>

Μέθοδος των Newton-Raphson

Εφαρμογή : Θα εφαρμόσουμε τη μέθοδο των Newton-Raphson για να προσεγγίσουμε τις λύσεις της **Μιγαδικής Εξίσωσης** : $\cosh(z) + 3 = 0$ με ακρίβεια πέντε δεκαδικών ψηφίων.

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> $with(Fractals)$:

> $with(Fractals[EscapeTime])$:

> $with(ImageTools)$:

>

$with(plots)$:

$\cosh(z) + 3$

$$\cosh(z) + 3$$

$P := unapply(convert(\cosh(z) + 3, \exp), z)$

$$P := z \mapsto \frac{e^{z}}{2} + \frac{e^{-z}}{2} + 3$$

$RIZES := \{evalf(solve(P(z), z))\}$

$$RIZES := \{-1.76275 + 3.14159\,I, 1.76275 + 3.14159\,I\}$$

$RIZES3D := [\mathrm{Re}, \mathrm{Im}, 0]\sim(RIZES)$

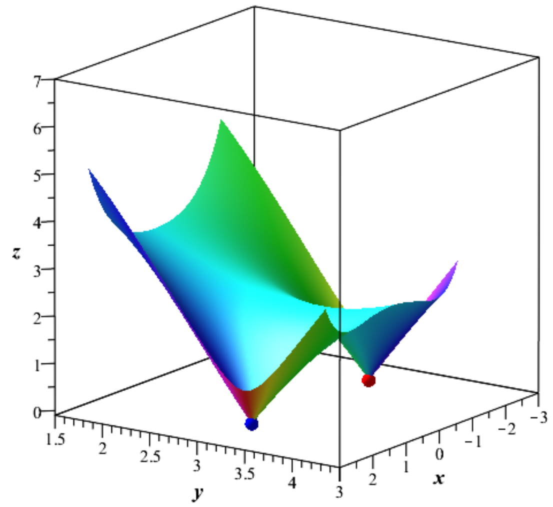
$$RIZES3D := \{[-1.76275, 3.14159, 0], [1.76275, 3.14159, 0]\}$$

$A := pointplot3d(RIZES3D, symbol = solidcircle, symbolsize = 20, color = [red, blue])$:

$B := complexplot3d(p, -2 - 4\cdot I..+2+4\cdot I, grid = [200, 200], style = surface)$:

$display(A, B, view = [-3..3, 3..3, -0.1..7], labels = [x, y, z], labelfont = [arial, bold, 14], orientation = [35, 75, 0], title =$ "ΣΥΝΑΡΤΗΣΗ P &RIZES3D", $titlefont = [arial, bold, 14])$

ΣΥΝΑΡΤΗΣΗ P &RIZES3D



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> $p := \cosh(z) + 3$

$$p := \cosh(z) + 3 \tag{1}$$

> $p1 := diff(p, z)$

$$p1 := \sinh(z) \tag{2}$$

> $\frac{p}{p1}$

$$\frac{\cosh(z) + 3}{\sinh(z)} \tag{3}$$

> $f := unapply\left(z - \frac{p}{p1}, z\right)$

(4)

$$f := z \mapsto z - \frac{\cosh(z) + 3}{\sinh(z)} \tag{4}$$

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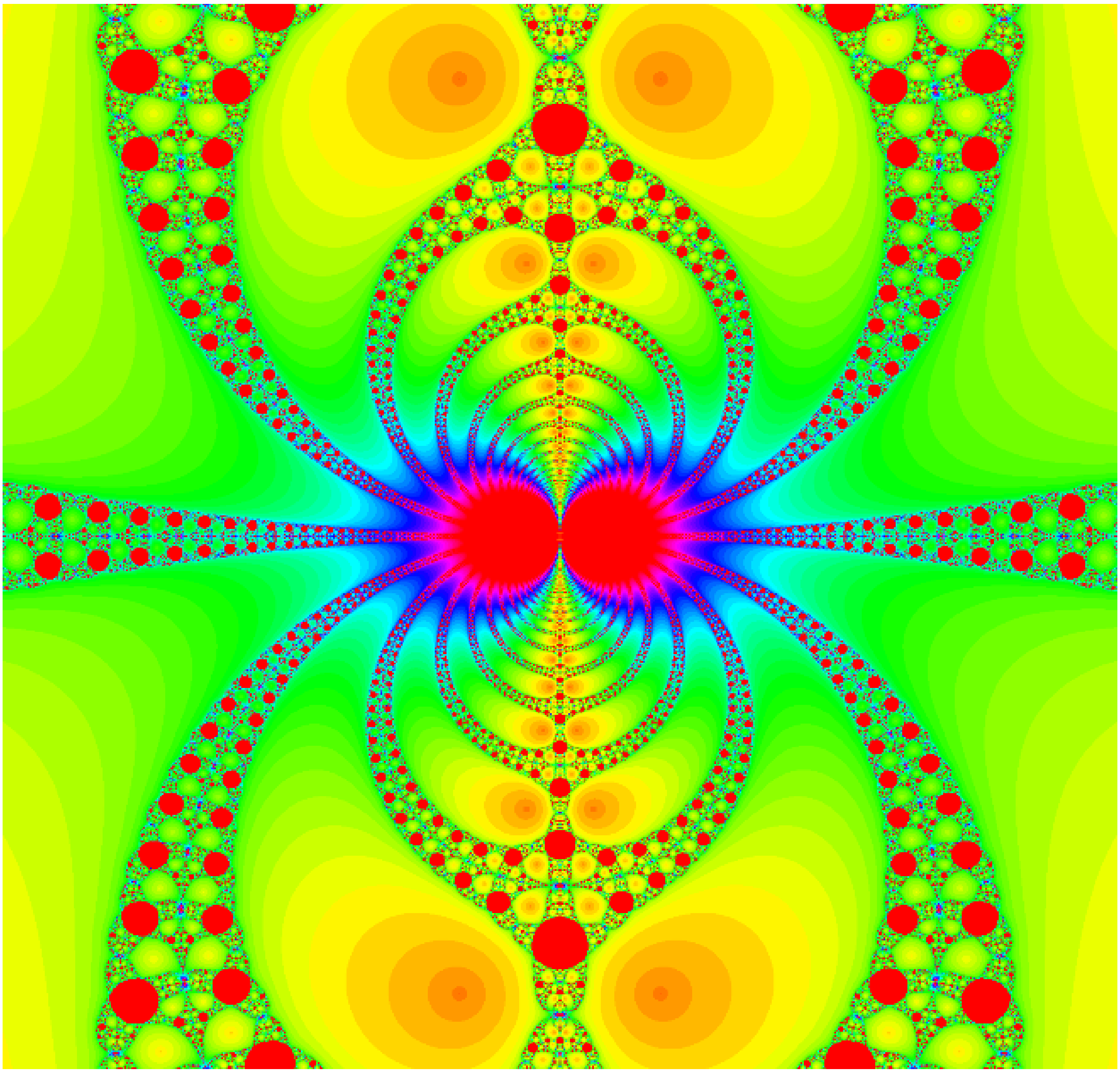

Newton Fractal

$p := \cosh(z) + 3$

$p1 := \mathit{diff}(p, z) = \sinh(z)$

$\frac{p}{p1} = \frac{\cosh(z) + 3}{\sinh(z)}$

$f := z \mapsto z - \frac{\cosh(z) + 3}{\sinh(z)}$

***Embed*(*Newton*(1500, −0.5 − 0.5·I, 0.5 + 0.5·I, *p*))**

ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ

> *convert*(cosh(*z*), exp)

(5)

$$\frac{e^{z}}{2}+\frac{e^{-z}}{2} \qquad \textbf{(5)}$$

> $RIZES := evalf(solve(\textbf{(5)}+3, z))$

$$RIZES := 1.76275+3.14159\,\mathrm{I},\ -1.76275+3.14159\,\mathrm{I} \qquad \textbf{(6)}$$

> $F := simplify(subs([x=x[k], y=y[k]], f(x+y\cdot\mathrm{I})))$

$$F := 1.76275+3.14159\,\mathrm{I} \qquad \textbf{(7)}$$

> $x[k+1] := \mathrm{Re}(F)$ assuming $x[k] :: real, y[k] :: real$

$$x_4 := 1.76275 \qquad \textbf{(8)}$$

> $y[k+1] := \mathrm{Im}(F)$ assuming $x[k] :: real, y[k] :: real$

$$y_4 := 3.14159 \qquad \textbf{(9)}$$

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ΑΝΑΖΗΤΗΣΗ ΤΗΣ ΡΙΖΑΣ 1.76275 + 3.14159 I

> $x[0] := 2.0$

$$x_0 := 2.00000 \qquad \textbf{(10)}$$

> $y[0] := 3.0$

$$y_0 := 3.00000 \qquad \textbf{(11)}$$

> $iter := 2$

$$iter := 2 \qquad \textbf{(12)}$$

> **for** k **from** 0 **to** $iter$ **do** $x[k+1] := \mathrm{Re}(F)$ assuming $x[k] :: real, y[k] :: real : y[k+1] :=$ $\mathrm{Im}(F)$ assuming $x[k] :: real, y[k] :: real$:**end do**

$$x_1 := 1.76275$$
$$y_1 := 3.14159$$
$$x_2 := 1.76275$$
$$y_2 := 3.14159$$
$$x_3 := 1.76275$$
$$y_3 := 3.14159 \qquad \textbf{(13)}$$

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Μετά από τρείς (3) επαναλήψεις προσεγγίσαμε την ζητούμενη ρίζα .!!!

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>

ANIMATE

> $with(plots)$:

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Οι Συντεταγμένες του Κέντρου του Στόχου (X, Y).

Παράθυρο ΖΟΥΜ: $(2\cdot a, 2\cdot b)$

$$bl := X[n] - a + (Y[n] - b)\cdot \mathrm{I}$$

$$ur := X[n] + a + (Y[n] + b)\cdot \mathrm{I}$$

> $X := \cos(\phi)$

$$X := \cos(\phi) \tag{14}$$

> $Y := 1 + \sin(\phi)$

$$Y := 1 + \sin(\phi) \tag{15}$$

> $a := 1$

$$a := 1 \tag{16}$$

> $b := 1$

$$b := 1 \tag{17}$$

> $bl := X - a + (Y - b)\cdot \mathrm{I}$

$$bl := \cos(\phi) - 1 + \mathrm{I}\sin(\phi) \tag{18}$$

> $ur := X + a + (Y + b)\cdot \mathrm{I}$

$$ur := \cos(\phi) + 1 + \mathrm{I}\,(2 + \sin(\phi)) \tag{19}$$

> $f := z \mapsto z - \dfrac{\cosh(z) + 3}{\sinh(z)}$

$$f := z \mapsto z - \frac{\cosh(z) + 3}{\sinh(z)} \tag{20}$$

> $P := pointplot3d([0, 0, 0], color = yellow, symbol = solidcircle, symbolsize = 10)$:

> $ANIM := display\Big(seq\Big(complexplot3d\big(f^{(8)}, bl\,..ur, view = -1\,..4, grid = [150, 150], style = patchnogrid, transparency = 0.0, labels = [x, y, z], labelfont = [arial, bold, 12], title =$ "ANIMATE-NEWTON 3D Επαναληπτική διαδικασία\nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ", $titlefont = [arial, bold, 14]\big), \phi = 0\,..6.30, 0.21\Big), insequence = true\Big)$:

> $display(P, ANIM, orientation = [-90, 0, 0], scaling = constrained, axes = none)$:

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