>

$$\begin{bmatrix} \vec{e1} \\ \vec{e2} \\ \vec{e3} \end{bmatrix} = \begin{bmatrix} \cos(a)\,\hat{i} + \cos(b)\,\hat{j} + \cos(c)\,\hat{k} \\ \cos(d)\,\hat{i} + \cos(e)\,\hat{j} + \cos(f)\,\hat{k} \\ \cos(g)\,\hat{i} + \cos(h)\,\hat{j} + \cos(i)\,\hat{k} \end{bmatrix}$$

$$\vec{e1} = \cos(a)\,\hat{i} + \cos(b)\,\hat{j} + \cos(c)\,\hat{k}$$

$$\vec{e2} = \cos(d)\,\hat{i} + \cos(e)\,\hat{j} + \cos(f)\,\hat{k}$$

$$\vec{e3} = \cos(g)\,\hat{i} + \cos(h)\,\hat{j} + \cos(i)\,\hat{k}$$

$$\begin{bmatrix} \hat{i} \\ \hat{j} \\ \hat{k} \end{bmatrix} = \begin{bmatrix} \cos(a)\,\vec{e1} + \cos(d)\,\vec{e2} + \cos(g)\,\vec{e3} \\ \cos(b)\,\vec{e1} + \cos(e)\,\vec{e2} + \cos(h)\,\vec{e3} \\ \cos(c)\,\vec{e1} + \cos(f)\,\vec{e2} + \cos(i)\,\vec{e3} \end{bmatrix}$$

$$\hat{i} = \cos(a)\,\vec{e1} + \cos(d)\,\vec{e2} + \cos(g)\,\vec{e3}$$

$$\hat{j} = \cos(b)\,\vec{e1} + \cos(e)\,\vec{e2} + \cos(h)\,\vec{e3}$$

$$\hat{k} = \cos(c)\,\vec{e1} + \cos(f)\,\vec{e2} + \cos(i)\,\vec{e3}$$

ΜΕΤΑΣΧΗΜΑΤΙΣΜΟΣ ΣΥΣΤΗΜΑΤΩΝ

Να χαράξουμε Ελικοειδή Καμπύλη (Κυκλική ή Ελλειπτική)
της οποίας τα δύο (2) άκρα Α1 , Β1 να περνούν από δύο (2) σημεία Α , Β αντίστοιχα στο χώρο .

ΕΠΙΒΟΛΗ ΔΙΕΓΕΡΣΗΣ SINUS ΣΤΟ ΣΗΜΕΙΟ B στην διεύθυνση του άξονα AB .

ΓΙΑ ΑΛΛΑΓΗ ΔΙΕΥΘΥΝΣΗΣ ΤΗΣ ΔΙΕΓΕΡΣΗΣ , ΑΛΛΑΖΟΥΜΕ ΤΟ : $1 - h \cdot \sin(\Theta)$ ΣΕ : $1 + h \cdot \sin(\Theta)$

```
> with(Physics[Vectors])
[&x, `+`, `.`, ChangeBasis, ChangeCoordinates, Component, Curl, DirectionalDiff,
    Divergence, Gradient, Identify, Laplacian,∇, Norm, Setup, diff]      (1)
> Setup(mathematicalnotation = true)
                    [mathematicalnotation = true]                        (2)
> with(plots) :
>
> a := 4
                    a := 4                                               (3)
> b := 2
                    b := 2                                               (4)
>
```

Αριθμός Ελικώσεων n

```
> n := 5
                    n := 5                                               (5)
```

Πλάτος Ταλάντωσης h

```
> h := 1
                    h := 1                                               (6)
```

> $xA := -2$

$$xA := -2 \tag{7}$$

> $yA := 2$

$$yA := 2 \tag{8}$$

> $zA := 1$

$$zA := 1 \tag{9}$$

> $xB := 1$

$$xB := 1 \tag{10}$$

> $yB := 4$

$$yB := 4 \tag{11}$$

> $zB := 7$

$$zB := 7 \tag{12}$$

>

> $A := [xA, yA, zA]$

$$A := [-2, 2, 1] \tag{13}$$

> $B := [xB, yB, zB]$

$$B := [1, 4, 7] \tag{14}$$

> $MHKOSAB := \sqrt{(A[1]-B[1])^2 + (A[2]-B[2])^2 + (A[3]-B[3])^2}$

$$MHKOSAB := 7 \tag{15}$$

> $BHMA := \frac{MHKOSAB}{n}$

$$BHMA := \frac{7}{5} \tag{16}$$

> $AB := spacecurve([A[1]+\lambda\cdot(B[1]-A[1]), A[2]+\lambda\cdot(B[2]-A[2]), A[3]+\lambda\cdot(B[3]-A[3])], \lambda=0..1):$

> $rA\_ := A[1]\cdot\_i + A[2]\cdot\_j + A[3]\cdot\_k$

$$\overrightarrow{rA} := -2\,\hat{i} + 2\,\hat{j} + \hat{k} \tag{17}$$

> $rB\_ := B[1]\cdot\_i + B[2]\cdot\_j + B[3]\cdot\_k$

$$\overrightarrow{rB} := \hat{i} + 4\,\hat{j} + 7\,\hat{k} \tag{18}$$

>

ΑΡΧΗ ΤΟΥ $[\overrightarrow{e1}, \overrightarrow{e2}, \overrightarrow{e3}]$ ΤΟ ΣΗΜΕΙΟ A , $\overrightarrow{e3} = \frac{\overrightarrow{AB}}{|\overrightarrow{AB}|}$

$$\boldsymbol{HELIX := \left[-a + a\cdot\cos(\phi), + b\cdot\sin(\phi), \frac{BHMA\cdot\phi}{2\cdot Pi}\right]}$$

>

> $e3\_ := \frac{(rB\_ - rA\_)}{Norm(rB\_ - rA\_)}$

$$\overrightarrow{e3} := \frac{(3\,\hat{i} + 2\,\hat{j} + 6\,\hat{k})\sqrt{49}}{49} \tag{19}$$

>

> $Norm(e3\_)$

$$1 \tag{20}$$

> $C := [xC, yC, zC]$

$$C := [xC, yC, zC] \tag{21}$$

> $rC\_ := C[1]\cdot\_i + C[2]\cdot\_j + C[3]\cdot\_k$

$$\overrightarrow{rC} := xC\,\hat{i} + yC\,\hat{j} + zC\,\hat{k} \tag{22}$$

> $simplify((rC\_ - rA\_).e3\_ = 0)$

$$\frac{3\,xC}{7} - \frac{4}{7} + \frac{2\,yC}{7} + \frac{6\,zC}{7} = 0 \tag{23}$$

> $numer(lhs((\mathbf{23}))) = 0$

$$3\,xC - 4 + 2\,yC + 6\,zC = 0 \tag{24}$$

>

SOS : ΕΠΑΛΗΘΕΥΕΤΑΙ και γιά : [xC=--4 , yC=2 , zC=2]

>

> $rC1\_ := -4\cdot\_i + 2\cdot\_j + 2\cdot\_k$

$$\overrightarrow{rC1} := -4\,\hat{i} + 2\,\hat{j} + 2\,\hat{k} \tag{25}$$

> $e1\_ := \dfrac{(rC1\_ - rA\_)}{Norm(rC1\_ - rA\_)}$

$$\overrightarrow{e1} := \frac{(-2\,\hat{i} + \hat{k})\sqrt{5}}{5} \tag{26}$$

>

> $Norm(e1\_)$

$$1 \tag{27}$$

> $\mathbf{e2\_ := e3\_ \times e1\_}$

$$\overrightarrow{e2} := \frac{2\sqrt{49}\sqrt{5}\,\hat{i}}{245} - \frac{3\sqrt{49}\sqrt{5}\,\hat{j}}{49} + \frac{4\sqrt{49}\sqrt{5}\,\hat{k}}{245} \tag{28}$$

> $Norm(e2\_)$

$$1 \tag{29}$$

>

$$\boldsymbol{HELIX := \left[-a + a\cdot\cos(\phi), +\, b\cdot\sin(\phi), \frac{BHMA\cdot\phi}{2\cdot \mathrm{Pi}}\right]}$$

>

> $(-a + a\cdot\cos(\phi))\cdot e1\_ + (b\cdot\sin(\phi))\cdot e2\_ + \left(\frac{BHMA}{2\cdot \mathrm{Pi}}\cdot\phi\cdot(1 + h\cdot\sin(\Theta))\right)\cdot e3\_ + A[1]\cdot\_i + A[2]\cdot\_j + A[3]\cdot\_k$

$$\hat{i}\left(\frac{8\sqrt{5}}{5} - \frac{8\cos(\phi)\sqrt{5}}{5} + \frac{4\sin(\phi)\sqrt{49}\sqrt{5}}{245} + \frac{3\,\phi\sqrt{49}}{70\,\pi} + \frac{3\,\phi\sin(\Theta)\sqrt{49}}{70\,\pi} - 2\right) + \hat{j}\left(-\frac{6\sin(\phi)\sqrt{49}\sqrt{5}}{49} + \frac{\phi\sqrt{49}}{35\,\pi} + \frac{\phi\sin(\Theta)\sqrt{49}}{35\,\pi} + 2\right) + \hat{k}\left(-\frac{4\sqrt{5}}{5}\right. \tag{30}$$

$$+\frac{4\cos(\phi)\sqrt{5}}{5}+\frac{8\sin(\phi)\sqrt{49}\sqrt{5}}{245}+\frac{3\phi\sqrt{49}}{35\pi}+\frac{3\phi\sin(\Theta)\sqrt{49}}{35\pi}+1\Big)$$

>

> **HELIXAB** := [Component((**30**), 1), Component((**30**), 2), Component((**30**), 3)]

$$HELIXAB := \left[\frac{8\sqrt{5}}{5}-\frac{8\cos(\phi)\sqrt{5}}{5}+\frac{4\sin(\phi)\sqrt{49}\sqrt{5}}{245}+\frac{3\phi\sqrt{49}}{70\pi}+\frac{3\phi\sin(\Theta)\sqrt{49}}{70\pi}-2, -\frac{6\sin(\phi)\sqrt{49}\sqrt{5}}{49}+\frac{\phi\sqrt{49}}{35\pi}+\frac{\phi\sin(\Theta)\sqrt{49}}{35\pi}+2, -\frac{4\sqrt{5}}{5}+\frac{4\cos(\phi)\sqrt{5}}{5}+\frac{8\sin(\phi)\sqrt{49}\sqrt{5}}{245}+\frac{3\phi\sqrt{49}}{35\pi}+\frac{3\phi\sin(\Theta)\sqrt{49}}{35\pi}+1\right] \quad \textbf{(31)}$$

>

TEST

> simplify(subs({φ = 0, Θ = 0}, HELIXAB)) = A

$$[-2, 2, 1] = [-2, 2, 1] \quad \textbf{(32)}$$

> simplify(subs({φ = n·2·Pi, Θ = 0}, HELIXAB)) = B

$$[1, 4, 7] = [1, 4, 7] \quad \textbf{(33)}$$

>

ΤΟ ΣΗΜΕΙΟ A στο [e1,e2,e3]

> A1 := [0, 0, 0]

$$A1 := [0, 0, 0] \quad \textbf{(34)}$$

>

ΤΟ ΣΗΜΕΙΟ B στο [e1,e2,e3]

> **B1 := [0, 0, MHKOSAB·(1 + h·sin(Θ))]**

$$B1 := [0, 0, 7+7\sin(\Theta)] \quad \textbf{(35)}$$

> A1B1 := [B1[1] + λ·(A1[1] − B1[1]), B1[2] + λ·(A1[2] − B1[2]), B1[3] + λ·(A1[3] − B1[3])]

$$A1B1 := [0, 0, 7+7\sin(\Theta)+\lambda(-7-7\sin(\Theta))] \quad \textbf{(36)}$$

> A1B1[1]

$$0 \quad \textbf{(37)}$$

> A1B1[2]

$$0 \quad \textbf{(38)}$$

> A1B1[3]

$$7+7\sin(\Theta)+\lambda(-7-7\sin(\Theta)) \quad \textbf{(39)}$$

>

> linAB := A1B1[1]·e1_ + A1B1[2]·e2_ + A1B1[3]·e3_ + **A[1]·_i + A[2]·_j + A[3]·_k**

$$linAB := \hat{i}\left(\frac{3\sqrt{49}}{7}+\frac{3\sin(\Theta)\sqrt{49}}{7}-\frac{3\lambda\sin(\Theta)\sqrt{49}}{7}-\frac{3\lambda\sqrt{49}}{7}-2\right)+\hat{j}\left(\frac{2\sqrt{49}}{7}+\frac{2\sin(\Theta)\sqrt{49}}{7}-\frac{2\lambda\sin(\Theta)\sqrt{49}}{7}-\frac{2\lambda\sqrt{49}}{7}+2\right)+\hat{k}\left(\frac{6\sqrt{49}}{7}\right. \quad \textbf{(40)}$$

$$+\frac{6\sin(\Theta)\sqrt{49}}{7}-\frac{6\lambda\sin(\Theta)\sqrt{49}}{7}-\frac{6\lambda\sqrt{49}}{7}+1\Big)$$

>

> ABxyz := [Component((40), 1), Component((40), 2), Component((40), 3)]

$$ABxyz := \left[\frac{3\sqrt{49}}{7}+\frac{3\sin(\Theta)\sqrt{49}}{7}-\frac{3\lambda\sin(\Theta)\sqrt{49}}{7}-\frac{3\lambda\sqrt{49}}{7}-2, \frac{2\sqrt{49}}{7}+\frac{2\sin(\Theta)\sqrt{49}}{7}-\frac{2\lambda\sin(\Theta)\sqrt{49}}{7}-\frac{2\lambda\sqrt{49}}{7}+2, \frac{6\sqrt{49}}{7}+\frac{6\sin(\Theta)\sqrt{49}}{7}-\frac{6\lambda\sin(\Theta)\sqrt{49}}{7}-\frac{6\lambda\sqrt{49}}{7}+1\right] \quad \textbf{(41)}$$

> evalf(subs({λ=0, Θ=0}, ABxyz))

[1.000000000, 4.000000000, 7.000000000] **(42)**

> evalf(subs({λ=1, Θ=0}, ABxyz))

[−2., 2., 1.] **(43)**

>

```
> animAB := animate(spacecurve, [ABxyz, λ = 0 ..1, color = GREEN, thickness = 1, linestyle
      = 1], Θ = 0 ..2·Pi, frames = 101) :
>
> AB := spacecurve([A[1] + λ·(B[1] − A[1]), A[2] + λ·(B[2] − A[2]), A[3] + λ·(B[3]
      − A[3])], λ = 0 ..1, color = red, linestyle = 4) :
> pA := pointplot3d(A, symbol = solidcircle, symbolsize = 10) :
> pB := pointplot3d(B, symbol = solidcircle, symbolsize = 10) :
>
> OO := pointplot3d([0, 0, 0], symbol = solidcircle, symbolsize = 10) :
> axX := spacecurve([x, 0, 0], x =-5.5 ..5.5, linestyle = 3, thickness = 1, color = blue) :
> axY := spacecurve([0, y, 0], y =-5.5 ..5.5, linestyle = 3, thickness = 1, color = blue) :
> axZ := spacecurve([0, 0, z], z = 0 ..7.5, linestyle = 3, thickness = 2, color = blue) :
> axZ1 := spacecurve([0, 0, z], z = 0 ..5.5, linestyle = 3, thickness = 2, color = blue) :
> ARaxX := arrow([5.5, 0, 0], [0.5, 0, 0], width = 0.1, head_length = 0.3, shape
      = cylindrical_arrow, color = blue) :
> ARaxY := arrow([0, 5.5, 0], [0, 0.5, 0], width = 0.1, head_length = 0.3, shape
      = cylindrical_arrow, color = blue) :
> ARaxZ := arrow([0, 0, 7.5], [0, 0, 0.5], width = 0.1, head_length = 0.3, shape
      = cylindrical_arrow, color = blue) :
> ARaxZ1 := arrow([0, 0, 5.5], [0, 0, 0.5], width = 0.1, head_length = 0.3, shape
      = cylindrical_arrow, color = blue) :
> tX := textplot3d([6.1, 0.0, 0, "x"], color = gold, font = [arial, bold, 14]) :
> tY := textplot3d([0, 6.1, 0, "y"], color = gold, font = [arial, bold, 14]) :
> tZ := textplot3d([0, 0, 8.2, "z"], color = gold, font = [arial, bold, 14]) :
> tZ1 := textplot3d([0, 0, 6.2, "z"], color = gold, font = [arial, bold, 14]) :
>
> tA := textplot3d([A[1] − 0.2, A[2], A[3] − 0.3, "A"], color = blue, font = [arial, bold,
      14]) :
> tB := textplot3d([B[1], B[2] + 0.3, B[3] + 0.3, "B"], color = blue, font = [arial, bold,
      14]) :
>
```

> $EL := animate\left(spacecurve, \left[HELIXAB, \phi = 0 ..n \cdot 2 \cdot \mathrm{Pi}, color = blue\right], \Theta = 0 ..2 \cdot \mathrm{Pi}, frames = 101\right) :$

>

ΕΠΙΒΟΛΗ ΔΙΕΓΕΡΣΗΣ ΣΤΟ ΣΗΜΕΙΟ B στην διεύθυνση του άξονα AB .

>

> $B1[1] \cdot e1\_ + B1[2] \cdot e2\_ + B1[3] \cdot e3\_ + A[1] \cdot \_i + A[2] \cdot \_j + A[3] \cdot \_k$

$$\hat{i}\left(\frac{3\sqrt{49}}{7} + \frac{3\sin(\Theta)\sqrt{49}}{7} - 2\right) + \hat{j}\left(\frac{2\sqrt{49}}{7} + \frac{2\sin(\Theta)\sqrt{49}}{7} + 2\right) + \hat{k}\left(\frac{6\sqrt{49}}{7} + \frac{6\sin(\Theta)\sqrt{49}}{7} + 1\right) \quad \textbf{(44)}$$

> $[Component(\textbf{(44)}, 1), Component(\textbf{(44)}, 2), Component(\textbf{(44)}, 3)]$

$$\left[\frac{3\sqrt{49}}{7} + \frac{3\sin(\Theta)\sqrt{49}}{7} - 2, \frac{2\sqrt{49}}{7} + \frac{2\sin(\Theta)\sqrt{49}}{7} + 2, \frac{6\sqrt{49}}{7} + \frac{6\sin(\Theta)\sqrt{49}}{7} + 1\right] \quad \textbf{(45)}$$

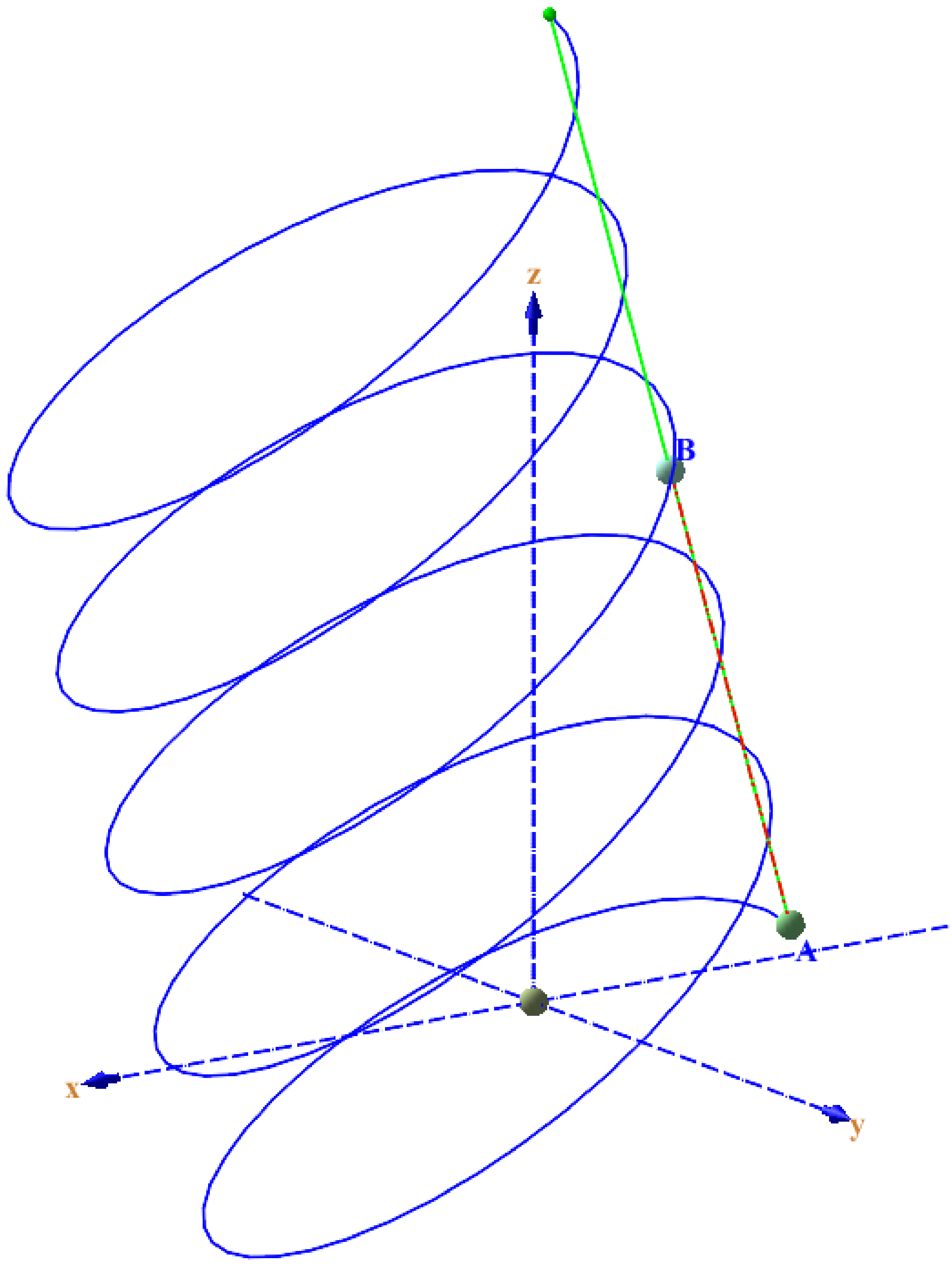
> $animB := animate\left(pointplot3d, [\textbf{(45)}, symbol = solidcircle, symbolsize = 5, color = green], \Theta = 0 ..2 \cdot \mathrm{Pi}, frames = 101, trace = 00\right) :$

>

>

> *display*(*animB*, *EL*, *animAB*, *OO*, *axX*, *axY*, *axZ*, *ARaxX*, *ARaxY*, *ARaxZ*, *tX*, *tY*, *tZ*, *pA*, *pB*, *AB*, *tA*, *tB*, *axes* = *none*, *scaling* = *constrained*, *title* = "ANIMATION\nΕΛΙΚΑ ΠΟΥ ΠΕΡΝΑΕΙ ΑΠΟ 2 ΣΗΜΕΙΑ ΣΤΟ ΧΩΡΟ\nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ", *titlefont* = [*arial*, *bold*, 14], *orientation* = [55, 75, 0]) :

ANIMATION
ΕΛΙΚΑ ΠΟΥ ΠΕΡΝΑΕΙ ΑΠΟ 2 ΣΗΜΕΙΑ ΣΤΟ ΧΩΡΟ
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ



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