

ΕΠΙΦΑΝΕΙΑ ΗΜΙΤΟΝΟΕΙΔΗΣ ΕΚ ΠΕΡΙΣΤΡΟΦΗΣ - SAGRADA FAMILIA

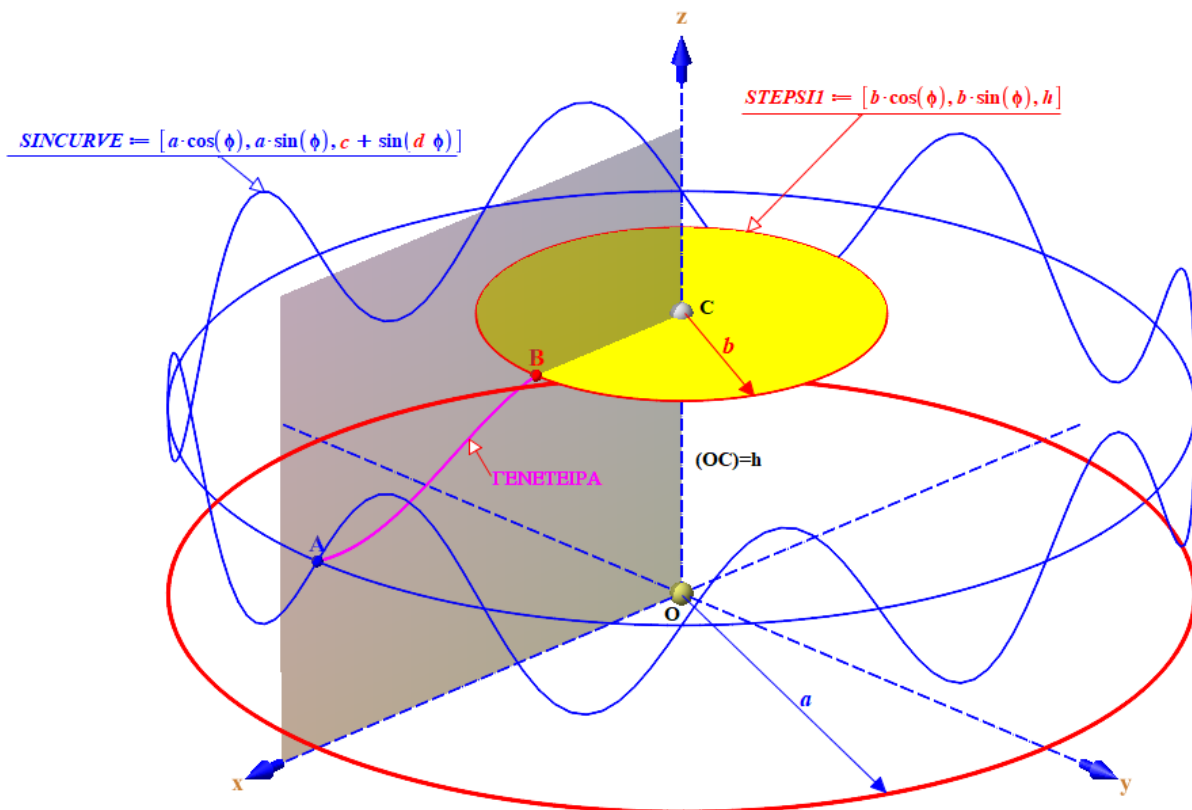
Επιφάνεια παραγόμενη από την περιστροφή της ημιτονοειδούς καμπύλης (AB),
που βρίσκεται πάνω στο κατακόρυφο επίπεδο Oz
και που τα άκρα της A,B συναντούν αντίστοιχα :

την ημιτονοειδή καμπύλη : $SINCURVE := [a \cdot \cos(\phi), a \cdot \sin(\phi), c + \sin(d \cdot \phi)]$

την κυκλική καμπύλη : $STEPSII := [b \cos(\phi), b \sin(\phi), h]$

(Εφαρμογή : $a := 5 ; b := 2 ; h := 3 ; c := 2 ; d := 8 ;$)

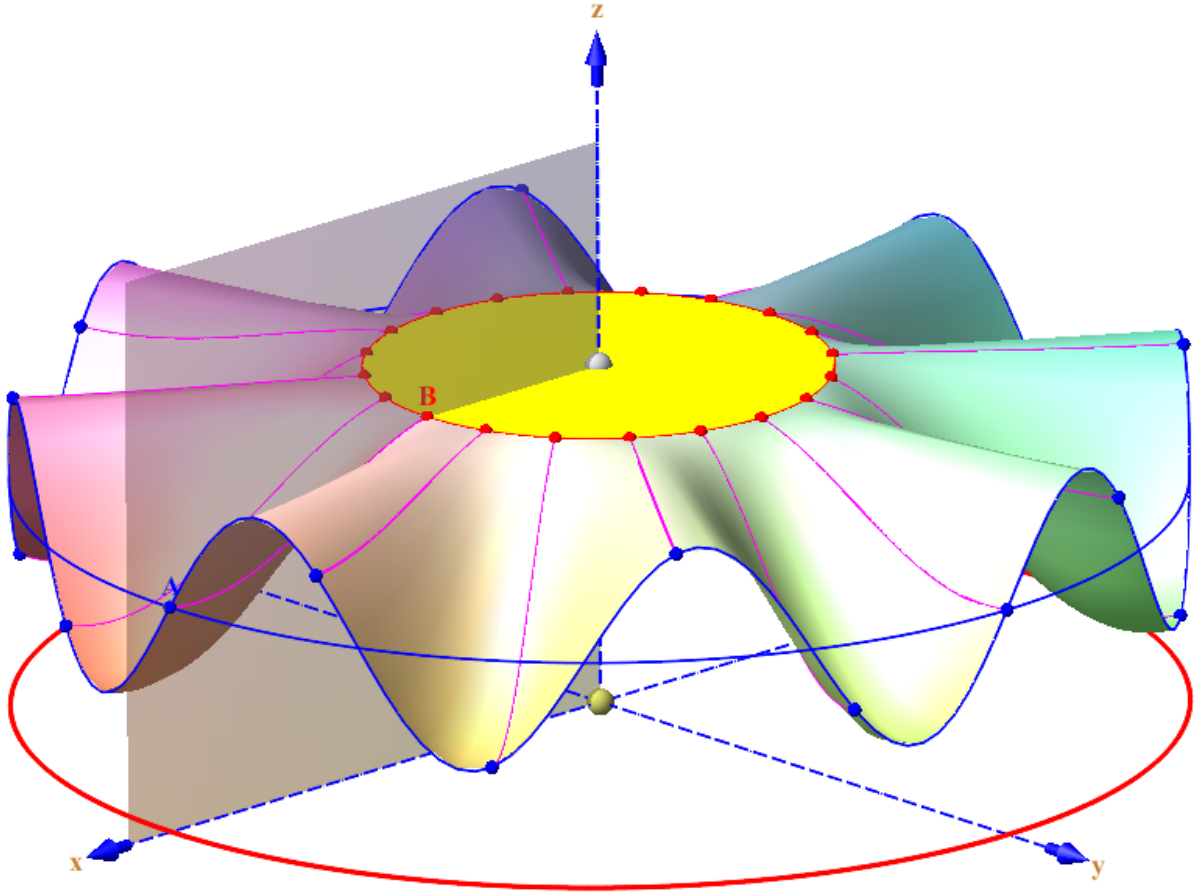
ΕΠΙΦΑΝΕΙΑ ΕΚ ΠΕΡΙΣΤΡΟΦΗΣ ΗΜΙΤΟΝΟΕΙΔΟΥΣ ΚΑΜΠΥΛΗΣ
SAGRADA FAMILIA
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ



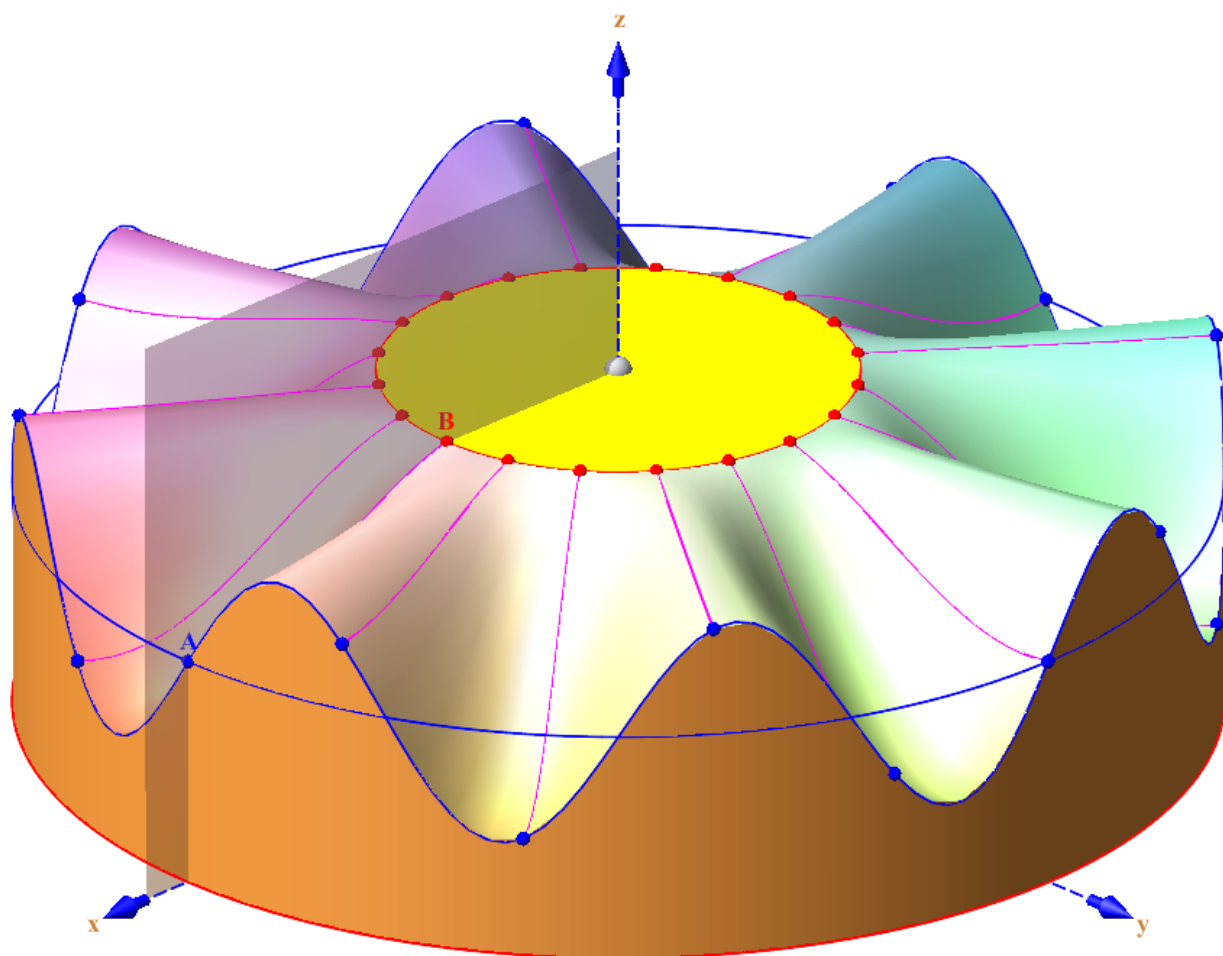
**ΕΠΙΦΑΝΕΙΑ ΕΚ ΠΕΡΙΣΤΡΟΦΗΣ ΗΜΙΤΟΝΟΕΙΔΟΥΣ ΚΑΜΠΥΛΗΣ
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Παραμετρική εξίσωση της παραγόμενης Επιφάνειας

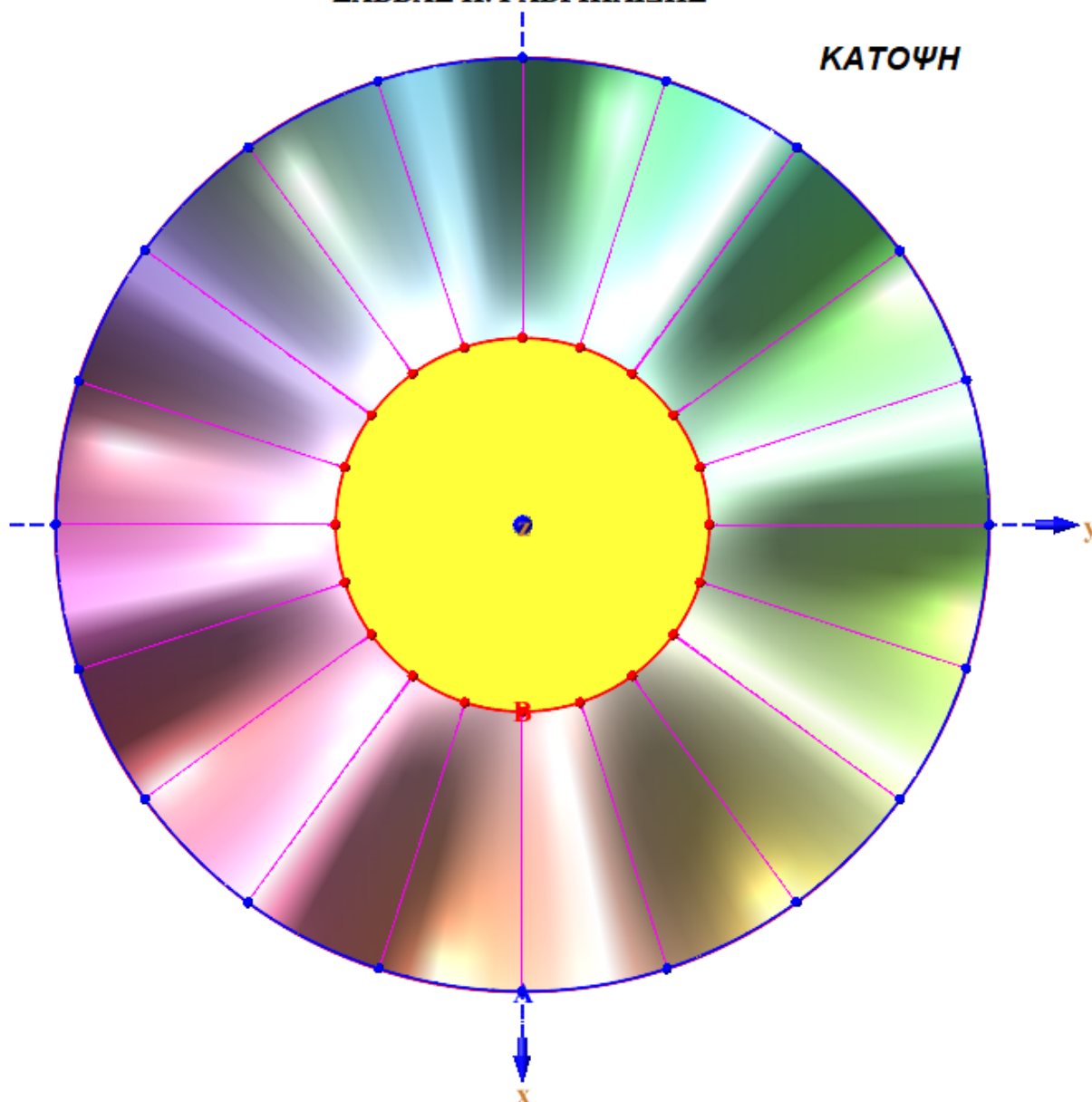
$$\left[t \cos(\phi), t \sin(\phi), \frac{\sin(2) \sin(8\phi) + 2 \sin(2) - 3 \sin(5)}{\sin(2) - \sin(5)} - \frac{(-1 + \sin(8\phi)) \sin(t)}{\sin(2) - \sin(5)} \right]$$



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ΕΠΙΦΑΝΕΙΑ ΕΚ ΠΕΡΙΣΤΡΟΦΗΣ ΗΜΙΤΟΝΟΕΙΔΟΥΣ ΚΑΜΠΥΛΗΣ
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```

>
> with(plots) :
> with(Physics[ Vectors])
[&x, '+', '\', ChangeBasis, ChangeCoordinates, Component, Curl, DirectionalDiff,
  Divergence, Gradient, Identify, Laplacian, ∇, Norm, Setup, diff]
> Setup(mathematicalnotation = true)
[mathematicalnotation = true]
> a := 5
a := 5
> b := 1
b := 1
> h := 4
h := 4
> c := h - 1

```

(1)

(2)

(3)

(4)

(5)

..

```

[ ] c := 3 (6)
> d := 4
[ ] d := 4 (7)
> K := 10
[ ] K := 10 (8)
>
> OO := pointplot3d([0, 0, 0], symbol=solidcircle, symbolsize=10) :
> axX := spacecurve([x, 0, 0], x=-(a+0.5)..(a+0.5), linestyle=3, thickness=1, color
= blue) :
> axY := spacecurve([0, y, 0], y=-(a+0.5)..(a+0.5), linestyle=3, thickness=1, color
= blue) :
> axZ := spacecurve([0, 0, z], z=0..h+K, linestyle=3, thickness=2, color=blue) :
>
> ARaxX := arrow([a+0.5, 0, 0], [0.5, 0, 0], width=0.1, head_length=0.3, shape
= cylindrical_arrow, color=blue) :
> ARaxY := arrow([0, a+0.5, 0], [0, 0.5, 0], width=0.1, head_length=0.3, shape
= cylindrical_arrow, color=blue) :
> ARaxZ := arrow([0, 0, h+K], [0, 0, 0.5], width=0.1, head_length=0.3, shape
= cylindrical_arrow, color=blue) :
>
> tX := textplot3d([a+1.1, 0.0, 0, "x"], color=gold, font=[arial, bold, 14]) :
> tY := textplot3d([0, a+1.1, 0, "y"], color=gold, font=[arial, bold, 14]) :
> tZ := textplot3d([0, 0, h+K+0.8, "z"], color=gold, font=[arial, bold, 14]) :
>
> CENTRE := pointplot3d([0, 0, h], symbol=solidcircle, symbolsize=10) :
> CYLINDRE := [a*cos(phi), a*sin(phi), z] :
> BASH := spacecurve([CYLINDRE[1], CYLINDRE[2], 0], phi=0..2*Pi, thickness=3, color
= red) :
> FSTEPSI := plot3d([R*cos(phi), R*sin(phi), h], R=0..b, phi=0..2*Pi, style=surface, color
= yellow, transparency=0.00) :
> EPIPEDO := [rho*cos(phi), rho*sin(phi), z]
EPIPEDO := [rho*cos(phi), rho*sin(phi), z] (9)
> SINCURVE := [a*cos(phi), a*sin(phi), c+sin(d*phi)]
SINCURVE := [5*cos(phi), 5*sin(phi), 3+sin(4*phi)] (10)
> STEPSI := [b*cos(phi), b*sin(phi), h]
STEPSI := [cos(phi), sin(phi), 4] (11)
> MESHGRAMMH := spacecurve([CYLINDRE[1], CYLINDRE[2], c], phi=0..2*Pi, thickness
= 1, linestyle=1, color=blue) :
> ASHMEIO := SINCURVE
ASHMEIO := [5*cos(phi), 5*sin(phi), 3+sin(4*phi)] (12)
> BSHMEIO := STEPSI
BSHMEIO := [cos(phi), sin(phi), 4] (13)
> simplify(subs(phi=0, ASHMEIO))
[5, 0, 3] (14)
> simplify(subs(phi=0, BSHMEIO))

```

[1, 0, 4]

(15)

> A := pointplot3d(subs($\phi = 0$, ASHMEIO), symbol=solidcircle, symbolsize=5, color=blue) :

> tA := textplot3d([(14)[1] + 0.1, (14)[2], (14)[3] + 0.2, "A"], font=[arial, bold, 14], color=blue) :

> B := pointplot3d(subs($\phi = 0$, BSHMEIO), symbol=solidcircle, symbolsize=5, color=red) :

> tB := textplot3d([(15)[1] + 0.1, (15)[2], (15)[3] + 0.2, "B"], font=[arial, bold, 14], color=red) :

>

> ABMHKOSGENETEIRAS :=
(simplify((SINCURVE[1] - STEPSI[1])² + (SINCURVE[2] - STEPSI[2])²) + (SINCURVE[3] - STEPSI[3])²)^{1/2}

$$ABMHKOSGENETEIRAS := \sqrt{16 + (-1 + \sin(4\phi))^2} \quad (16)$$

> evalf(subs($\phi = 0$, ABMHKOSGENETEIRAS))

$$4.123105626 \quad (17)$$

> evalf(seq(subs($\phi = \frac{3 \cdot \text{Pi}}{n}$, ABMHKOSGENETEIRAS), n = 1 .. 16))

$$4.123105624, 4.123105626, 4.123105624, 4.123105626, 4.000299421, 4.123105626, 4.378918066, 4.472135955, 4.413847619, 4.303610346, 4.200337885, 4.123105626, 4.071687692, 4.039862327, 4.021184030, 4.010708970 \quad (18)$$

> MAXL := evalf(subs($\phi = \frac{3 \cdot \text{Pi}}{16}$, ABMHKOSGENETEIRAS))

$$MAXL := 4.010708970 \quad (19)$$

> evalf(seq(subs($\phi = \frac{\text{Pi}}{n}$, ABMHKOSGENETEIRAS), n = 1 .. 16))

$$4.123105624, 4.123105626, 4.413847619, 4.123105626, 4.021184030, 4.002243020, 4.000078575, 4.000000000, 4.000028850, 4.000299421, 4.001020667, 4.002243020, 4.003914923, 4.005945269, 4.008238339, 4.010708970 \quad (20)$$

> MINL := evalf(subs($\phi = \frac{\text{Pi}}{16}$, ABMHKOSGENETEIRAS))

$$MINL := 4.010708970 \quad (21)$$

>

> PLAN := animate(plot3d, [EPIPEDO, $\rho = 0 \dots a + 0.2$, $z = 0 \dots h$, style=surface, transparency=0.50], $\phi = 0 \dots 2 \cdot \text{Pi}$, frames=21, trace=00) :

> SINCURVEA := spacecurve(SINCURVE, $\phi = 0 \dots 2 \cdot \text{Pi}$, color=blue, thickness=1) :

> STEPSIA := spacecurve(STEPSI, $\phi = 0 \dots 2 \cdot \text{Pi}$, color=red, thickness=1) :

> SHMEIO1 := animate(pointplot3d, [SINCURVE, symbol=solidcircle, symbolsize=5, color=blue], $\phi = 0 \dots 2 \cdot \text{Pi}$, frames=21, trace=20) :

> SHMEIO2 := animate(pointplot3d, [STEPSI, symbol=solidcircle, symbolsize=5, color=red], $\phi = 0 \dots 2 \cdot \text{Pi}$, frames=21, trace=20) :

>

>

ΝΑ ΒΡΟΥΜΕ ΜΙΑ ΗΜΙΤΟΝΟΕΙΑΗ ΚΑΜΠΥΛΗ ΠΑΝΩ ΣΤΟ

ΕΠΙΠΕΔΟ ΠΟΥ ΝΑ ΠΕΡΝΑΕΙ ΑΠΟ ΤΑ ΣΗΜΕΙΑ Α,Β .

$$EPIPEDO := [\rho \cos(\phi), \rho \sin(\phi), z]$$

$$ASHMEIO := [5 \cos(\phi), 5 \sin(\phi), 2 + \sin(8\phi)]$$

$$BSHMEIO := [2 \cos(\phi), 2 \sin(\phi), 3]$$

$$\begin{aligned} > rA_ := ASHMEIO[1] \cdot _i + ASHMEIO[2] \cdot _j + ASHMEIO[3] \cdot _k \\ & \quad \vec{rA} := 5 \cos(\phi) \hat{i} + 5 \sin(\phi) \hat{j} + (3 + \sin(4\phi)) \hat{k} \end{aligned} \quad (22)$$

$$\begin{aligned} > rB_ := BSHMEIO[1] \cdot _i + BSHMEIO[2] \cdot _j + BSHMEIO[3] \cdot _k \\ & \quad \vec{rB} := \cos(\phi) \hat{i} + \sin(\phi) \hat{j} + 4 \hat{k} \end{aligned} \quad (23)$$

ΚΛΙΣΗ ΤΗΣ ΓΕΝΕΤΕΙΡΑΣ ΩΣ ΠΡΟΣ x,y,z .

$$\begin{aligned} > simplify\left(\frac{(rB_ - rA_)}{ABMHKOSGENETEIRAS} \cdot _i\right) = \cos(\Phi_x) \\ & \quad - \frac{4 \cos(\phi)}{\sqrt{-\cos(4\phi)^2 - 2 \sin(4\phi) + 18}} = \cos(\Phi_x) \end{aligned} \quad (24)$$

$$\begin{aligned} > \Phi_x = convert(solve(evalf(subs(\phi = 0, (24)))), \Phi_x), degrees) \\ & \quad \Phi_x = 165.9637565 \text{ degrees} \end{aligned} \quad (25)$$

$$\begin{aligned} > \Phi_x = convert\left(solve\left(evalf\left(subs\left(\phi = \frac{\text{Pi}}{2}, (24)\right)\right), \Phi_x\right), degrees\right) \\ & \quad \Phi_x = 90.00000000 \text{ degrees} \end{aligned} \quad (26)$$

$$\begin{aligned} > simplify\left(\frac{(rB_ - rA_)}{ABMHKOSGENETEIRAS} \cdot _j\right) = \cos(\Phi_y) \\ & \quad - \frac{4 \sin(\phi)}{\sqrt{-\cos(4\phi)^2 - 2 \sin(4\phi) + 18}} = \cos(\Phi_y) \end{aligned} \quad (27)$$

$$\begin{aligned} > \Phi_y = convert(solve(evalf(subs(\phi = 0, (27)))), \Phi_y), degrees) \\ & \quad \Phi_y = 90.00000000 \text{ degrees} \end{aligned} \quad (28)$$

$$\begin{aligned} > \Phi_y = convert\left(solve\left(evalf\left(subs\left(\phi = \frac{\text{Pi}}{2}, (27)\right)\right), \Phi_y\right), degrees\right) \\ & \quad \Phi_y = 165.9637565 \text{ degrees} \end{aligned} \quad (29)$$

$$\begin{aligned} > simplify\left(\frac{(rB_ - rA_)}{ABMHKOSGENETEIRAS} \cdot _k\right) = \cos(\Phi_z) \\ & \quad - \frac{-1 + \sin(4\phi)}{\sqrt{-\cos(4\phi)^2 - 2 \sin(4\phi) + 18}} = \cos(\Phi_z) \end{aligned} \quad (30)$$

$$\begin{aligned} > \Phi_z = convert(solve(evalf(subs(\phi = 0, (30)))), \Phi_z), degrees) \\ & \quad \Phi_z = 75.96375655 \text{ degrees} \end{aligned} \quad (31)$$

$$\begin{aligned} > \Phi_z = convert\left(solve\left(evalf\left(subs\left(\phi = \frac{\text{Pi}}{2}, (30)\right)\right), \Phi_z\right), degrees\right) \\ & \quad \Phi_z = 75.96375655 \text{ degrees} \end{aligned} \quad (32)$$

ΗΜΙΤΟΝΟΕΙΔΗΣ ΚΑΜΠΥΛΗ ΠΑΝΩ ΣΤΟ ΕΠΙΠΕΔΟ

$$\begin{aligned} > CURVE := subs(\{\rho = \rho(t), z = z(t)\}, EPIPEDO) \\ & \quad CURVE := [\rho(t) \cos(\phi), \rho(t) \sin(\phi), z(t)] \end{aligned} \quad (33)$$

$$\begin{aligned} > sinusCURVE := subs(\{\rho(t) = t, z(t) = C[1] + C[2] \cdot \sin(t)\}, CURVE) \\ & \quad sinusCURVE := [t \cos(\phi), t \sin(\phi), C_1 + C_2 \sin(t)] \end{aligned} \quad (34)$$

ΗΜΙΤΟΝΟΕΙΔΗΣ ΚΑΜΠΥΛΗ ΠΑΝΩ ΣΤΟ ΕΠΙΠΕΔΟ ΠΟΥ ΝΑ ΠΕΡΝΑΕΙ ΑΠΟ ΤΑ

ΣΗΜΕΙΑ A, B.

$$\begin{aligned} EPIPEDO &:= [\rho \cos(\phi), \rho \sin(\phi), z] \\ ASHMEIO &:= [5 \cos(\phi), 5 \sin(\phi), 2 + \sin(8\phi)] \\ BSHMEIO &:= [2 \cos(\phi), 2 \sin(\phi), 3] \end{aligned}$$

$$\begin{aligned} > \frac{(sinusCURVE[1])}{\cos(\phi)} &= \frac{(ASHMEIO[1])}{\cos(\phi)} \\ & \quad t = 5 \end{aligned} \quad (35)$$

$$\begin{aligned} > sinusCURVE[2] &= ASHMEIO[2] \\ & \quad t \sin(\phi) = 5 \sin(\phi) \end{aligned} \quad (36)$$

$$\begin{aligned} > sinusCURVE[3] &= ASHMEIO[3] \\ & \quad C_1 + C_2 \sin(t) = 3 + \sin(4\phi) \end{aligned} \quad (37)$$

$$\begin{aligned} > subs((35), (37)) \\ & \quad C_1 + C_2 \sin(5) = 3 + \sin(4\phi) \end{aligned} \quad (38)$$

$$\begin{aligned} > \frac{(sinusCURVE[1])}{\cos(\phi)} &= \frac{(BSHMEIO[1])}{\cos(\phi)} \\ & \quad t = 1 \end{aligned} \quad (39)$$

$$\begin{aligned} > sinusCURVE[2] &= BSHMEIO[2] \\ & \quad t \sin(\phi) = \sin(\phi) \end{aligned} \quad (40)$$

$$\begin{aligned} > sinusCURVE[3] &= BSHMEIO[3] \\ & \quad C_1 + C_2 \sin(t) = 4 \end{aligned} \quad (41)$$

$$\begin{aligned} > subs((39), (41)) \\ & \quad C_1 + C_2 \sin(1) = 4 \end{aligned} \quad (42)$$

$$\begin{aligned} > eq1 &:= (38) \\ & \quad eq1 := C_1 + C_2 \sin(5) = 3 + \sin(4\phi) \end{aligned} \quad (43)$$

$$\begin{aligned} > eq2 &:= (42) \\ & \quad eq2 := C_1 + C_2 \sin(1) = 4 \end{aligned} \quad (44)$$

$$\begin{aligned} > SOL &:= solve([eq1, eq2], [C[1], C[2]]) \\ & \quad SOL := \left[\left[C_1 = \frac{\sin(1) \sin(4\phi) + 3 \sin(1) - 4 \sin(5)}{\sin(1) - \sin(5)}, C_2 = -\frac{-1 + \sin(4\phi)}{\sin(1) - \sin(5)} \right] \right] \end{aligned} \quad (45)$$

$$> SOL[1]$$

$$\left[C_1 = \frac{\sin(1) \sin(4\phi) + 3 \sin(1) - 4 \sin(5)}{\sin(1) - \sin(5)}, C_2 = -\frac{-1 + \sin(4\phi)}{\sin(1) - \sin(5)} \right] \quad (46)$$

> ACURVE := subs(SOL[1], sinusCURVE)

$$ACURVE := \left[t \cos(\phi), t \sin(\phi), \frac{\sin(1) \sin(4\phi) + 3 \sin(1) - 4 \sin(5)}{\sin(1) - \sin(5)} - \frac{(-1 + \sin(4\phi)) \sin(t)}{\sin(1) - \sin(5)} \right] \quad (47)$$

>

ΕΛΕΙΧΟΣ.

$$EPIPEDO := [\rho \cos(\phi), \rho \sin(\phi), z]$$

$$ASHMEIO := [5 \cos(\phi), 5 \sin(\phi), 2 + \sin(8\phi)]$$

$$BSHMEIO := [2 \cos(\phi), 2 \sin(\phi), 3]$$

> evalf(subs([φ=0, t=2], ACURVE))
[2., 0., 4.037673083] (48)

> evalf(subs([φ=0, t=5], ACURVE))
[5., 0., 2.999999999] (49)

>

> ABCURVE := evalf(subs(φ=0, ACURVE))
ABCURVE := [1. t, 0., 3.532618750 + 0.5554335885 sin(t)] (50)

> ABCURVE1 := spacecurve(ABCURVE, t=b..a, thickness=2, color=magenta) :

> animCURVE := animate(spacecurve, [ACURVE, t=b..a, color=red, thickness=1], φ=0..2·Pi, frames=21, trace=20) :

> SINUSSTEGASTRO := plot3d(ACURVE, t=b..a, φ=0..2·Pi, style=surface, numpoints=10000, transparency=0.00) :

> ACYLINDRE := plot3d(CYLINDRE, φ=0..2·Pi, z=0..SINCURVE[3], numpoints=10000, style=surface, transparency=0.00, color=plum) :

>

ΚΩΝΟΣ ΥΨΟΥΣ : Κ

> H := h + K
H := 14 (51)

>

> [b + λ·(0 - b), 0 + λ·(0 - 0), **h** + λ·(H - h)]
[1 - λ, 0, 4 + 10 λ] (52)

> [(52)[1]·cos(φ), (52)[1]·sin(φ), (52)[3]]
[(1 - λ) cos(φ), (1 - λ) sin(φ), 4 + 10 λ] (53)

> KVNOS := plot3d((53), λ=0..1, φ=0..2·Pi, style=surface, color=wheat, transparency=0.00) :

>

> display(KVNOS, ACYLINDRE, SINUSSTEGASTRO, animCURVE, ABCURVE1, A, B, tA, tB, MESHGRAMMH, OO, axX, axY, axZ, ARaxX, ARaxY, ARaxZ, tX, tY, tZ, CENTRE, BASH, FSTEPSI, PLAN, SINCURVEA, STEPSIA, SHMEIO1, SHMEIO2, lightmodel=light4,

orientation = [45, 65, 0], axes = none, scaling = constrained, title = " ΕΠΙΦΑΝΕΙΑ ΕΚ ΠΕΡΙΣΤΡΟΦΗΣ ΗΜΙΤΟΝΟΕΙΔΟΥΣ ΚΑΜΠΥΛΗΣ \nSAGRADA FAMILIA \nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ", titlefont = [arial, bold, 14]) :

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