridlines) :$

> $RIZ := pointplot([[\text{Re}(RIZES[1]), \text{Im}(RIZES[1])], [\text{Re}(RIZES[2]), \text{Im}(RIZES[2])], [\text{Re}(RIZES[3]), \text{Im}(RIZES[3])]], symbol = solidcircle, symbolsize = 20, color = [red, blue, gold]) :$

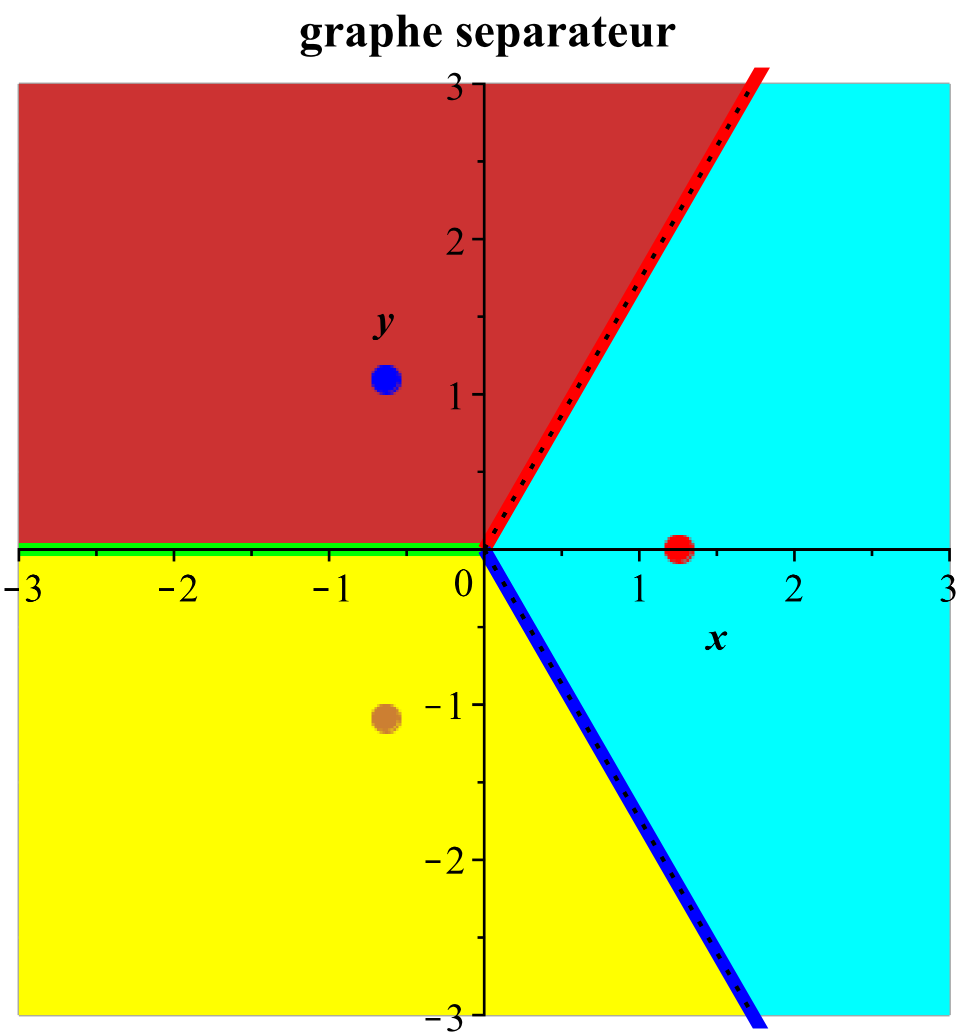
> $A := inequal(y - y2 > 0, x = -3..3, y = 0..3.0, scaling = constrained, color = orange, transparency = 0.50) :$

> $B := inequal(y - y3 < 0, x = -3..3, y = -3.0..0, scaling = constrained, color = yellow, transparency = 0.75) :$

> $C := inequal(y - y2 < 0, x = -3..3, y = 0..3.0, scaling = constrained, color = cyan, transparency = 0.25)$:

> $E := inequal(y - y3 > 0, x = -3..3, y = -3.0..0, scaling = constrained, color = cyan, transparency = 0.25)$:

> $display(LIN, RIZ, A, B, C, E, title = \text{"graphe separateur"}, titlefont = [arial, bold, 14], labels = [x, y], labelfont = [arial, bold, 12], scaling = constrained, view = [-3..3, -3..3])$



> $APLOT := display(LIN, RIZ, A, B, C, E, title = \text{"graphe separateur"}, titlefont = [arial, bold, 14], labels = [x, y], labelfont = [arial, bold, 12], scaling = constrained, view = [-3..3, -3..3])$:

ΑΝΑΖΗΤΗΣΗ ΡΙΖΑΣ : $-0.62996 + 1.09112\,I$

> $h := 0.1$

$$h := 0.10000 \qquad (7)$$

> $F := unapply\left(z - h \cdot \frac{z^3 - 2}{3 z^2}, z\right)$

$$F := z \mapsto z - \frac{0.03333\,(z^3 - 2)}{z^2} \qquad (8)$$

> $F1 := simplify(subs([x = x[k], y = y[k]], F(x + y \cdot I)))$

$$F1 := \frac{-2.90000\, x_k y_k^2 + 2.90000\, I x_k^2 y_k + 0.96667\, x_k^3 - 0.96667\, I y_k^3 + 0.06667}{(x_k + I y_k)^2} \qquad (9)$$

```
> x[k+1] := Re(F1) assuming x[k] :: real, y[k] :: real
```

$$x_{k+1} := \frac{1.93333\, x_k^3 y_k^2 + 0.96667\, x_k y_k^4 + 0.96667\, x_k^5 + 0.06667\, x_k^2 - 0.06667\, y_k^2}{\left(x_k^2 + y_k^2\right)^2} \quad \textbf{(10)}$$

```
> y[k+1] := Im(F1) assuming x[k] :: real, y[k] :: real
```

$$y_{k+1} := \frac{y_k \left(1.93333\, x_k^2 y_k^2 + 0.96667\, x_k^4 - 0.13333\, x_k + 0.96667\, y_k^4\right)}{\left(x_k^2 + y_k^2\right)^2} \quad \textbf{(11)}$$

```
> x[0] := 1
```

$$x_0 := 1 \quad \textbf{(12)}$$

```
> y[0] := 3
```

$$y_0 := 3 \quad \textbf{(13)}$$

```
> iter := 132
```

$$iter := 132 \quad \textbf{(14)}$$

```
> for k from 0 to iter do x[k+1] := evalf(Re(F1)) assuming x[k] :: real, y[k] :: real : y[k
    +1] := evalf(Im(F1)) assuming x[k] :: real, y[k] :: real :end do:
> PP1 := plot([seq([x[n], y[n]], n = 0 ..132)], style = line, gridlines) :
> AR1 := arrow(⟨x[7], y[7]⟩, ⟨x[8] - x[7], y[8] - y[7]⟩, color = green, width = 0.02,
    head_length = 0.1, length = 0.2) :
> anim1 := display(seq(pointplot([x[k], y[k]], symbol = solidcircle, symbolsize = 20, color
    = blue), k = 1 ..60), insequence = true) :
>
```

Με (133) {iter+1} επαναλήψεις βρήκαμε την ΡΙΖΑ .!!!

```
>
```

ΑΝΑΖΗΤΗΣΗ ΡΙΖΑΣ : $-0.62996 + 1.09112\,\mathrm{I}$

ME R , S

```
> W := simplify(subs([R = R[a], S = S[a]], F(R + S·I)))
```

$$W := \frac{-2.90000\, R_a S_a^2 + 2.90000\, \mathrm{I}\, R_a^2 S_a + 0.96667\, R_a^3 - 0.96667\, \mathrm{I}\, S_a^3 + 0.06667}{\left(R_a + \mathrm{I}\, S_a\right)^2} \quad \textbf{(15)}$$

```
> R[a+1] := Re(W) assuming R[a] :: real, S[a] :: real
```

$$R_{a+1} := \frac{1.93333\, R_a^3 S_a^2 + 0.96667\, R_a S_a^4 + 0.96667\, R_a^5 + 0.06667\, R_a^2 - 0.06667\, S_a^2}{\left(R_a^2 + S_a^2\right)^2} \quad \textbf{(16)}$$

```
> S[a+1] := Im(W) assuming R[a] :: real, S[a] :: real
```

$$S_{a+1} := \frac{S_a \left(1.93333\, R_a^2 S_a^2 + 0.96667\, R_a^4 - 0.13333\, R_a + 0.96667\, S_a^4\right)}{\left(R_a^2 + S_a^2\right)^2} \quad \textbf{(17)}$$

```
> R[0] := -3
```

$$R_0 := -3 \quad \textbf{(18)}$$

```
> S[0] := 3
```

$$S_0 := 3 \quad \textbf{(19)}$$

```
> for a from 0 to iter do R[a+1] := Re(W) assuming R[a] :: real, S[a] :: real : S[a
    +1] := Im(W) assuming R[a] :: real, S[a] :: real :end do:
```

> $PP1a := plot([seq([R[a], S[a]], a = 0..132)], style = line, gridlines)$:

> $AR1a := arrow(\langle R[7], S[7]\rangle, \langle R[8] - R[7], S[8] - S[7]\rangle, color = green, width = 0.02, head\_length = 0.1, length = 0.2)$:

> $anim1a := display(seq(pointplot([R[a], S[a]], symbol = solidcircle, symbolsize = 20, color = blue), a = 1..60), insequence = true)$:

>

ΑΝΑΖΗΤΗΣΗ ΡΙΖΑΣ : − 0.62996 − 1.09112 I

> $G := simplify(subs([X = X[m], Y = Y[m]], F(X + Y \cdot I)))$

$$G := \frac{-2.90000\, X_m Y_m^2 + 2.90000\, \mathrm{I} X_m^2 Y_m + 0.96667\, X_m^3 - 0.96667\, \mathrm{I}\, Y_m^3 + 0.06667}{(X_m + \mathrm{I}\, Y_m)^2} \quad \textbf{(20)}$$

> $X[m+1] := \mathrm{Re}(G)$ assuming $X[m] :: real, Y[m] :: real$

$$X_{m+1} := \frac{1.93333\, X_m^3 Y_m^2 + 0.96667\, X_m Y_m^4 + 0.96667\, X_m^5 + 0.06667\, X_m^2 - 0.06667\, Y_m^2}{(X_m^2 + Y_m^2)^2} \quad \textbf{(21)}$$

> $Y[m+1] := \mathrm{Im}(G)$ assuming $X[m] :: real, Y[m] :: real$

$$Y_{m+1} := \frac{Y_m\left(1.93333\, X_m^2 Y_m^2 + 0.96667\, X_m^4 - 0.13333\, X_m + 0.96667\, Y_m^4\right)}{(X_m^2 + Y_m^2)^2} \quad \textbf{(22)}$$

> $X[0] := 1$

$$X_0 := 1 \quad \textbf{(23)}$$

> $Y[0] := -3$

$$Y_0 := -3 \quad \textbf{(24)}$$

> $iter := 132$

$$iter := 132 \quad \textbf{(25)}$$

> **for** m **from** 0 **to** $iter$ **do** $X[m+1] := \mathrm{Re}(G)$ assuming $X[m] :: real, Y[m] :: real$: $Y[m+1] := \mathrm{Im}(G)$ assuming $X[m] :: real, Y[m] :: real$:**end do**:

> $PP2 := plot([seq([X[n], Y[n]], n = 0..132)], style = line, gridlines)$:

> $AR2 := arrow(\langle X[7], Y[7]\rangle, \langle X[8] - X[7], Y[8] - Y[7]\rangle, color = green, width = 0.02, head\_length = 0.1, length = 0.2)$:

> $anim2 := display(seq(pointplot([X[m], Y[m]], symbol = solidcircle, symbolsize = 20, color = gold), m = 1..60), insequence = true)$:

>

ΑΝΑΖΗΤΗΣΗ ΡΙΖΑΣ : − 0.62996 − 1.09112 I

ME r,s

> $WW := simplify(subs([r = r[b], s = s[b]], F(r + s \cdot I)))$

$$WW := \frac{-2.90000\, r_b s_b^2 + 2.90000\, \mathrm{I}\, r_b^2 s_b + 0.96667\, r_b^3 - 0.96667\, \mathrm{I}\, s_b^3 + 0.06667}{(r_b + \mathrm{I}\, s_b)^2} \quad \textbf{(26)}$$

> $r[b+1] := \mathrm{Re}(WW)$ assuming $r[b] :: real, s[b] :: real$

(27)

$$r_{b+1} := \frac{1.93333\, r_b^3 s_b^2 + 0.96667\, r_b s_b^4 + 0.96667\, r_b^5 + 0.06667\, r_b^2 - 0.06667\, s_b^2}{\left(r_b^2 + s_b^2\right)^2} \quad \textbf{(27)}$$

> $s[b+1] := \mathrm{Im}(WW)$ assuming $r[b] :: real, s[b] :: real$

$$s_{b+1} := \frac{s_b \left(1.93333\, r_b^2 s_b^2 + 0.96667\, r_b^4 - 0.13333\, r_b + 0.96667\, s_b^4\right)}{\left(r_b^2 + s_b^2\right)^2} \quad \textbf{(28)}$$

> $r[0] := -3$

$$r_0 := -3 \quad \textbf{(29)}$$

> $s[0] := -3$

$$s_0 := -3 \quad \textbf{(30)}$$

> $iter := 132$

$$iter := 132 \quad \textbf{(31)}$$

> **for** b **from** 0 **to** $iter$ **do** $r[b+1] := \mathrm{Re}(WW)$ assuming $r[b] :: real, s[b] :: real : s[b+1] := \mathrm{Im}(WW)$ assuming $r[b] :: real, s[b] :: real$:**end do**:

> $PP2a := plot([seq([r[b], s[b]], b = 0..132)], style = line, gridlines) :$

> $AR2a := arrow(\langle r[7], s[7]\rangle, \langle r[8] - r[7], s[8] - s[7]\rangle, color = green, width = 0.02, head\_length = 0.1, length = 0.2) :$

> $anim2a := display(seq(pointplot([r[b], s[b]], symbol = solidcircle, symbolsize = 20, color = gold), b = 1..60), insequence = true) :$

>

ΑΝΑΖΗΤΗΣΗ ΤΗΣ ΡΙΖΑΣ : 1.25992 ME p,q

> $H := simplify(subs([p = p[l], q = q[l]], F(p + q \cdot I)))$

$$H := \frac{-2.90000\, p_l q_l^2 + 2.90000\, \mathrm{I}\, p_l^2 q_l + 0.96667\, p_l^3 - 0.96667\, \mathrm{I}\, q_l^3 + 0.06667}{\left(p_l + \mathrm{I}\, q_l\right)^2} \quad \textbf{(32)}$$

> $p[l+1] := \mathrm{Re}(H)$ assuming $p[l] :: real, q[l] :: real$

$$p_{l+1} := \frac{1.93333\, p_l^3 q_l^2 + 0.96667\, p_l q_l^4 + 0.96667\, p_l^5 + 0.06667\, p_l^2 - 0.06667\, q_l^2}{\left(p_l^2 + q_l^2\right)^2} \quad \textbf{(33)}$$

> $q[l+1] := \mathrm{Im}(H)$ assuming $p[l] :: real, q[l] :: real$

$$q_{l+1} := \frac{q_l \left(1.93333\, p_l^2 q_l^2 + 0.96667\, p_l^4 - 0.13333\, p_l + 0.96667\, q_l^4\right)}{\left(p_l^2 + q_l^2\right)^2} \quad \textbf{(34)}$$

> $p[0] := 3$

$$p_0 := 3 \quad \textbf{(35)}$$

> $q[0] := 2$

$$q_0 := 2 \quad \textbf{(36)}$$

> $iter := 132$

$$iter := 132 \quad \textbf{(37)}$$

> **for** l **from** 0 **to** $iter$ **do** $p[l+1] := evalf(\mathrm{Re}(H))$ assuming $p[l] :: real, q[l] :: real : q[l+1] := evalf(\mathrm{Im}(H))$ assuming $p[l] :: real, q[l] :: real$:**end do**:

> $PP3 := plot([seq([p[l], q[l]], l = 0..132)], style = line, gridlines) :$

> $AR3 := arrow(\langle p[7], q[7]\rangle, \langle p[8] - p[7], q[8] - q[7]\rangle, color = green, width = 0.02, head\_length = 0.1, length = 0.2) :$

> $anim3 := display(seq(pointplot([p[l], q[l]], symbol = solidcircle, symbolsize = 20, color = red), l = 1..60), insequence = true) :$

>

ΑΝΑΖΗΤΗΣΗ ΤΗΣ ΡΙΖΑΣ : 1.25992

Με P,Q

> $J := simplify(subs([P = P[n], Q = Q[n]], F(P + Q\cdot I)))$

$$J := \frac{-2.90000\, P_n Q_n^2 + 2.90000\, I P_n^2 Q_n + 0.96667\, P_n^3 - 0.96667\, I Q_n^3 + 0.06667}{(P_n + I Q_n)^2} \quad \textbf{(38)}$$

> $P[n+1] := \mathrm{Re}(J)$ assuming $P[n] :: real, Q[n] :: real$

$$P_{n+1} := \frac{1.93333\, P_n^3 Q_n^2 + 0.96667\, P_n Q_n^4 + 0.96667\, P_n^5 + 0.06667\, P_n^2 - 0.06667\, Q_n^2}{(P_n^2 + Q_n^2)^2} \quad \textbf{(39)}$$

> $Q[n+1] := \mathrm{Im}(J)$ assuming $P[n] :: real, Q[n] :: real$

$$Q_{n+1} := \frac{Q_n \left(1.93333\, P_n^2 Q_n^2 + 0.96667\, P_n^4 - 0.13333\, P_n + 0.96667\, Q_n^4\right)}{(P_n^2 + Q_n^2)^2} \quad \textbf{(40)}$$

> $P[0] := 3$

$$P_0 := 3 \quad \textbf{(41)}$$

> $Q[0] := -2$

$$Q_0 := -2 \quad \textbf{(42)}$$

> $iter := 132$

$$iter := 132 \quad \textbf{(43)}$$

> **for** n **from** 0 **to** $iter$ **do** $P[n+1] := \mathrm{Re}(J)$ assuming $P[n] :: real, Q[n] :: real : Q[n+1] := \mathrm{Im}(J)$ assuming $P[n] :: real, Q[n] :: real$: **end do**:

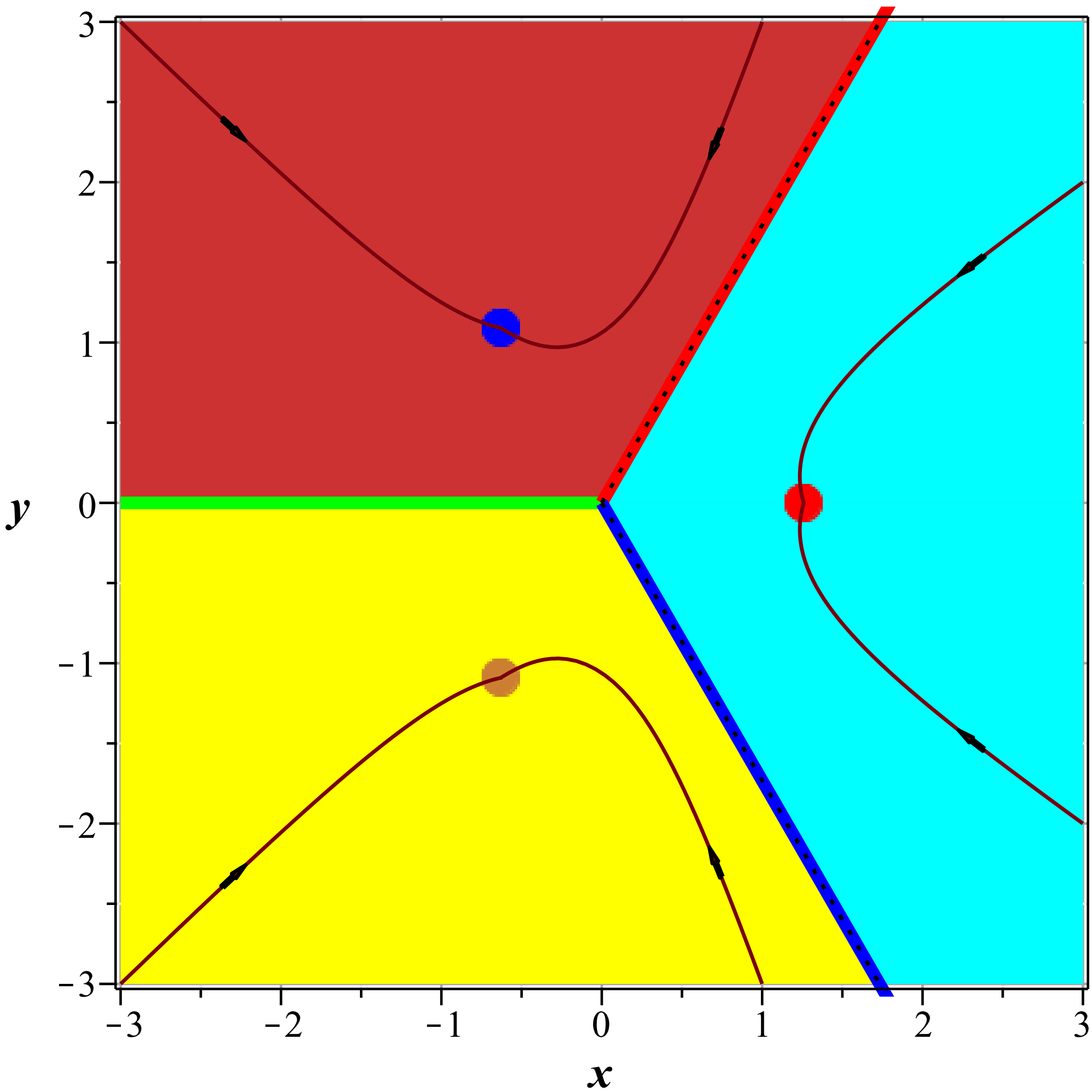
> $PP4 := plot([seq([P[n], Q[n]], n = 0..132)], style = line, gridlines) :$

> $AR4 := arrow(\langle P[7], Q[7]\rangle, \langle P[8] - P[7], Q[8] - Q[7]\rangle, color = green, width = 0.02, head\_length = 0.1, length = 0.2) :$

> $anim3a := display(seq(pointplot([P[n], Q[n]], symbol = solidcircle, symbolsize = 20, color = red), n = 1..60), insequence = true) :$

> $display(APLOT, RIZ, PP1, AR1, PP1a, AR1a, PP2, AR2, PP2a, AR2a, PP3, AR3, PP4, AR4, scaling = constrained, view = [-3..3, -3..3], title$ = "ΡΙΖΕΣ-ΓΡΑΦΗΜΑ ΔΙΑΧΩΡΙΣΜΟΥ-ΡΟΗ ΑΝΑΖΗΤΗΣΗΣ\nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ", $titlefont = [arial, bold, 14], labels = [x, y], labelfont = [arial, bold, 14], axes = boxed)$

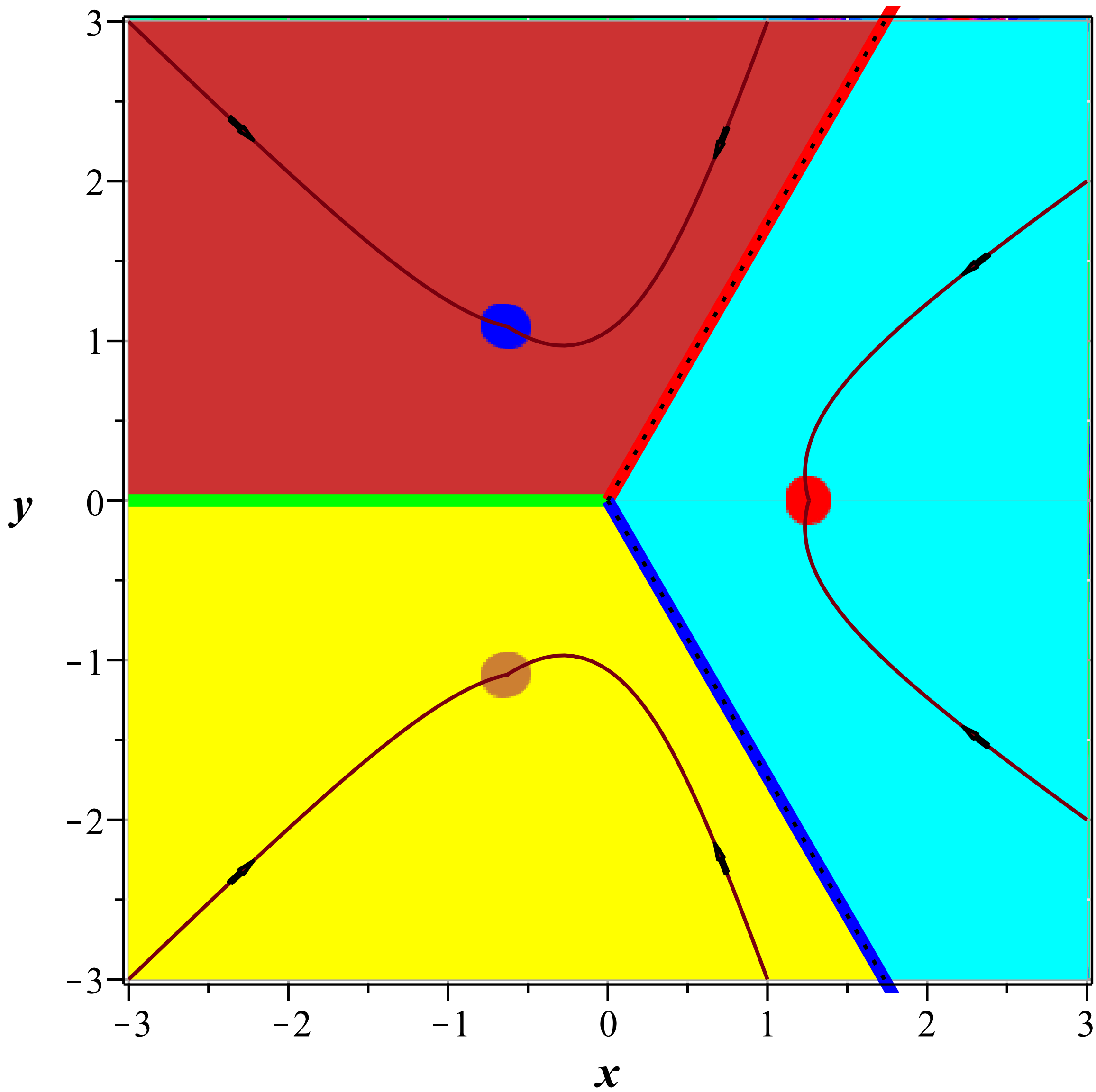
ΡΙΖΕΣ-ΓΡΑΦΗΜΑ ΔΙΑΧΩΡΙΣΜΟΥ-ΡΟΗ ΑΝΑΖΗΤΗΣΗΣ
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ



```
> with(FileTools) :
> SABBAS := JoinPath(["C:", "SPGABRIHLIDHS", "IMAGES",
      "FRACTAL-NEWTON-2.jpg"]) :
> display(APLOT, RIZ, PP1, AR1, PP1a, AR1a, PP2, AR2, PP2a, AR2a, PP3, AR3, PP4, AR4,
      anim1, anim1a, anim2, anim2a, anim3, anim3a, scaling = constrained, view = [-3 ..3, -3
      ..3], title
      = "FRACTAL-NEWTON\nΡΙΖΕΣ-ΓΡΑΦΗΜΑ ΔΙΑΧΩΡΙΣΜΟΥ-ΡΟΗ
      ΑΝΑΖΗΤΗΣΗΣ\nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ", titlefont = [arial, bold, 14], labels = [x,
      y], labelfont = [arial, bold, 14], axes = boxed, background = SABBAS)
```

FRACTAL-NEWTON
ΡΙΖΕΣ-ΓΡΑΦΗΜΑ ΔΙΑΧΩΡΙΣΜΟΥ-ΡΟΗ ΑΝΑΖΗΤΗΣΗΣ

ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ



ΣΗΜ. Από την $Embed(Newton(3000, bl, ur, p, iterationlimit = 18, output = color))$ **με** $bl\,..ur = view$ **αποθηκεύουμε την ΕΙΚΟΝΑ και την ανακαλούμε με :** $SABBAS := JoinPath(["C:", "SPGABRIHLIDHS", "IMAGES", "FRACTAL-NEWTON-2.jpg"])$

>