

>

Θέμα :

**Σχηματική αναπαράσταση ενός Ανιχνευτή
Βαρυτικών Κυμάτων ,
ως ελατηρίου με δύο ταυτοικές μάζες στις
άκρες του .**

$$\begin{bmatrix} \vec{e1} \\ \vec{e2} \\ \vec{e3} \end{bmatrix} = \begin{bmatrix} \cos(a) \hat{i} + \cos(b) \hat{j} + \cos(c) \hat{k} \\ \cos(d) \hat{i} + \cos(e) \hat{j} + \cos(f) \hat{k} \\ \cos(g) \hat{i} + \cos(h) \hat{j} + \cos(i) \hat{k} \end{bmatrix}$$

$$\vec{e1} = \cos(a) \hat{i} + \cos(b) \hat{j} + \cos(c) \hat{k}$$

$$\vec{e2} = \cos(d) \hat{i} + \cos(e) \hat{j} + \cos(f) \hat{k}$$

$$\vec{e3} = \cos(g) \hat{i} + \cos(h) \hat{j} + \cos(i) \hat{k}$$

$$\begin{bmatrix} \hat{i} \\ \hat{j} \\ \hat{k} \end{bmatrix} = \begin{bmatrix} \cos(a) \vec{e1} + \cos(d) \vec{e2} + \cos(g) \vec{e3} \\ \cos(b) \vec{e1} + \cos(e) \vec{e2} + \cos(h) \vec{e3} \\ \cos(c) \vec{e1} + \cos(f) \vec{e2} + \cos(i) \vec{e3} \end{bmatrix}$$

$$\hat{i} = \cos(a) \vec{e1} + \cos(d) \vec{e2} + \cos(g) \vec{e3}$$

$$\hat{j} = \cos(b) \vec{e1} + \cos(e) \vec{e2} + \cos(h) \vec{e3}$$

$$\hat{k} = \cos(c) \vec{e1} + \cos(f) \vec{e2} + \cos(i) \vec{e3}$$

ΜΕΤΑΣΧΗΜΑΤΙΣΜΟΣ ΣΥΣΤΗΜΑΤΩΝ

$$A_x = A_{e1} \cos(a) + A_{e2} \cos(d) + A_{e3} \cos(g)$$

$$A_y = A_{e1} \cos(b) + A_{e2} \cos(e) + A_{e3} \cos(h)$$

$$A_z = A_{e1} \cos(c) + A_{e2} \cos(f) + A_{e3} \cos(i)$$

Να χαράξουμε Ελικοειδή Καμπύλη (Κυκλική ή Ελλειπτική)
της οποίας τα δύο (2) άκρα A1 , B1 να περνούν από δύο (2) σημεία A , B αντίστοιχα στο χώρο .

**ΓΙΑ ΑΛΛΑΓΗ ΔΙΕΥΘΥΝΣΗΣ ΤΗΣ ΔΙΕΓΕΡΣΗΣ , ΑΛΛΑΖΟΥΜΕ ΤΟ : $1 - h \cdot \sin(\Theta)$
ΣΕ : $1 + h \cdot \sin(\Theta)$**

>

```
> with(Physics[Vectors])
[&x, `+`, `;`, ChangeBasis, ChangeCoordinates, Component, Curl, DirectionalDiff,
 Divergence, Gradient, Identify, Laplacian, ∇, Norm, Setup, diff]
> Setup(mathematicalnotation=true)
```

[mathematicalnotation = true] (2)

> $\text{with}(\text{plots}) :$

> $a := 4$

$$a := 4 \quad (3)$$

> $b := 2$

$$b := 2 \quad (4)$$

Αριθμός Ελικώσεων n

> $n := 5$

$$n := 5 \quad (5)$$

Πλάτος Ταλάντωσης h

> $h := 0.5$

$$h := 0.5 \quad (6)$$

> $xA := -2$

$$xA := -2 \quad (7)$$

> $yA := 2$

$$yA := 2 \quad (8)$$

> $zA := 1$

$$zA := 1 \quad (9)$$

> $xB := 1$

$$xB := 1 \quad (10)$$

> $yB := 4$

$$yB := 4 \quad (11)$$

> $zB := 7$

$$zB := 7 \quad (12)$$

> $xK := \frac{(xA + xB)}{2}$

$$xK := -\frac{1}{2} \quad (13)$$

> $yK := \frac{(yA + yB)}{2}$

$$yK := 3 \quad (14)$$

> $zK := \frac{(zA + zB)}{2}$

$$zK := 4 \quad (15)$$

> $K := [xK, yK, zK]$

$$K := \left[-\frac{1}{2}, 3, 4 \right] \quad (16)$$

> $MHKOSAK := \sqrt{(A[1] - K[1])^2 + (A[2] - K[2])^2 + (A[3] - K[3])^2}$

$$MHKOSAK := \frac{7}{2} \quad (17)$$

```

> A := [xA, yA, zA]                                A := [-2, 2, 1]          (18)
=>
> B := [xB, yB, zB]                                B := [1, 4, 7]          (19)
=>
> MHKOSAB :=  $\sqrt{(A[1]-B[1])^2 + (A[2]-B[2])^2 + (A[3]-B[3])^2}$ 
=>
> MHKOSAB := 7                                     (20)
=>
> BHMA :=  $\frac{MHKOSAB}{n}$                          BHMA :=  $\frac{7}{5}$            (21)
=>
> AB := spacecurve([A[1] + λ · (B[1] - A[1]), A[2] + λ · (B[2] - A[2]), A[3] + λ · (B[3] - A[3])], λ = 0 .. 1) :
=>
> rA_ := A[1] · _i + A[2] · _j + A[3] · _k
=>
> rA := -2  $\hat{i}$  + 2  $\hat{j}$  +  $\hat{k}$                    (22)
=>
> rB_ := B[1] · _i + B[2] · _j + B[3] · _k
=>
> rB :=  $\hat{i}$  + 4  $\hat{j}$  + 7  $\hat{k}$                    (23)
=>
>



ΑΡΧΗ ΤΟΥ  $\vec{e1}, \vec{e2}, \vec{e3}$  ΤΟ ΣΗΜΕΙΟ  $A$ ,  $\vec{e3} = \frac{\vec{AB}}{|\vec{AB}|}$


=>
> HELIX :=  $\left[ -a + a \cdot \cos(\phi), + b \cdot \sin(\phi), \frac{BHMA \cdot \phi}{2 \cdot \text{Pi}} \right]$ 
=>
> e3_ :=  $\frac{(rB_ - rA_)}{Norm(rB_ - rA_)}$ 
=>
> e3 :=  $\frac{(3 \hat{i} + 2 \hat{j} + 6 \hat{k}) \sqrt{49}}{49}$           (24)
=>
> Norm(e3_)                                         1
=>
> C := [xC, yC, zC]                                C := [xC, yC, zC]          (26)
=>
> rC_ := C[1] · _i + C[2] · _j + C[3] · _k
=>
> rC := xC  $\hat{i}$  + yC  $\hat{j}$  + zC  $\hat{k}$                   (27)
=>
> simplify((rC_ - rA_).e3_ = 0)
=>
>  $\frac{3 \cdot xC}{7} - \frac{4}{7} + \frac{2 \cdot yC}{7} + \frac{6 \cdot zC}{7} = 0$           (28)
=>
> numer(lhs((28))) = 0
=>
> 3 xC - 4 + 2 yC + 6 zC = 0                      (29)
=>

SOS : ΕΠΑΛΗΘΕΥΤΑΙ και γιά : [xC=-4, yC=2, zC=2]

```

$$> rCl_1 := -4 \cdot i + 2 \cdot j + 2 \cdot k \quad \overrightarrow{rCl} := -4 \hat{i} + 2 \hat{j} + 2 \hat{k} \quad (30)$$

$$> e1_1 := \frac{(rCl_1 - rA_1)}{\text{Norm}(rCl_1 - rA_1)} \quad \overrightarrow{e1} := \frac{(-2 \hat{i} + \hat{k}) \sqrt{5}}{5} \quad (31)$$

$$> \text{Norm}(e1_1) \quad 1 \quad (32)$$

$$> e2_1 := e3_1 \times e1_1 \quad \overrightarrow{e2} := \frac{2 \sqrt{49} \sqrt{5} \hat{i}}{245} - \frac{3 \sqrt{49} \sqrt{5} \hat{j}}{49} + \frac{4 \sqrt{49} \sqrt{5} \hat{k}}{245} \quad (33)$$

$$> \text{Norm}(e2_1) \quad 1 \quad (34)$$

$$\text{HELIX} := \left[-a + a \cdot \cos(\phi), + b \cdot \sin(\phi), \frac{BHMA \cdot \phi}{2 \cdot \text{Pi}} \right]$$

ΕΠΙΒΟΛΗ ΔΙΕΓΕΡΣΗΣ SINUS ΣΤΟ ΣΗΜΕΙΟ B στην διεύθυνση του άξονα BA .

$$> (-a + a \cdot \cos(\phi)) \cdot e1_1 + (b \cdot \sin(\phi)) \cdot e2_1 + \left(\frac{BHMA}{2 \cdot \text{Pi}} \cdot \phi \cdot (0.5 - h \cdot \sin(\Theta)) \right) \cdot e3_1 + K[1] \cdot i + K[2] \cdot j + K[3] \cdot k$$

$$(1.600000000 \sqrt{5} - 1.600000000 \cos(\phi) \sqrt{5} + 0.01632653061 \sin(\phi) \sqrt{49} \sqrt{5} + 0.006820926132 \phi \sqrt{49} - 0.006820926132 \phi \sin(\Theta) \sqrt{49} - 0.5000000000) \hat{i} + (-0.1224489796 \sin(\phi) \sqrt{49} \sqrt{5} + 0.004547284088 \phi \sqrt{49} - 0.004547284088 \phi \sin(\Theta) \sqrt{49} + 3.) \hat{j} + (-0.8000000000 \sqrt{5} + 0.8000000000 \cos(\phi) \sqrt{5} + 0.03265306122 \sin(\phi) \sqrt{49} \sqrt{5} + 0.01364185226 \phi \sqrt{49} - 0.01364185226 \phi \sin(\Theta) \sqrt{49} + 4.) \hat{k}$$

$$(35)$$

$$> \text{HELIXBK} := [\text{Component}(35, 1), \text{Component}(35, 2), \text{Component}(35, 3)]$$

$$\text{HELIXBK} := [1.600000000 \sqrt{5} - 1.600000000 \cos(\phi) \sqrt{5} + 0.01632653061 \sin(\phi) \sqrt{49} \sqrt{5} + 0.006820926132 \phi \sqrt{49}]$$

$$(36)$$

$$\begin{aligned}
& -0.006820926132 \phi \sin(\Theta) \sqrt{49} - 0.5000000000, -0.1224489796 \sin(\phi) \sqrt{49} \sqrt{5} \\
& + 0.004547284088 \phi \sqrt{49} - 0.004547284088 \phi \sin(\Theta) \sqrt{49} + 3., \\
& -0.8000000000 \sqrt{5} + 0.8000000000 \cos(\phi) \sqrt{5} + 0.03265306122 \sin(\phi) \sqrt{49} \sqrt{5} \\
& + 0.01364185226 \phi \sqrt{49} - 0.01364185226 \phi \sin(\Theta) \sqrt{49} + 4.]
\end{aligned}$$

>

ΕΠΙΒΟΛΗ ΔΙΕΓΕΡΣΗΣ SINUS ΣΤΟ ΣΗΜΕΙΟ A στην διεύθυνση του άξονα AB .

>

$$\begin{aligned}
& > (-a + a \cdot \cos(-\phi)) \cdot e1_+ + (b \cdot \sin(-\phi)) \cdot e2_+ + \left(\frac{BHMA}{2 \cdot \text{Pi}} \cdot \phi \cdot (-0.5 \right. \\
& \quad \left. + h \cdot \sin(\Theta)) \right) \cdot e3_+ + K[1] \cdot i + K[2] \cdot j + K[3] \cdot k \\
& (1.600000000 \sqrt{5} - 1.600000000 \cos(\phi) \sqrt{5} - 0.01632653061 \sin(\phi) \sqrt{49} \sqrt{5} \quad (37) \\
& - 0.006820926132 \phi \sqrt{49} + 0.006820926132 \phi \sin(\Theta) \sqrt{49} - 0.5000000000) \hat{i} \\
& + (0.1224489796 \sin(\phi) \sqrt{49} \sqrt{5} - 0.004547284088 \phi \sqrt{49} \\
& + 0.004547284088 \phi \sin(\Theta) \sqrt{49} + 3.) \hat{j} + (-0.8000000000 \sqrt{5} \\
& + 0.8000000000 \cos(\phi) \sqrt{5} - 0.03265306122 \sin(\phi) \sqrt{49} \sqrt{5} \\
& - 0.01364185226 \phi \sqrt{49} + 0.01364185226 \phi \sin(\Theta) \sqrt{49} + 4.) \hat{k}
\end{aligned}$$

>

> *HELIXAK* := [Component((37), 1), Component((37), 2), Component((37), 3)]

$$\begin{aligned}
& HELIXAK := [1.600000000 \sqrt{5} - 1.600000000 \cos(\phi) \sqrt{5} \quad (38) \\
& - 0.01632653061 \sin(\phi) \sqrt{49} \sqrt{5} - 0.006820926132 \phi \sqrt{49} \\
& + 0.006820926132 \phi \sin(\Theta) \sqrt{49} - 0.5000000000, 0.1224489796 \sin(\phi) \sqrt{49} \sqrt{5} \\
& - 0.004547284088 \phi \sqrt{49} + 0.004547284088 \phi \sin(\Theta) \sqrt{49} + 3., \\
& -0.8000000000 \sqrt{5} + 0.8000000000 \cos(\phi) \sqrt{5} - 0.03265306122 \sin(\phi) \sqrt{49} \sqrt{5} \\
& - 0.01364185226 \phi \sqrt{49} + 0.01364185226 \phi \sin(\Theta) \sqrt{49} + 4.]
\end{aligned}$$

>

> *animHELIXBK* := *animate*(*spacecurve*, [*HELIXBK*, $\phi = 0 .. n \cdot 2 \cdot \text{Pi}$, *color* = blue], $\Theta = 0 .. 2 \cdot \text{Pi}$, *frames* = 101) :

> *animHELIXAK* := *animate*(*spacecurve*, [*HELIXAK*, $\phi = 0 .. n \cdot 2 \cdot \text{Pi}$, *color* = blue], $\Theta = 0 .. 2 \cdot \text{Pi}$, *frames* = 101) :

>

ΤΟ ΣΗΜΕΙΟ A στο [e1,e2,e3]

$$> A1 := [0, 0, MHKOSAB \cdot (0 + h \cdot \sin(\Theta))] \quad (39)$$

$$A1 := [0, 0, 3.5 \sin(\Theta)]$$

>

ΤΟ ΣΗΜΕΙΟ B στο [e1,e2,e3]

$$> B1 := [0, 0, MHKOSAB \cdot (1 - h \cdot \sin(\Theta))]$$

```


$$BI := [0, 0, 7 - 3.5 \sin(\Theta)] \quad (40)$$


$$> AIBI := [BI[1] + \lambda \cdot (AI[1] - BI[1]), BI[2] + \lambda \cdot (AI[2] - BI[2]), BI[3] + \lambda \cdot (AI[3] - BI[3])]$$


$$AIBI := [0, 0, 7 - 3.5 \sin(\Theta) + \lambda (7.0 \sin(\Theta) - 7)] \quad (41)$$


$$> AIBI[1] \quad 0 \quad (42)$$


$$> AIBI[2] \quad 0 \quad (43)$$


$$> AIBI[3] \quad 7 - 3.5 \sin(\Theta) + \lambda (7.0 \sin(\Theta) - 7) \quad (44)$$


$$>$$


$$> linAB := AIBI[1] \cdot e1_+ + AIBI[2] \cdot e2_+ + AIBI[3] \cdot e3_+ + A[1] \cdot \underline{i} + A[2] \cdot \underline{j} + A[3] \cdot \underline{k}$$


$$linAB := (0.4285714286 \sqrt{49} - 0.2142857143 \sin(\Theta) \sqrt{49} + 0.4285714287 \lambda \sin(\Theta) \sqrt{49} - 0.4285714286 \lambda \sqrt{49} - 2.) \hat{i}$$


$$+ (0.2857142857 \sqrt{49} - 0.1428571429 \sin(\Theta) \sqrt{49} + 0.2857142858 \lambda \sin(\Theta) \sqrt{49} - 0.2857142857 \lambda \sqrt{49} + 2.) \hat{j}$$


$$+ (0.8571428571 \sqrt{49} - 0.4285714286 \sin(\Theta) \sqrt{49} + 0.8571428574 \lambda \sin(\Theta) \sqrt{49} - 0.8571428571 \lambda \sqrt{49} + 1.) \hat{k} \quad (45)$$


$$>$$


$$> ABxyz := [Component(45), 1), Component(45), 2), Component(45), 3)]$$


$$ABxyz := [0.4285714286 \sqrt{49} - 0.2142857143 \sin(\Theta) \sqrt{49} + 0.4285714287 \lambda \sin(\Theta) \sqrt{49} - 0.4285714286 \lambda \sqrt{49} - 2., 0.2857142857 \sqrt{49} - 0.1428571429 \sin(\Theta) \sqrt{49} + 0.2857142858 \lambda \sin(\Theta) \sqrt{49} - 0.2857142857 \lambda \sqrt{49} + 2., 0.8571428571 \sqrt{49} - 0.4285714286 \sin(\Theta) \sqrt{49} + 0.8571428574 \lambda \sin(\Theta) \sqrt{49} - 0.8571428571 \lambda \sqrt{49} + 1.] \quad (46)$$


$$> evalf(subs(\{\lambda=0, \Theta=0\}, ABxyz))$$


$$[1.000000000, 4.000000000, 7.000000000] \quad (47)$$


$$> evalf(subs(\{\lambda=1, \Theta=0\}, ABxyz))$$


$$[-2., 2., 1.] \quad (48)$$


$$>$$


$$> animAB := animate(spacecurve, [ABxyz, \lambda=0..1, color=gold, thickness=3, linestyle=1], \Theta=0..2\cdot Pi, frames=101) :$$


$$>$$


$$> AB := spacecurve([A[1] + \lambda \cdot (B[1] - A[1]), A[2] + \lambda \cdot (B[2] - A[2]), A[3] + \lambda \cdot (B[3] - A[3])], \lambda=0..1, color=red, linestyle=4) :$$


$$> pA := pointplot3d(A, symbol=solidcircle, symbolsize=10) :$$


$$> pB := pointplot3d(B, symbol=solidcircle, symbolsize=10) :$$


$$> pK := pointplot3d(K, symbol=solidcircle, symbolsize=10) :$$


$$>$$


$$> OO := pointplot3d([0, 0, 0], symbol=solidcircle, symbolsize=10) :$$


```

```

> axX := spacecurve([x, 0, 0], x = -5.5 .. 5.5, linestyle = 3, thickness = 1, color = blue) :
> axY := spacecurve([0, y, 0], y = -6.5 .. 6.5, linestyle = 3, thickness = 1, color = blue) :
> axZ := spacecurve([0, 0, z], z = 0 .. 7.5, linestyle = 3, thickness = 2, color = blue) :
> axZ1 := spacecurve([0, 0, z], z = 0 .. 5.5, linestyle = 3, thickness = 2, color = blue) :
> ARaxX := arrow([5.5, 0, 0], [0.5, 0, 0], width = 0.1, head_length = 0.3, shape
    = cylindrical_arrow, color = blue) :
> ARaxY := arrow([0, 6.5, 0], [0, 0.5, 0], width = 0.1, head_length = 0.3, shape
    = cylindrical_arrow, color = blue) :
> ARaxZ := arrow([0, 0, 7.5], [0, 0, 0.5], width = 0.1, head_length = 0.3, shape
    = cylindrical_arrow, color = blue) :
> ARaxZ1 := arrow([0, 0, 5.5], [0, 0, 0.5], width = 0.1, head_length = 0.3, shape
    = cylindrical_arrow, color = blue) :
> tX := textplot3d([6.1, 0.0, 0, "x"], color = gold, font = [arial, bold, 14]) :
> tY := textplot3d([0, 7.2, 0, "y"], color = gold, font = [arial, bold, 14]) :
> tZ := textplot3d([0, 0, 8.2, "z"], color = gold, font = [arial, bold, 14]) :
> tZ1 := textplot3d([0, 0, 6.2, "z"], color = gold, font = [arial, bold, 14]) :
>
>

```

ΕΠΙΒΟΛΗ ΔΙΕΓΕΡΣΗΣ ΣΤΟ ΣΗΜΕΙΟ Β στην διεύθυνση του άξονα ΒΑ .

```

> B1[1]·e1_ + B1[2]·e2_ + B1[3]·e3_ + A[1]·_i + A[2]·_j + A[3]·_k
(0.4285714286 √49 - 0.2142857143 sin(Θ) √49 - 2.) ^ i + (0.2857142857 √49
- 0.1428571429 sin(Θ) √49 + 2.) ^ j + (0.8571428571 √49
- 0.4285714286 sin(Θ) √49 + 1.) ^ k
> [Component((49), 1), Component((49), 2), Component((49), 3)]
[0.4285714286 √49 - 0.2142857143 sin(Θ) √49 - 2., 0.2857142857 √49
- 0.1428571429 sin(Θ) √49 + 2., 0.8571428571 √49 - 0.4285714286 sin(Θ) √49
+ 1.]
> animB := animate(pointplot3d, [(50), symbol = solidcircle, symbolsize = 15, color = green],
Θ = 0 .. 2·Pi, frames = 101, trace = 00) :
>
>
```

ΕΠΙΒΟΛΗ ΔΙΕΓΕΡΣΗΣ ΣΤΟ ΣΗΜΕΙΟ Α στην διεύθυνση του άξονα ΑΒ .

```

> A1[1]·e1_ + A1[2]·e2_ + A1[3]·e3_ + A[1]·_i + A[2]·_j + A[3]·_k
(0.2142857143 sin(Θ) √49 - 2.) ^ i + (0.1428571429 sin(Θ) √49 + 2.) ^ j
+ (0.4285714286 sin(Θ) √49 + 1.) ^ k
> [Component((51), 1), Component((51), 2), Component((51), 3)]

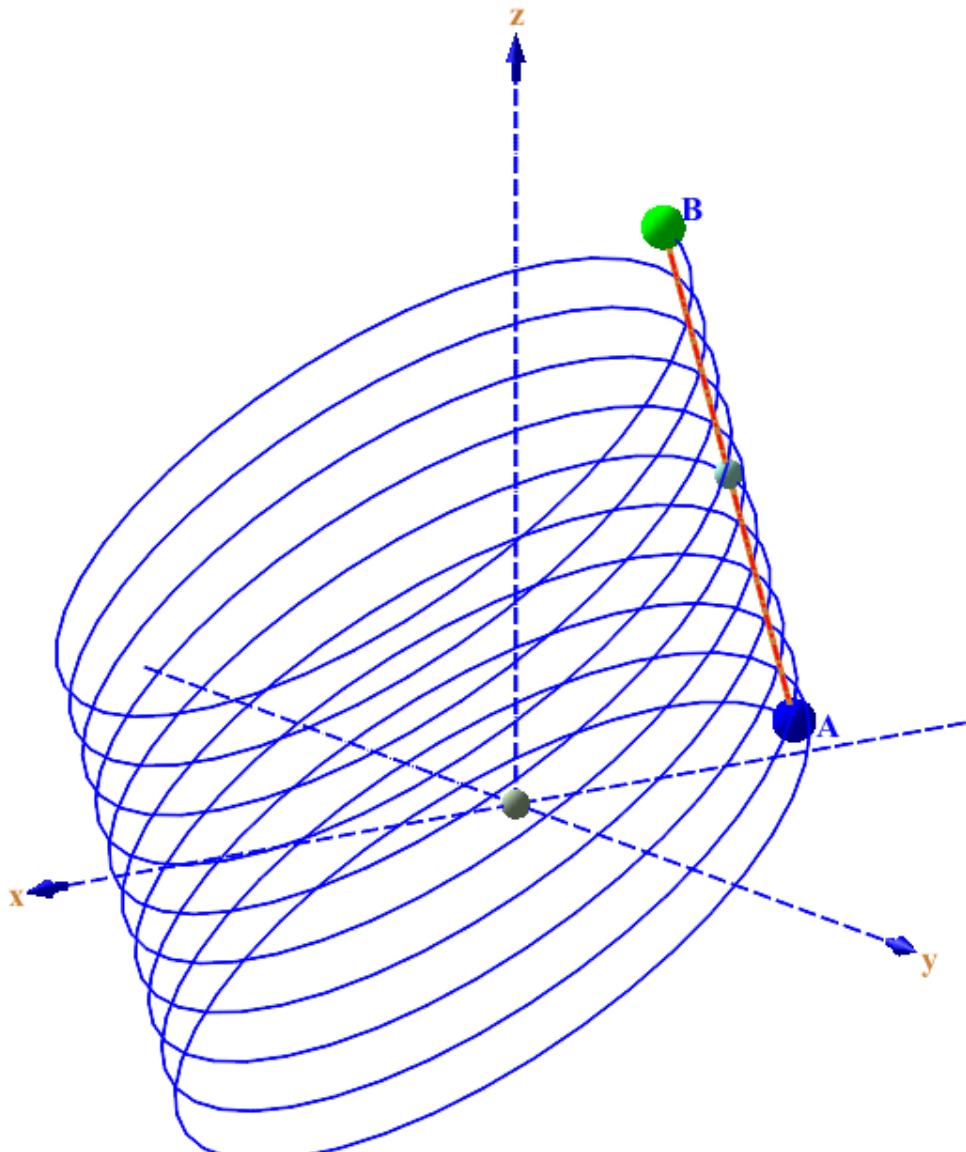
```

(52)

$$[0.2142857143 \sin(\Theta) \sqrt{49} - 2., 0.1428571429 \sin(\Theta) \sqrt{49} + 2., \\ 0.4285714286 \sin(\Theta) \sqrt{49} + 1.] \quad (52)$$

```
> animA := animate(pointplot3d, [(52), symbol=solidcircle, symbolsize=15, color=blue], Θ
= 0 .. 2·Pi, frames = 101, trace = 00) :
>
> display(animHELIXBK, animHELIXAK, animA, animB, animAB, OO, axX, axY, axZ, ARaxX,
ARaxY, ARaxZ, tX, tY, tZ, pA, pB, pK, AB, tA, tB, axes = none, scaling = constrained, title
= "ANIMATION\nΣΧΗΜΑΤΙΚΗ ΑΝΑΠΑΡΑΣΤΑΣΗ ΑΝΙΧΝΕΥΤΗ ΒΑΡΥΤΙΚΩΝ
ΚΥΜΑΤΩΝ\nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ", titlefont = [arial, bold, 14], orientation
= [55, 75, 0]) :
```

ANIMATION
ΣΧΗΜΑΤΙΚΗ ΑΝΑΠΑΡΑΣΤΑΣΗ ΑΝΙΧΝΕΥΤΗ ΒΑΡΥΤΙΚΩΝ ΚΥΜΑΤΩΝ
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ



```
>
>
```