```
> with(plots) :
> with(Physics[Vectors])
```

[&x, `+`, `.`, ChangeBasis, ChangeCoordinates, Component, Curl, DirectionalDiff, Divergence, Gradient, Identify, Laplacian, ∇, Norm, Setup, diff] **(1)**

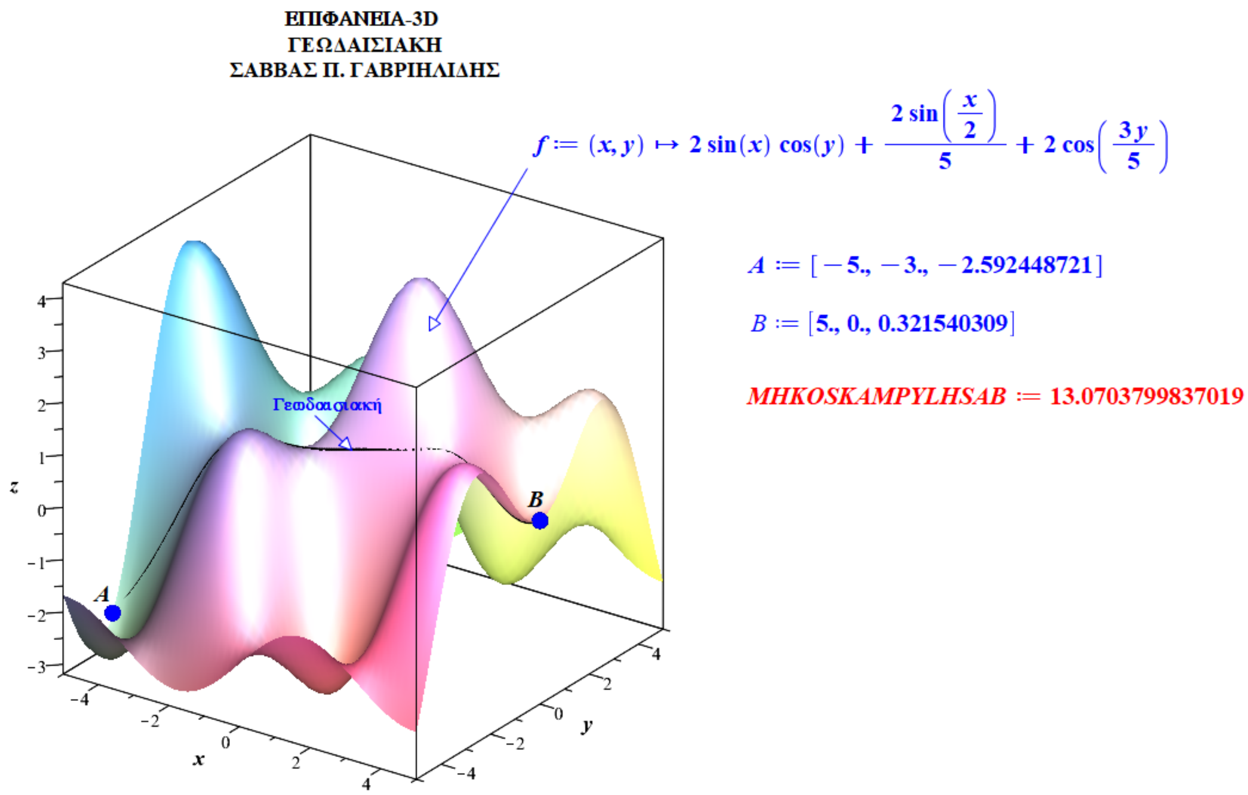
```
> Setup(mathematicalnotation = true)
```

[mathematicalnotation = true] **(2)**

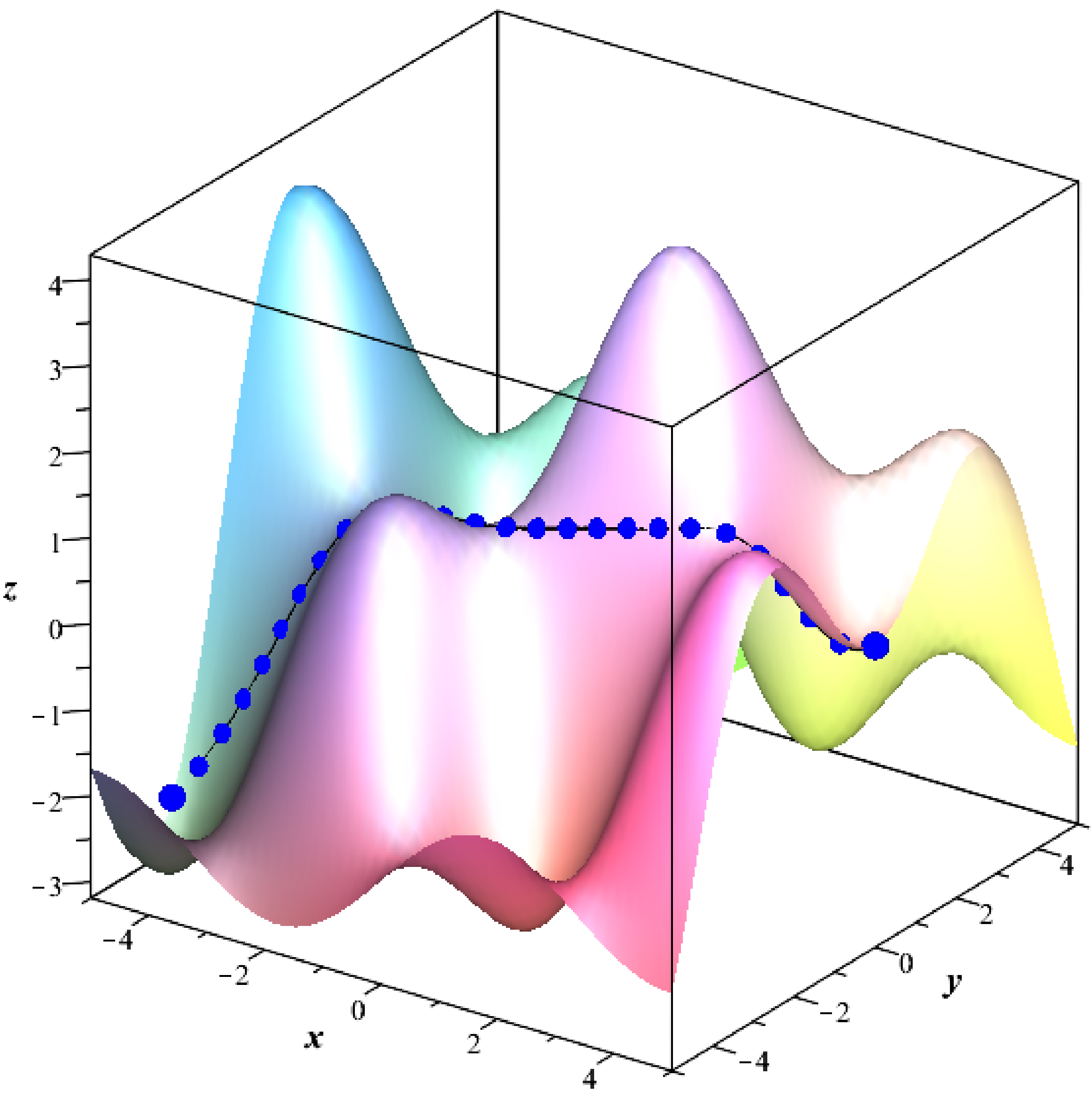
```
>
```

Θέμα : Να βρούμε την συντομότερη διαδρομή (Γεωδαισιακή καμπύλη) ανάμεσα στα σημεία A , B της επιφάνειας που ορίζεται από την εξίσωση :

$$f := (x, y) \rightarrow 2\cdot\sin(x)\cdot\cos(y) + \frac{2}{5}\cdot\sin\left(\frac{x}{2}\right) + 2\cdot\cos\left(\frac{3\cdot y}{5}\right)$$



ΕΠΙΦΑΝΕΙΑ-3D
ΓΕΩΔΑΙΣΙΑΚΗ-ANIMATION
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ



> $f := (x, y) \to 2 \cdot \sin(x) \cdot \cos(y) + \frac{2}{5} \cdot \sin\left(\frac{x}{2}\right) + 2 \cdot \cos\left(\frac{3 \cdot y}{5}\right)$

$$f := (x, y) \mapsto 2 \sin(x) \cos(y) + \frac{2 \sin\left(\frac{x}{2}\right)}{5} + 2 \cos\left(\frac{3\, y}{5}\right) \quad \textbf{(3)}$$

> $A := evalf([-5, -3, f(-5, -3)])$

$$A := [-5., -3., -2.592448721] \quad \textbf{(4)}$$

> $B := evalf([5, 0, f(5, 0)])$

$$B := [5., 0., 0.321540309] \quad \textbf{(5)}$$

> $P2 := pointplot3d(A, symbol = solidsphere, symbolsize = 20, color = blue)$:

> $P3 := pointplot3d(B, symbol = solidsphere, symbolsize = 20, color = blue)$:

> $P1 := plot3d(f(x, y), x = -5 ..5, y = -5 ..5, style = surface)$:

> display(P1, P2, P3, orientation = [-55, 65], axes = boxed, lightmodel = light1) :

> r_ := x·_i + y·_j + f(x, y)·_k

$$\vec{r} := x\,\hat{i} + y\,\hat{j} + \left(2\sin(x)\cos(y) + \frac{2\sin\left(\frac{x}{2}\right)}{5} + 2\cos\left(\frac{3y}{5}\right)\right)\hat{k} \tag{6}$$

> Hrb := [Component(r_, 1), Component(r_, 2), Component(r_, 3)]

$$Hrb := \left[x, y, 2\sin(x)\cos(y) + \frac{2\sin\left(\frac{x}{2}\right)}{5} + 2\cos\left(\frac{3y}{5}\right)\right] \tag{7}$$

> E := simplify(diff(r_, x).diff(r_, x))

$$E := 4\cos(x)^2\cos(y)^2 + \frac{4\cos(x)\cos(y)\cos\left(\frac{x}{2}\right)}{5} + \frac{\cos\left(\frac{x}{2}\right)^2}{25} + 1 \tag{8}$$

> F := diff(r_, x).diff(r_, y)

$$F := -4\cos(x)\cos(y)\sin(x)\sin(y) - \frac{12\cos(x)\cos(y)\sin\left(\frac{3y}{5}\right)}{5} - \frac{2\cos\left(\frac{x}{2}\right)\sin(x)\sin(y)}{5} - \frac{6\cos\left(\frac{x}{2}\right)\sin\left(\frac{3y}{5}\right)}{25} \tag{9}$$

> G := simplify(diff(r_, y).diff(r_, y))

$$G := 1 + \left(-2\sin(x)\sin(y) - \frac{6\sin\left(\frac{3y}{5}\right)}{5}\right)^2 \tag{10}$$

> E1 := subs({x = x(t), y = y(t)}, E)

$$E1 := 4\cos(x(t))^2\cos(y(t))^2 + \frac{4\cos(x(t))\cos(y(t))\cos\left(\frac{x(t)}{2}\right)}{5} + \frac{\cos\left(\frac{x(t)}{2}\right)^2}{25} + 1 \tag{11}$$

> F1 := subs({x = x(t), y = y(t)}, F)

$$F1 := -4\cos(x(t))\cos(y(t))\sin(x(t))\sin(y(t)) - \frac{12\cos(x(t))\cos(y(t))\sin\left(\frac{3y(t)}{5}\right)}{5} - \frac{2\cos\left(\frac{x(t)}{2}\right)\sin(x(t))\sin(y(t))}{5} - \frac{6\cos\left(\frac{x(t)}{2}\right)\sin\left(\frac{3y(t)}{5}\right)}{25} \tag{12}$$

> G1 := subs({x = x(t), y = y(t)}, G)

$$G1 := 1 + \left(-2\sin(x(t))\sin(y(t)) - \frac{6\sin\left(\frac{3y(t)}{5}\right)}{5}\right)^2 \tag{13}$$

>

ΔΙΑΦΟΡΙΚΕΣ ΕΞΙΣΩΣΕΙΣ ΓΕΩΔΑΙΣΙΑΚΩΝ :

$$\begin{cases} \dfrac{d}{dt}\left(E\cdot\dot{u}+F\cdot\dot{v}\right)=\dfrac{1}{2}\left(E_u\cdot\dot{u}^2+2F_u\cdot\dot{u}\cdot\dot{v}+G_u\,\dot{v}^2\right) \\ \dfrac{d}{dt}\left(F\cdot\dot{u}+G\cdot\dot{v}\right)=\dfrac{1}{2}\left(E_v\cdot\dot{u}^2+2F_v\cdot\dot{u}\cdot\dot{v}+G_v\,\dot{v}^2\right) \end{cases}$$

\>

\> $ode1 := diff((E1\cdot diff(x(t),t)+F1\cdot diff(y(t),t)),t)=\frac{1}{2}\cdot\left(diff(E1,x(t))\cdot(diff(x(t),t))^2+2\cdot diff(F1,x(t))\cdot diff(x(t),t)\cdot diff(y(t),t)+diff(G1,x(t))\cdot(diff(y(t),t))^2\right):$

\> $ode2 := diff((F1\cdot diff(x(t),t)+G1\cdot diff(y(t),t)),t)=\frac{1}{2}\cdot\left(diff(E1,y(t))\cdot(diff(x(t),t))^2+2\cdot diff(F1,y(t))\cdot diff(x(t),t)\cdot diff(y(t),t)+diff(G1,y(t))\cdot(diff(y(t),t))^2\right):$

\> $sys := ode1, ode2:$

\> $ics := x(0)=-5, y(0)=-3, x(1)=5, y(1)=0$

$ics := x(0)=-5, y(0)=-3, x(1)=5, y(1)=0$ **(14)**

\> $SOL := dsolve(\{sys, ics\}, \{x(t), y(t)\}, numeric, output=listprocedure, abserr=10^{-5})$

$SOL :=$ [t = **proc**(t) ... **end proc**, $x(t)$ = **proc**(t) ... **end proc**, $\dot{x}(t)$ = **proc**(t) ... **end proc**, $y(t)$ = **proc**(t) ... **end proc**, $\dot{y}(t)$ = **proc**(t) ... **end proc**] **(15)**

\> $SOL(0)$

$[t(0)=0, x(t)(0)=-5.00000000000000, (\dot{x}(t))(0)=6.47248803720619, y(t)(0)=-3.00000000000000, (\dot{y}(t))(0)=8.48904622181207]$ **(16)**

\> $SOL(1)$

$[t(1)=1, x(t)(1)=5.00000000000000, (\dot{x}(t))(1)=8.00980608658472, y(t)(1)=0., (\dot{y}(t))(1)=9.80327836631790]$ **(17)**

\> $rhs(SOL[2](t))$

$x(t)(t)$ **(18)**

\>

\> $KAMPYLH := subs(\{x=rhs(SOL[2](t)), y=rhs(SOL[4](t))\}, Hrb)$

$$KAMPYLH := \left[x(t)(t), y(t)(t), 2\sin(x(t)(t))\cos(y(t)(t))+\frac{2\sin\left(\frac{x(t)(t)}{2}\right)}{5}+2\cos\left(\frac{3\,y(t)(t)}{5}\right)\right]$$ **(19)**

\> $SHMEIA := evalf(seq(KAMPYLH, t=0..1, 0.01)):$

\> $SHMEIA[1]$

$[-5.00000000000000, -3.00000000000000, -2.59244872047682]$ **(20)**

\> $SHMEIA[1][1]$

-5.00000000000000 **(21)**

\> $SHMEIA[1][2]$

-3.00000000000000 **(22)**

> *SHMEIA*[1][3]

$$-2.59244872047682 \qquad (23)$$

> *SHMEIA*[2][1]

$$-4.93314276911214 \qquad (24)$$

> *SHMEIA*[2][2]

$$-2.91981605726261 \qquad (25)$$

> *SHMEIA*[2][3]

$$-2.51384624246586 \qquad (26)$$

> $MHKOSKAMPYLHSAB := \sum_{i=1}^{100} \mathrm{sqrt}\left((SHMEIA[i][1] - SHMEIA[i+1][1])^2 + (SHMEIA[i][2] - SHMEIA[i+1][2])^2 + (SHMEIA[i][3] - SHMEIA[i+1][3])^2 \right)$

$$MHKOSKAMPYLHSAB := 13.0703799837019 \qquad (27)$$

> **animSHMEIA ≔ animate(pointplot3d, [KAMPYLH , color = blue, symbol = solidsphere, symbolsize = 15], t = 0 ..1, frames = 41, trace = 20) :**

> *KAMPYLHAB* ≔ *spacecurve*(*KAMPYLH*, *t* = 0 ..1, *thickness* = 2, *color* = *black*, *linestyle* = 1) :

> *display*(*P1*, *P2*, *P3*, *KAMPYLHAB*, *animSHMEIA*, *labels* = [*x*, *y*, *z*], *labelfont* = [*arial*, *bold*, 14], *title* = "ΕΠΙΦΑΝΕΙΑ-3D \nΓΕΩΔΑΙΣΙΑΚΗ-ANIMATION\nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ", *titlefont* = [*arial*, *bold*, 14], *orientation* = [-55, 65], *axes* = *boxed*, *lightmodel* = *light1*) :

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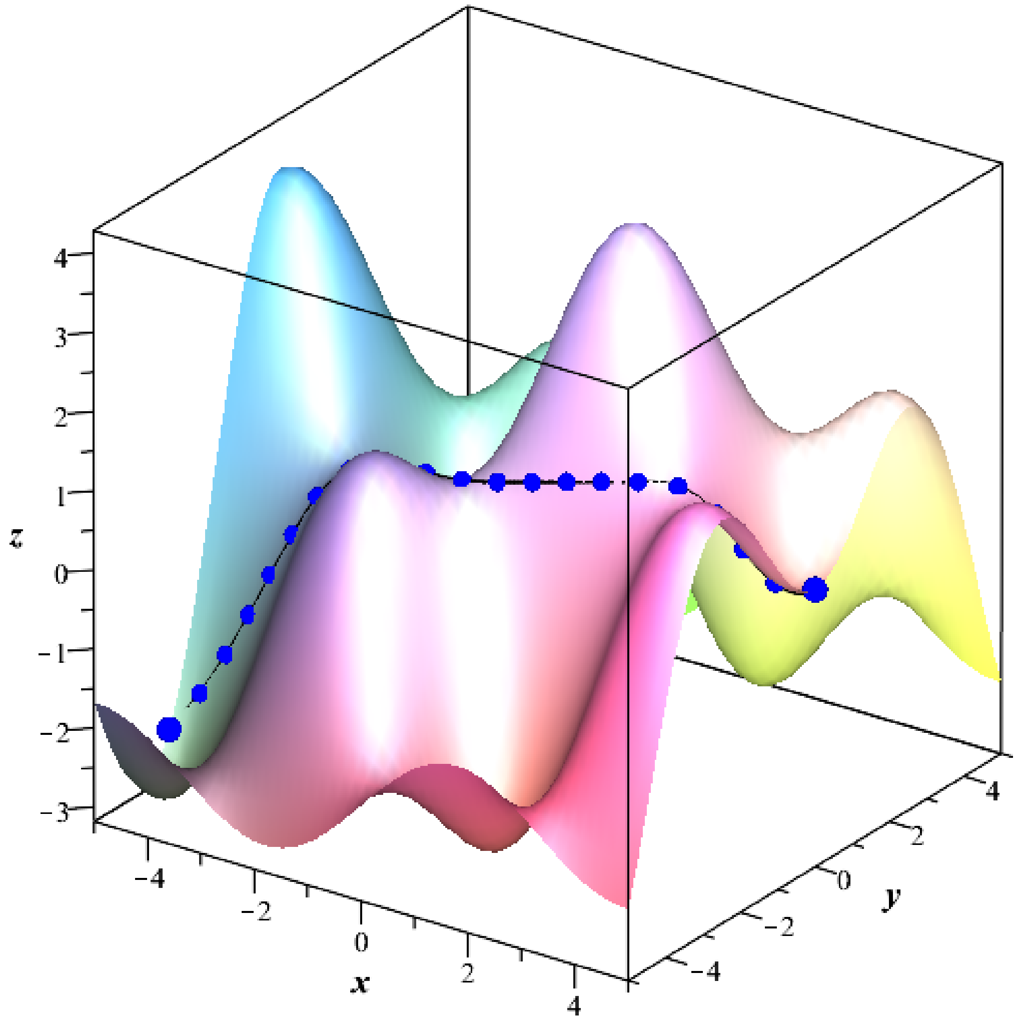

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