```
>
> with(plots) :
> with(Physics[Vectors])
```

[&x, `+`, `.`, ChangeBasis, ChangeCoordinates, Component, Curl, DirectionalDiff, Divergence, Gradient, Identify, Laplacian, ∇, Norm, Setup, diff] **(1)**

```
> Setup(mathematicalnotation = true)
```

[mathematicalnotation = true] **(2)**

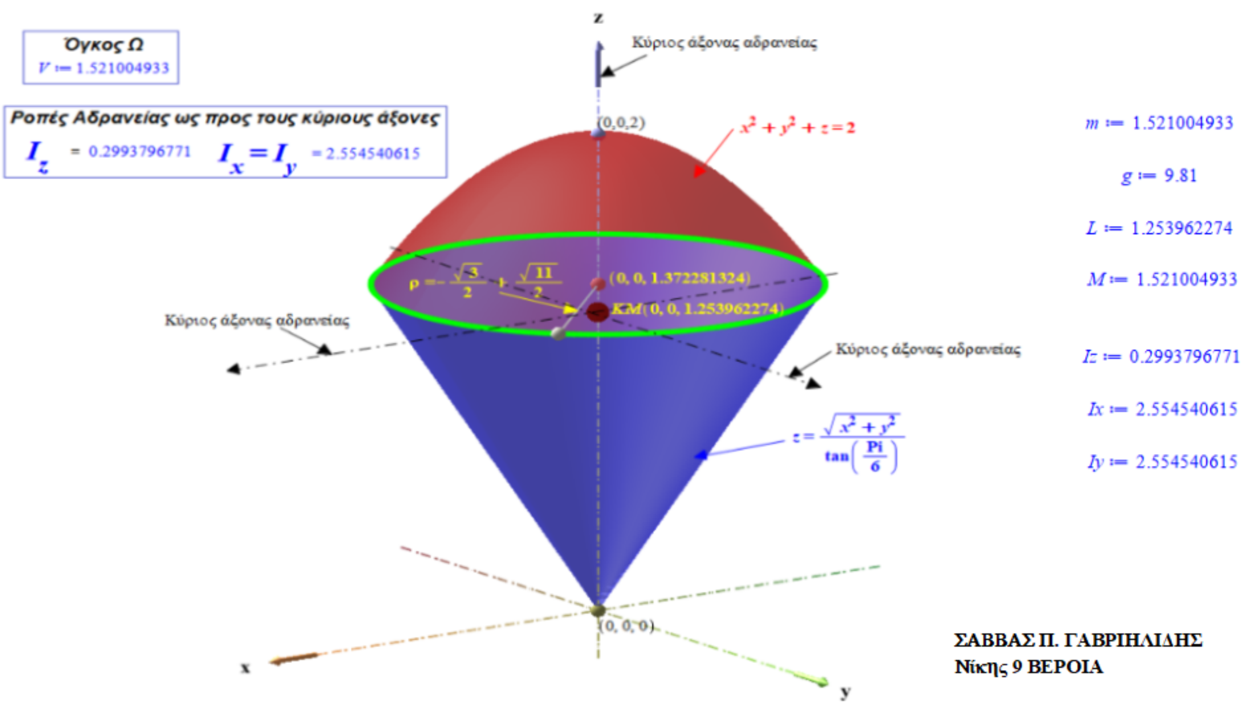
ΕΦΑΡΜΟΓΗ

$$\tau_{G\xi} = m\cdot g\cdot L\cdot \cos(\psi(t))\sin(\theta(t))$$

$$\tau_{G\eta} = -m\cdot g\cdot L\cdot \sin(\theta(t))\sin(\psi(t))$$

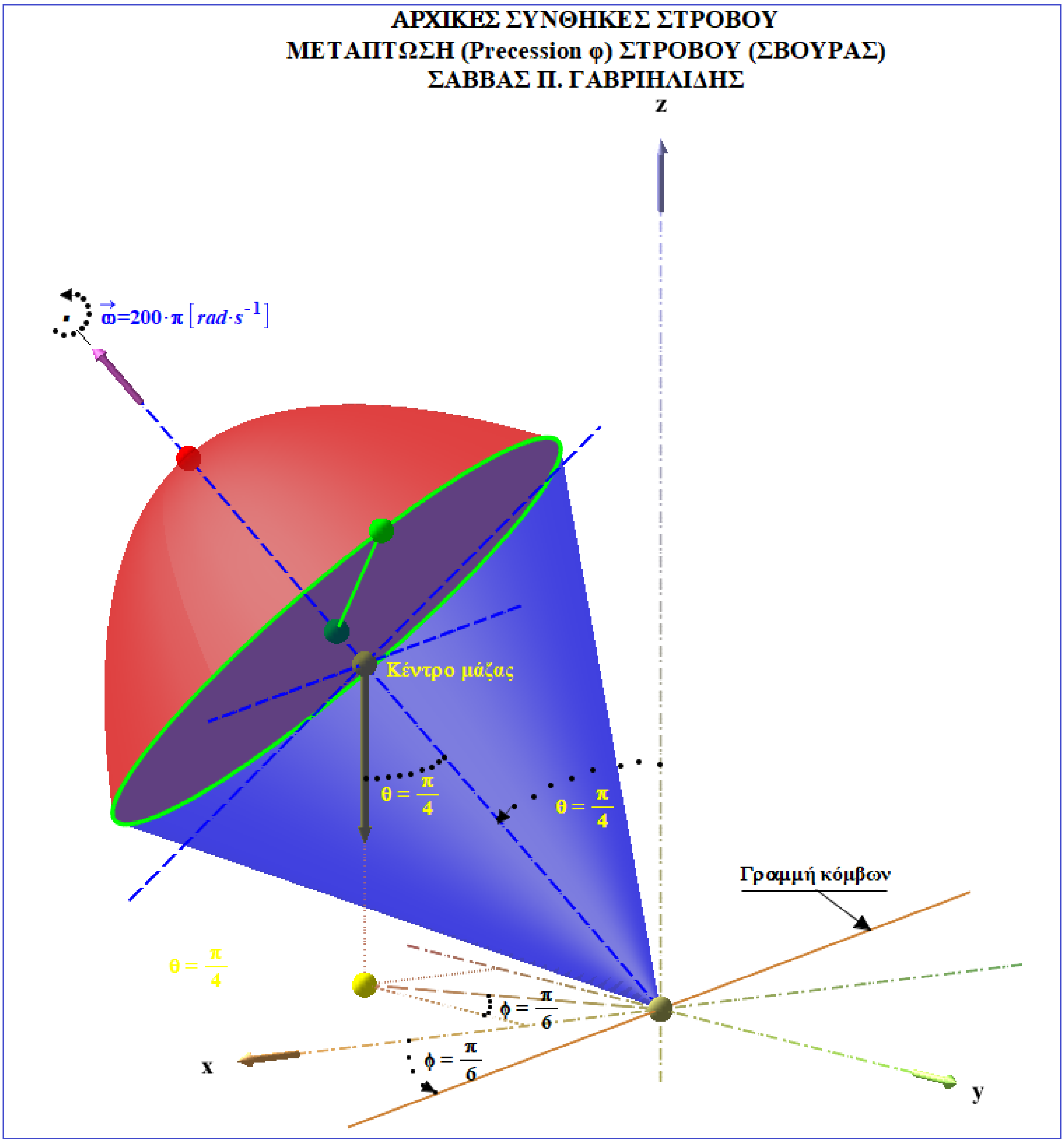
$$\tau_{G\zeta} = 0$$

Γωνιακή ταχύτητα περιστροφής του δίσκου του Γυροσκοπίου περί τον άξονα Οζ : D(ψ)(0)= 100στροφές/sec=200·π $[rad\cdot s^{-1}]$

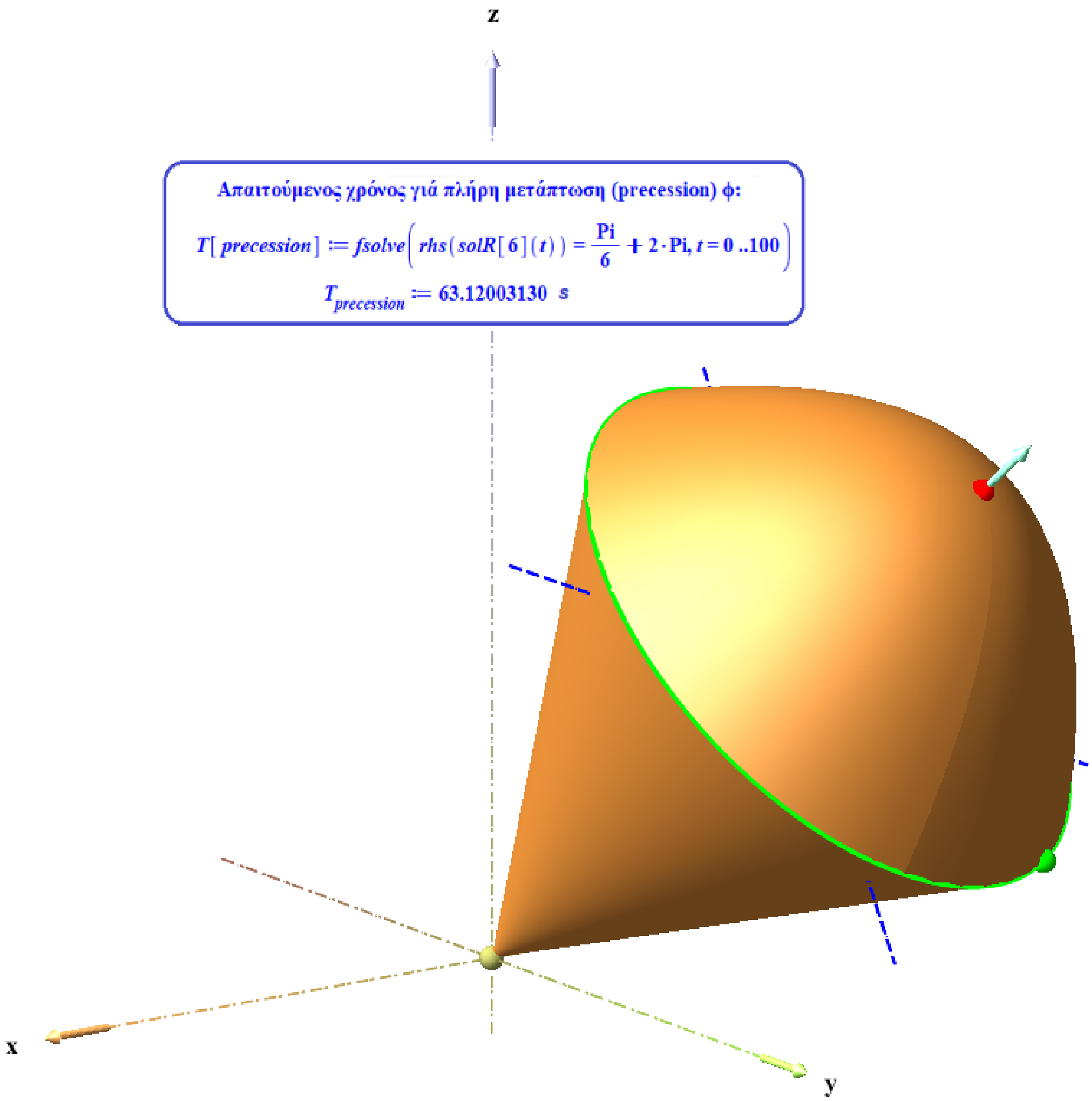


Αρχικές Συνθήκες :

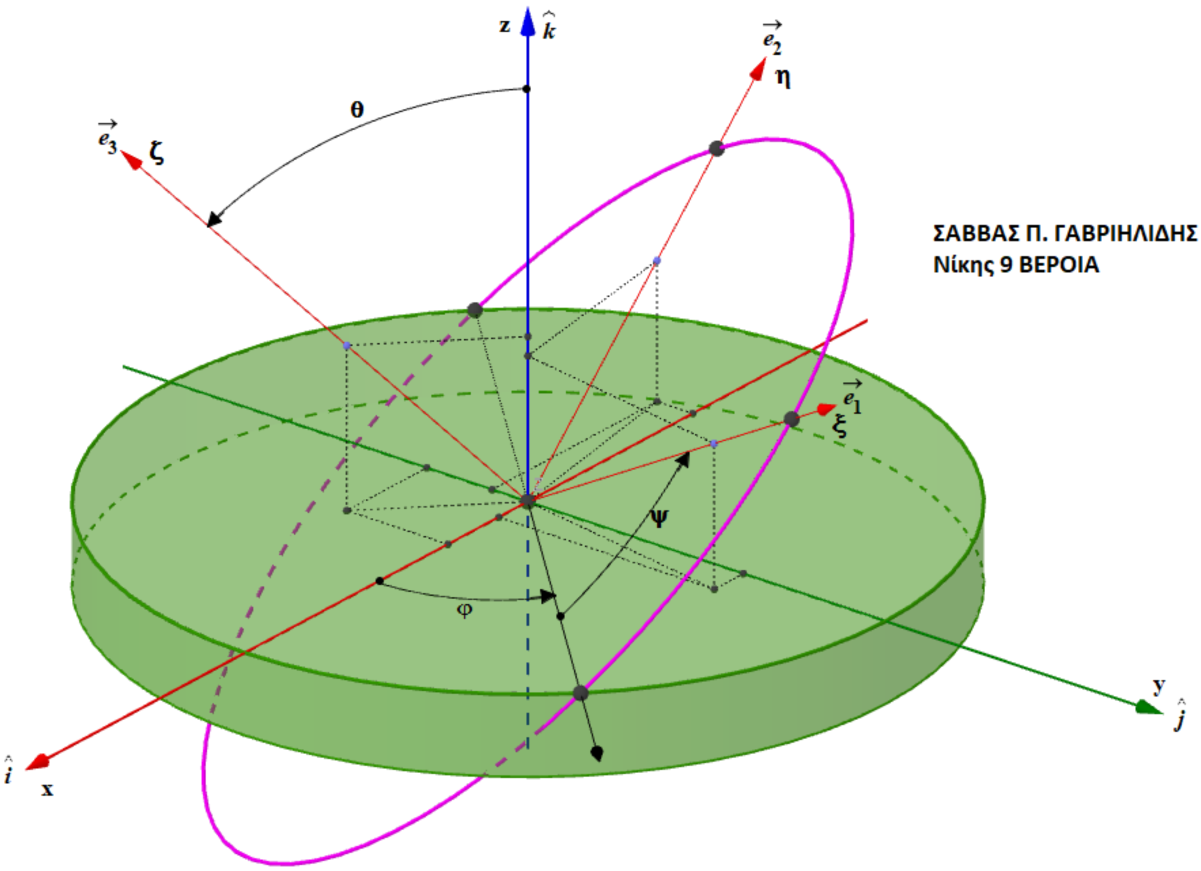
$icsR := \varphi(0) = \frac{Pi}{6}, D(\varphi)(0) = 0, \theta(0) = \frac{Pi}{4}, D(\theta)(0) = 0, \psi(0) = 0, D(\psi)(0) = 2\cdot Pi\cdot 100$


ΑΡΧΙΚΕΣ ΣΥΝΘΗΚΕΣ ΣΤΡΟΒΟΥ
ΜΕΤΑΠΤΩΣΗ (Precession φ) ΣΤΡΟΒΟΥ (ΣΒΟΥΡΑΣ)
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ
z
$\vec{\omega}=200\cdot\pi\,[rad\cdot s^{-1}]$
Κέντρο μάζας
$\theta = \frac{\pi}{4}$
$\theta = \frac{\pi}{4}$
Γραμμή κόμβων
$\theta = \frac{\pi}{4}$
$\phi = \frac{\pi}{6}$
x
$\phi = \frac{\pi}{6}$
y

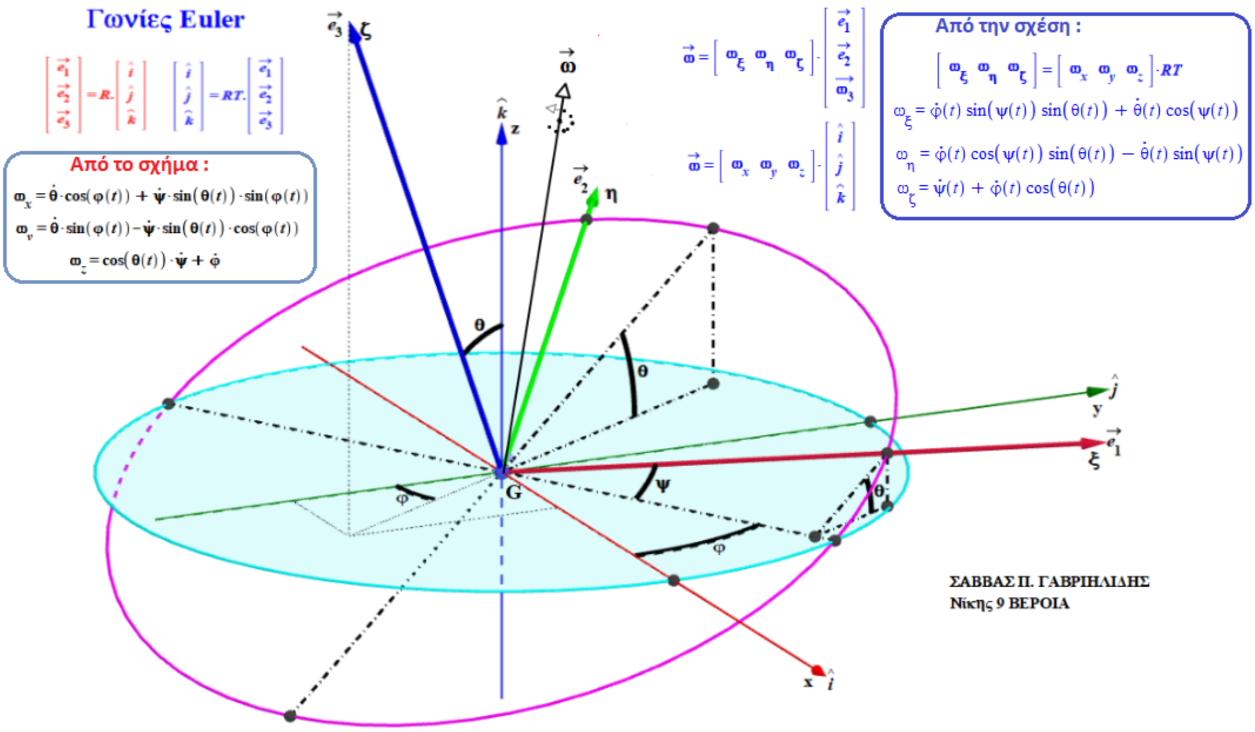
ΜΕΤΑΠΤΩΣΗ (Precession φ) ΣΤΡΟΒΟΥ (ΣΒΟΥΡΑΣ)
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ



Γωνίες Euler



ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ
Νίκης 9 ΒΕΡΟΙΑ



$$R := \begin{bmatrix} \cos(\varphi(t))\cdot\cos(\psi(t)) - \cos(\theta(t))\cdot\sin(\varphi(t))\cdot\sin(\psi(t)) & \cos(\psi(t))\cdot\sin(\varphi(t)) + \cos(\theta(t))\cdot\cos(\varphi(t))\cdot\sin(\psi(t)) & \sin(\theta(t))\cdot\sin(\psi(t)) \\ -\cos(\theta(t))\cdot\cos(\psi(t))\cdot\sin(\varphi(t)) - \cos(\varphi(t))\cdot\sin(\psi(t)) & \cos(\theta(t))\cdot\cos(\varphi(t))\cdot\cos(\psi(t)) - \sin(\varphi(t))\cdot\sin(\psi(t)) & \cos(\psi(t))\cdot\sin(\theta(t)) \\ \sin(\theta(t))\cdot\sin(\varphi(t)) & -\cos(\varphi(t))\cdot\sin(\theta(t)) & \cos(\theta(t)) \end{bmatrix} :$$

$$RT := \begin{bmatrix} \cos(\varphi(t))\cdot\cos(\psi(t)) - \cos(\theta(t))\cdot\sin(\varphi(t))\cdot\sin(\psi(t)) & -\cos(\theta(t))\cdot\cos(\psi(t))\cdot\sin(\varphi(t)) - \cos(\varphi(t))\cdot\sin(\psi(t)) & \sin(\theta(t))\cdot\sin(\varphi(t)) \\ \cos(\psi(t))\cdot\sin(\varphi(t)) + \cos(\theta(t))\cdot\cos(\varphi(t))\cdot\sin(\psi(t)) & \cos(\theta(t))\cdot\cos(\varphi(t))\cdot\cos(\psi(t)) - \sin(\varphi(t))\cdot\sin(\psi(t)) & -\cos(\varphi(t))\cdot\sin(\theta(t)) \\ \sin(\theta(t))\cdot\sin(\psi(t)) & \cos(\psi(t))\cdot\sin(\theta(t)) & \cos(\theta(t)) \end{bmatrix} :$$

Προκύπτει από το σχήμα (Γωνίες Euler) κατά σειρά περιστροφής :

(**Είναι** : $\vec{e}_3 = \sin(\theta(t))\sin(\varphi(t))\,\hat{i} - \cos(\varphi(t))\sin(\theta(t))\,\hat{j} + \cos(\theta(t))\,\hat{k}$)

$$\vec{\omega} = \dot{\varphi}(t)\cdot\hat{k} + \dot{\theta}(t)\cdot\left(\cos(\varphi(t))\cdot\hat{i} + \sin(\varphi(t))\cdot\hat{j}\right) + \dot{\psi}(t)\cdot\vec{e}_3 =$$

$$= \dot{\varphi}(t)\cdot\hat{k} + \dot{\theta}(t)\cdot\left(\cos(\varphi(t))\cdot\hat{i} + \sin(\varphi(t))\cdot\hat{j}\right) + \dot{\psi}(t)\cdot\left(\sin(\theta(t))\cdot\sin(\varphi(t))\cdot\hat{i} - \cos(\varphi(t))\cdot\sin(\theta(t))\cdot\hat{j} + \cos(\theta(t))\cdot\hat{k}\right) =$$

$$= \dot{\theta}(t)\cdot\cos(\varphi(t))\cdot\hat{i} + \dot{\psi}(t)\cdot\sin(\theta(t))\cdot\sin(\varphi(t))\cdot\hat{i} + \dot{\theta}(t)\cdot\sin(\varphi(t))\cdot\hat{j} - \dot{\psi}(t)\cdot\cos(\varphi(t))\cdot\sin(\theta(t))\cdot\hat{j} + \dot{\varphi}(t)\cdot\hat{k} + \dot{\psi}(t)\cdot\cos(\theta(t))\cdot\hat{k} =$$

$$= \left(\dot{\theta}(t)\cdot\cos(\varphi(t)) + \dot{\psi}(t)\cdot\sin(\theta(t))\cdot\sin(\varphi(t))\right)\cdot\hat{i} + \left(\dot{\theta}(t)\cdot\sin(\varphi(t)) - \dot{\psi}(t)\cdot\sin(\theta(t))\cdot\cos(\varphi(t))\right)\cdot\hat{j} + \left(\dot{\psi}(t)\cdot\cos(\theta(t)) + \dot{\varphi}(t)\right)\cdot\hat{k}$$

Θεωρούμε ένα στερεό σώμα με κέντρο μάζας G και κύριους Αδρανειακούς άξονες **G(ξηζ)**, οι οποίοι είναι **ενσωματωμένοι** στο σώμα με αρχή το G .
Έστω $\vec{\omega}$ η γωνιακή ταχύτητα του σώματος .(Η ίδια και των ενσωματωμένων αξόνων). $\vec{\omega} = \omega_\xi \cdot \vec{e_1} + \omega_\eta \cdot \vec{e_2} + \omega_\zeta \cdot \vec{e_3}$

Είναι : I_ξ, I_η, I_ζ οι κύριες Αδρανειακές ροπές του σώματος .

Η **Στροφορμή** : $\vec{h_G} = I_\xi \cdot \omega_\xi \cdot \vec{e_1} + I_\eta \cdot \omega_\eta \cdot \vec{e_2} + I_\zeta \cdot \omega_\zeta \cdot \vec{e_3}$

Οι δρώσες Εξωτερικές Ροπές : $\vec{\tau_G} = \tau_{G\xi} \cdot \vec{e_1} + \tau_{G\eta} \cdot \vec{e_2} + \tau_{G\zeta} \cdot \vec{e_3}$

Τότε : $\frac{d}{dt}\vec{h_G} = \vec{\tau_G}$

Είναι : $\dot{\vec{e_1}} = \vec{\omega} \times \vec{e_1} = \left(\omega_\xi \cdot \vec{e_1} + \omega_\eta \cdot \vec{e_2} + \omega_\zeta \cdot \vec{e_3}\right) \times \vec{e_1} = -\omega_\eta \cdot \vec{e_3} + \omega_\zeta \cdot \vec{e_2}$

$\dot{\vec{e_2}} = \vec{\omega} \times \vec{e_2} = \left(\omega_\xi \cdot \vec{e_1} + \omega_\eta \cdot \vec{e_2} + \omega_\zeta \cdot \vec{e_3}\right) \times \vec{e_2} = \omega_\xi \cdot \vec{e_3} - \omega_\zeta \cdot \vec{e_1}$

$\dot{\vec{e_3}} = \vec{\omega} \times \vec{e_3} = \left(\omega_\xi \cdot \vec{e_1} + \omega_\eta \cdot \vec{e_2} + \omega_\zeta \cdot \vec{e_3}\right) \times \vec{e_3} = -\omega_\xi \cdot \vec{e_2} + \omega_\eta \cdot \vec{e_1}$

$$\frac{d}{dt}\left(I_\xi \cdot \omega_\xi \cdot \vec{e_1} + I_\eta \cdot \omega_\eta \cdot \vec{e_2} + I_\zeta \cdot \omega_\zeta \cdot \vec{e_3}\right) = I_\xi \cdot \dot{\omega}_\xi \cdot \vec{e_1} + I_\eta \cdot \dot{\omega}_\eta \cdot \vec{e_2} + I_\zeta \cdot \dot{\omega}_\zeta \cdot \vec{e_3} + I_\xi \cdot \omega_\xi \cdot \dot{\vec{e_1}} + I_\eta \cdot \omega_\eta \cdot \dot{\vec{e_2}} + I_\zeta \cdot \omega_\zeta \cdot \dot{\vec{e_3}} =$$

$$= I_\xi \cdot \dot{\omega}_\xi \cdot \vec{e_1} + I_\eta \cdot \dot{\omega}_\eta \cdot \vec{e_2} + I_\zeta \cdot \dot{\omega}_\zeta \cdot \vec{e_3} + I_\xi \cdot \omega_\xi \cdot \left(\vec{\omega} \times \vec{e_1}\right) + I_\eta \cdot \omega_\eta \cdot \left(\vec{\omega} \times \vec{e_2}\right) + I_\zeta \cdot \omega_\zeta \cdot \left(\vec{\omega} \times \vec{e_3}\right) =$$

$$= I_\xi \cdot \dot{\omega}_\xi \cdot \vec{e_1} + I_\eta \cdot \dot{\omega}_\eta \cdot \vec{e_2} + I_\zeta \cdot \dot{\omega}_\zeta \cdot \vec{e_3} + I_\xi \cdot \omega_\xi \cdot \left(-\omega_\eta \cdot \vec{e_3} + \omega_\zeta \cdot \vec{e_2}\right) + I_\eta \cdot \omega_\eta \cdot \left(\omega_\xi \cdot \vec{e_3} - \omega_\zeta \cdot \vec{e_1}\right) + I_\zeta \cdot \omega_\zeta \cdot \left(-\omega_\xi \cdot \vec{e_2} + \omega_\eta \cdot \vec{e_1}\right) =$$

$$= \left[I_\xi \cdot \dot{\omega}_\xi \cdot \vec{e_1} - I_\eta \cdot \omega_\eta \cdot \omega_\zeta \cdot \vec{e_1} + I_\zeta \cdot \omega_\zeta \cdot \omega_\eta \cdot \vec{e_1}\right] + \left[I_\eta \cdot \dot{\omega}_\eta \cdot \vec{e_2} + I_\xi \cdot \omega_\xi \cdot \omega_\zeta \cdot \vec{e_2} - I_\zeta \cdot \omega_\zeta \cdot \omega_\xi \cdot \vec{e_2}\right] + \left[I_\zeta \cdot \dot{\omega}_\zeta \cdot \vec{e_3} - I_\xi \cdot \omega_\xi \cdot \omega_\eta \cdot \vec{e_3} + I_\eta \cdot \omega_\eta \cdot \omega_\xi \cdot \vec{e_3}\right] =$$

$$= \left[I_\xi \cdot \dot{\omega}_\xi - I_\eta \cdot \omega_\eta \cdot \omega_\zeta + I_\zeta \cdot \omega_\zeta \cdot \omega_\eta\right] \cdot \vec{e_1} + \left[I_\eta \cdot \dot{\omega}_\eta + I_\xi \cdot \omega_\xi \cdot \omega_\zeta - I_\zeta \cdot \omega_\zeta \cdot \omega_\xi\right] \cdot \vec{e_2} + \left[I_\zeta \cdot \dot{\omega}_\zeta - I_\xi \cdot \omega_\xi \cdot \omega_\eta + I_\eta \cdot \omega_\eta \cdot \omega_\xi\right] \cdot \vec{e_3} =$$

$$= \left[I_\xi \cdot \dot{\omega}_\xi - \left(I_\eta - I_\zeta\right) \cdot \omega_\eta \cdot \omega_\zeta\right] \cdot \vec{e_1} + \left[I_\eta \cdot \dot{\omega}_\eta + \left(I_\xi - I_\zeta\right) \cdot \omega_\xi \cdot \omega_\zeta\right] \cdot \vec{e_2} + \left[I_\zeta \cdot \dot{\omega}_\zeta - \left(I_\xi - I_\eta\right) \cdot \omega_\eta \cdot \omega_\xi\right] \cdot \vec{e_3}$$

Γενικευμένες εξισώσεις Euler :

$$I_\xi \cdot \dot{\omega}_\xi - \left(I_\eta - I_\zeta\right) \cdot \omega_\eta \cdot \omega_\zeta = \tau_{G\xi}$$
$$I_\eta \cdot \dot{\omega}_\eta - \left(I_\zeta - I_\xi\right) \cdot \omega_\xi \cdot \omega_\zeta = \tau_{G\eta}$$
$$I_\zeta \cdot \dot{\omega}_\zeta - \left(I_\xi - I_\eta\right) \cdot \omega_\eta \cdot \omega_\xi = \tau_{G\zeta}$$

\>

ΡΟΠΗ ως προς Ο≡Ω των Εξωτερικών Δυνάμεων :

$$\vec{\tau_G} = m \cdot g \cdot L \cdot \left(\hat{k} \times \vec{e_3}\right) = m \cdot g \cdot L \cdot \left(\left(\sin(\theta(t))\sin(\psi(t))\vec{e_1} + \cos(\psi(t))\sin(\theta(t))\vec{e_2} + \cos(\theta(t))\vec{e_3}\right) \times \vec{e_3}\right) =$$
$$= m \cdot g \cdot L \cdot \left(-\sin(\theta(t))\sin(\psi(t)) \cdot \vec{e_2} + \cos(\psi(t))\sin(\theta(t)) \cdot \vec{e_1}\right)$$

$$\tau_{G\xi} = m \cdot g \cdot L \cdot \cos(\psi(t))\sin(\theta(t))$$
$$\tau_{G\eta} = -m \cdot g \cdot L \cdot \sin(\theta(t))\sin(\psi(t))$$
$$\tau_{G\zeta} = 0$$

\> $\omega_x = diff(\theta(t), t) \cdot \cos(\varphi(t)) + diff(\psi(t), t) \cdot \sin(\theta(t)) \cdot \sin(\varphi(t))$

$$\omega_x = \dot{\theta}(t)\cos(\varphi(t)) + \dot{\psi}(t)\sin(\theta(t))\sin(\varphi(t)) \quad (3)$$

\> $\omega_y = diff(\theta(t), t) \cdot \sin(\varphi(t)) - diff(\psi(t), t) \cdot \sin(\theta(t)) \cdot \cos(\varphi(t))$

$$\omega_y = \dot{\theta}(t)\sin(\varphi(t)) - \dot{\psi}(t)\sin(\theta(t))\cos(\varphi(t)) \quad (4)$$

\> $\omega_z = diff(\psi(t), t) \cdot \cos(\theta(t)) + diff(\varphi(t), t)$

$$\omega_z = \dot{\psi}(t)\cos(\theta(t)) + \dot{\varphi}(t) \quad (5)$$

\>

\> $R := [[\cos(\varphi(t)) \cdot \cos(\psi(t)) - \cos(\theta(t)) \cdot \sin(\varphi(t)) \cdot \sin(\psi(t)), \cos(\psi(t)) \cdot \sin(\varphi(t)) + \cos(\theta(t)) \cdot \cos(\varphi(t)) \cdot \sin(\psi(t)), \sin(\theta(t)) \cdot \sin(\psi(t))],$
$[-\cos(\theta(t)) \cdot \cos(\psi(t)) \cdot \sin(\varphi(t)) - \cos(\varphi(t)) \cdot \sin(\psi(t)), \cos(\theta(t)) \cdot \cos(\varphi(t)) \cdot \cos(\psi(t)) - \sin(\varphi(t)) \cdot \sin(\psi(t)), \cos(\psi(t)) \cdot \sin(\theta(t))],$

$[\sin(\theta(t))\cdot\sin(\varphi(t)), -\cos(\varphi(t))\cdot\sin(\theta(t)), \cos(\theta(t))]]$:

> $RT := [[\cos(\varphi(t))\cdot\cos(\psi(t))-\cos(\theta(t))\cdot\sin(\varphi(t))\cdot\sin(\psi(t)), -\cos(\theta(t))\cdot\cos(\psi(t))\cdot\sin(\varphi(t))-\cos(\varphi(t))\cdot\sin(\psi(t)), \sin(\theta(t))\cdot\sin(\varphi(t))],$
$[\cos(\psi(t))\cdot\sin(\varphi(t))+\cos(\theta(t))\cdot\cos(\varphi(t))\cdot\sin(\psi(t)), \cos(\theta(t))\cdot\cos(\varphi(t))\cdot\cos(\psi(t))-\sin(\varphi(t))\cdot\sin(\psi(t)), -\cos(\varphi(t))\cdot\sin(\theta(t))],$
$[\sin(\theta(t))\cdot\sin(\psi(t)), \cos(\psi(t))\cdot\sin(\theta(t)), \cos(\theta(t))]]$:

> $simplify(R \cdot RT)$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (6)$$

> $\begin{bmatrix} e\_[1] \\ e\_[2] \\ e\_[3] \end{bmatrix} = R.\begin{bmatrix} \_i \\ \_j \\ \_k \end{bmatrix}$

$$\begin{bmatrix} \vec{e}_1 \\ \vec{e}_2 \\ \vec{e}_3 \end{bmatrix} = [[(\cos(\varphi(t))\cos(\psi(t)) - \cos(\theta(t))\sin(\varphi(t))\sin(\psi(t)))\hat{i} + (\cos(\psi(t))\sin(\varphi(t)) + \cos(\theta(t))\cos(\varphi(t))\sin(\psi(t)))\hat{j} + \sin(\theta(t))\sin(\psi(t))\hat{k}], [(-\cos(\theta(t))\cos(\psi(t))\sin(\varphi(t)) - \cos(\varphi(t))\sin(\psi(t)))\hat{i} + (\cos(\theta(t))\cos(\varphi(t))\cos(\psi(t)) - \sin(\varphi(t))\sin(\psi(t)))\hat{j} + \cos(\psi(t))\sin(\theta(t))\hat{k}], [\sin(\theta(t))\sin(\varphi(t))\hat{i} - \cos(\varphi(t))\sin(\theta(t))\hat{j} + \cos(\theta(t))\hat{k}]] \quad (7)$$

> $\begin{bmatrix} \_i \\ \_j \\ \_k \end{bmatrix} = RT.\begin{bmatrix} e\_[1] \\ e\_[2] \\ e\_[3] \end{bmatrix}$

$$\begin{bmatrix} \hat{i} \\ \hat{j} \\ \hat{k} \end{bmatrix} = [[(\cos(\varphi(t))\cos(\psi(t)) - \cos(\theta(t))\sin(\varphi(t))\sin(\psi(t)))\vec{e}_1 + (-\cos(\theta(t))\cos(\psi(t))\sin(\varphi(t)) - \cos(\varphi(t))\sin(\psi(t)))\vec{e}_2 + \sin(\theta(t))\sin(\varphi(t))\vec{e}_3], [(\cos(\psi(t))\sin(\varphi(t)) + \cos(\theta(t))\cos(\varphi(t))\sin(\psi(t)))\vec{e}_1 + (\cos(\theta(t))\cos(\varphi(t))\cos(\psi(t)) - \sin(\varphi(t))\sin(\psi(t)))\vec{e}_2 - \cos(\varphi(t))\sin(\theta(t))\vec{e}_3], [\sin(\theta(t))\sin(\psi(t))\vec{e}_1 + \cos(\psi(t))\sin(\theta(t))\vec{e}_2 + \cos(\theta(t))\vec{e}_3]] \quad (8)$$

>

Από το σχήμα :

$$\omega_x = \dot{\theta}\cdot\cos(\varphi(t)) + \dot{\psi}\cdot\sin(\theta(t))\cdot\sin(\varphi(t))$$
$$\omega_y = \dot{\theta}\cdot\sin(\varphi(t)) - \dot{\psi}\cdot\sin(\theta(t))\cdot\cos(\varphi(t))$$
$$\omega_z = \cos(\theta(t))\cdot\dot{\psi} + \dot{\varphi}$$

Από την σχέση :

$$\begin{bmatrix}\omega_\xi & \omega_\eta & \omega_\zeta\end{bmatrix} = \begin{bmatrix}\omega_x & \omega_y & \omega_z\end{bmatrix}\cdot RT$$
$$\omega_\xi = \dot{\varphi}(t)\sin(\psi(t))\sin(\theta(t)) + \dot{\theta}(t)\cos(\psi(t))$$
$$\omega_\eta = \dot{\varphi}(t)\cos(\psi(t))\sin(\theta(t)) - \dot{\theta}(t)\sin(\psi(t))$$
$$\omega_\zeta = \dot{\psi}(t) + \dot{\varphi}(t)\cos(\theta(t))$$

> *simplify*([**rhs((3))** **rhs((4))** **rhs((5))**].**RT**)

$$[\sin(\theta(t))\sin(\psi(t))\dot{\varphi}(t) + \cos(\psi(t))\dot{\theta}(t), \sin(\theta(t))\cos(\psi(t))\dot{\varphi}(t) - \sin(\psi(t))\dot{\theta}(t), \dot{\psi}(t) + \cos(\theta(t))\dot{\varphi}(t)] \quad (9)$$

> ω_ξ = **(9)**[1]

$$\omega_\xi = \sin(\theta(t))\sin(\psi(t))\dot{\varphi}(t) + \cos(\psi(t))\dot{\theta}(t) \quad (10)$$

> ω_η = **(9)**[2]

$$\omega_\eta = \sin(\theta(t))\cos(\psi(t))\dot{\varphi}(t) - \sin(\psi(t))\dot{\theta}(t) \quad (11)$$

> ω_ζ = **(9)**[3]

$$\omega_\zeta = \dot{\psi}(t) + \cos(\theta(t))\dot{\varphi}(t) \quad (12)$$

>

Οι Γενικευμένες Εξισώσεις Euler εκφραζόμενες συναρτήσει των Γωνιών Euler γίνονται :

Αλλάζουμε την γραμματοσειρά γιά το *I* στα Ελληνικά Ι .!!!

ΜΕ ΕΞΩΤΕΡΙΚΕΣ ΡΟΠΕΣ

>

> I[ξ]·*diff*(ω[ξ](t), t) − (I[η] − I[ζ])·ω[η](t)·ω[ζ](t) = $\tau_{G\xi}$

$$I_\xi\dot{\omega}_\xi(t) - (I_\eta - I_\zeta)\,\omega_\eta(t)\,\omega_\zeta(t) = \tau_{G\xi} \quad (13)$$

> I[η]·*diff*(ω[η](t), t) − (I[ζ] − I[ξ])·ω[ξ](t)·ω[ζ](t) = $\tau_{G\eta}$

$$I_\eta\dot{\omega}_\eta(t) - (I_\zeta - I_\xi)\,\omega_\xi(t)\,\omega_\zeta(t) = \tau_{G\eta} \quad (14)$$

> I[ζ]·*diff*(ω[ζ](t), t) − (I[ξ] − I[η])·ω[η](t)·ω[ξ](t) = $\tau_{G\zeta}$

$$I_\zeta\dot{\omega}_\zeta(t) - (I_\xi - I_\eta)\,\omega_\eta(t)\,\omega_\xi(t) = \tau_{G\zeta} \quad (15)$$

> *simplify*(*subs*([*diff*(ω[ξ](t), t) = *diff*(*rhs*(**(10)**), t), ω[ξ](t) = *rhs*(**(10)**), ω[η](t) = *rhs*(**(11)**), ω[ζ](t) = *rhs*(**(12)**)], **(13)**))

(16)

$$\ddot{\varphi}(t)\sin(\theta(t))\sin(\psi(t))\,I_{\xi}+\ddot{\theta}(t)\cos(\psi(t))\,I_{\xi}-\cos(\theta(t))\sin(\theta(t))\cos(\psi(t))\left(I_{\eta}-I_{\zeta}\right)\dot{\varphi}(t)^{2}+\left(\cos(\theta(t))\sin(\psi(t))\left(I_{\xi}+I_{\eta}-I_{\zeta}\right)\dot{\theta}(t)+\sin(\theta(t))\,\dot{\psi}(t)\cos(\psi(t))\left(I_{\xi}-I_{\eta}+I_{\zeta}\right)\right)\dot{\varphi}(t)-\dot{\psi}(t)\,\dot{\theta}(t)\sin(\psi(t))\left(I_{\xi}-I_{\eta}+I_{\zeta}\right)=\tau_{G\xi} \qquad \textbf{(16)}$$

```
> simplify(subs([diff(ω[η](t), t) = diff(rhs((11)), t), ω[ξ](t) = rhs((10)), ω[η](t) = rhs((11)), ω[ζ](t) = rhs((12))], (14)))
```

$$\ddot{\varphi}(t)\sin(\theta(t))\cos(\psi(t))\,I_{\eta}-\ddot{\theta}(t)\sin(\psi(t))\,I_{\eta}+\cos(\theta(t))\sin(\theta(t))\sin(\psi(t))\left(I_{\xi}-I_{\zeta}\right)\dot{\varphi}(t)^{2}+\left(\cos(\theta(t))\cos(\psi(t))\left(I_{\xi}+I_{\eta}-I_{\zeta}\right)\dot{\theta}(t)+\sin(\theta(t))\,\dot{\psi}(t)\sin(\psi(t))\left(I_{\xi}-I_{\eta}-I_{\zeta}\right)\right)\dot{\varphi}(t)+\dot{\psi}(t)\,\dot{\theta}(t)\cos(\psi(t))\left(I_{\xi}-I_{\eta}-I_{\zeta}\right)=\tau_{G\eta} \qquad \textbf{(17)}$$

```
> simplify(subs([diff(ω[ζ](t), t) = diff(rhs((12)), t), ω[ξ](t) = rhs((10)), ω[η](t) = rhs((11)), ω[ζ](t) = rhs((12))], (15)))
```

$$\ddot{\varphi}(t)\cos(\theta(t))\,I_{\zeta}+\ddot{\psi}(t)\,I_{\zeta}+\cos(\psi(t))\sin(\psi(t))\left(\cos(\theta(t))-1\right)\left(\cos(\theta(t))+1\right)\left(I_{\xi}-I_{\eta}\right)\dot{\varphi}(t)^{2}-2\left(\left(I_{\xi}-I_{\eta}\right)\cos(\psi(t))^{2}-\frac{I_{\xi}}{2}+\frac{I_{\eta}}{2}+\frac{I_{\zeta}}{2}\right)\sin(\theta(t))\,\dot{\theta}(t)\,\dot{\varphi}(t)+\dot{\theta}(t)^{2}\cos(\psi(t))\sin(\psi(t))\left(I_{\xi}-I_{\eta}\right)=\tau_{G\zeta} \qquad \textbf{(18)}$$

```
>
```

Γιά $I_{\xi} = I_{\eta} = I$ οι εξισώσεις γίνονται :

```
> eq1R := subs([I[ξ] = I, I[η] = I], (16))
```

$$eq1R := \ddot{\varphi}(t)\sin(\theta(t))\sin(\psi(t))\,I+\ddot{\theta}(t)\cos(\psi(t))\,I-\cos(\theta(t))\sin(\theta(t))\cos(\psi(t))\left(I-I_{\zeta}\right)\dot{\varphi}(t)^{2}+\left(\cos(\theta(t))\sin(\psi(t))\left(2I-I_{\zeta}\right)\dot{\theta}(t)+\sin(\theta(t))\,\dot{\psi}(t)\cos(\psi(t))\,I_{\zeta}\right)\dot{\varphi}(t)-\sin(\psi(t))\,\dot{\theta}(t)\,\dot{\psi}(t)\,I_{\zeta}=\tau_{G\xi} \qquad \textbf{(19)}$$

```
> eq2R := subs([I[ξ] = I, I[η] = I], (17))
```

$$eq2R := \ddot{\varphi}(t)\sin(\theta(t))\cos(\psi(t))\,I-\ddot{\theta}(t)\sin(\psi(t))\,I+\cos(\theta(t))\sin(\theta(t))\sin(\psi(t))\left(I-I_{\zeta}\right)\dot{\varphi}(t)^{2}+\left(\cos(\theta(t))\cos(\psi(t))\left(2I-I_{\zeta}\right)\dot{\theta}(t)-\sin(\theta(t))\,\dot{\psi}(t)\sin(\psi(t))\,I_{\zeta}\right)\dot{\varphi}(t)-\cos(\psi(t))\,\dot{\theta}(t)\,\dot{\psi}(t)\,I_{\zeta}=\tau_{G\eta} \qquad \textbf{(20)}$$

```
> eq3R := simplify(subs([I[ξ] = I, I[η] = I], (18)))
```

$$eq3R := I_{\zeta}\left(\ddot{\psi}(t)-\dot{\theta}(t)\sin(\theta(t))\,\dot{\varphi}(t)+\cos(\theta(t))\,\ddot{\varphi}(t)\right)=\tau_{G\zeta} \qquad \textbf{(21)}$$

```
>
> eq1F := subs([I[ξ] = I, I[η] = I], (13))
```

$$eq1F := I\,\dot{\omega}_{\xi}(t)-\left(I-I_{\zeta}\right)\omega_{\eta}(t)\,\omega_{\zeta}(t)=\tau_{G\xi} \qquad \textbf{(22)}$$

```
> eq2F := subs([I[ξ] = I, I[η] = I], (14))
```

$$eq2F := I\,\dot{\omega}_{\eta}(t)-\left(I_{\zeta}-I\right)\omega_{\xi}(t)\,\omega_{\zeta}(t)=\tau_{G\eta} \qquad \textbf{(23)}$$

```
> eq3F := simplify(subs([I[ξ] = I, I[η] = I], (15)))
```

$$eq3F := I_{\zeta}\,\dot{\omega}_{\zeta}(t)=\tau_{G\zeta} \qquad \textbf{(24)}$$

```
>
```

> $m := 1.521004933$

$$m := 1.521004933 \quad (25)$$

> $g := 9.81$

$$g := 9.81 \quad (26)$$

> $L := 1.253962274$

$$L := 1.253962274 \quad (27)$$

> $m \cdot g \cdot L$

$$18.71044431 \quad (28)$$

> $M := m$

$$M := 1.521004933 \quad (29)$$

>

> $Iz := 0.2993796771$

$$Iz := 0.2993796771 \quad (30)$$

> $Ix := 2.554540615$

$$Ix := 2.554540615 \quad (31)$$

> $Iy := Ix$

$$Iy := 2.554540615 \quad (32)$$

>

> $EQ1R := subs\left(\left[\mathrm{I} = \mathbf{(31)}, \mathrm{I}[\zeta] = \mathbf{(30)}, \tau_{G\xi} = \mathbf{(28)} \cdot \cos(\psi(t)) \cdot \sin(\theta(t))\right], \mathbf{(19)}\right)$

$$EQ1R := 2.554540615 \sin(\theta(t)) \sin(\psi(t)) \ddot{\varphi}(t) + 2.554540615 \cos(\psi(t)) \ddot{\theta}(t) - 2.255160938 \cos(\theta(t)) \sin(\theta(t)) \cos(\psi(t)) \dot{\varphi}(t)^2 + \left(4.809701553 \cos(\theta(t)) \sin(\psi(t)) \dot{\theta}(t) + 0.2993796771 \sin(\theta(t)) \dot{\psi}(t) \cos(\psi(t))\right) \dot{\varphi}(t) - 0.2993796771 \dot{\psi}(t) \sin(\psi(t)) \dot{\theta}(t) = 18.71044431 \cos(\psi(t)) \sin(\theta(t)) \quad (33)$$

> $EQ2R := subs\left(\left[\mathrm{I} = \mathbf{(31)}, \mathrm{I}[\zeta] = \mathbf{(30)}, \tau_{G\eta} = -\mathbf{(28)} \cdot \sin(\psi(t)) \cdot \sin(\theta(t))\right], \mathbf{(20)}\right)$

$$EQ2R := 2.554540615 \sin(\theta(t)) \cos(\psi(t)) \ddot{\varphi}(t) - 2.554540615 \sin(\psi(t)) \ddot{\theta}(t) + 2.255160938 \cos(\theta(t)) \sin(\theta(t)) \sin(\psi(t)) \dot{\varphi}(t)^2 + \left(4.809701553 \cos(\theta(t)) \cos(\psi(t)) \dot{\theta}(t) - 0.2993796771 \sin(\theta(t)) \dot{\psi}(t) \sin(\psi(t))\right) \dot{\varphi}(t) - 0.2993796771 \dot{\psi}(t) \cos(\psi(t)) \dot{\theta}(t) = -18.71044431 \sin(\theta(t)) \sin(\psi(t)) \quad (34)$$

> $EQ3R := subs\left(\left[\mathrm{I} = \mathbf{(31)}, \mathrm{I}[\zeta] = \mathbf{(30)}, \tau_{G\zeta} = 0\right], \mathbf{(21)}\right)$

$$EQ3R := 0.2993796771 \ddot{\psi}(t) - 0.2993796771 \dot{\theta}(t) \sin(\theta(t)) \dot{\varphi}(t) + 0.2993796771 \cos(\theta(t)) \ddot{\varphi}(t) = 0 \quad (35)$$

>

Αρχικές Συνθήκες :

> $icsR := \varphi(0) = \frac{\mathrm{Pi}}{6}, \mathrm{D}(\varphi)(0) = 0, \theta(0) = \frac{\mathrm{Pi}}{4}, \mathrm{D}(\theta)(0) = 0, \psi(0) = 0, \mathrm{D}(\psi)(0) = 2 \cdot \mathrm{Pi} \cdot 100$

$$icsR := \varphi(0) = \frac{\pi}{6}, \mathrm{D}(\varphi)(0) = 0, \theta(0) = \frac{\pi}{4}, \mathrm{D}(\theta)(0) = 0, \psi(0) = 0, \mathrm{D}(\psi)(0) = 200\,\pi \quad (36)$$

>

Αριθμητική Επίλυση των Διαφορικών Εξισώσεων (33),(34),(35).
maxfun=5000000 , για t=0..10 !!!!!

maxfun=15.000.000 ,για t=0..31 !!!!!!

Με maxfun=0 έχουμε αποτελεσματικότητα .!!!

> $sysR := EQ1R, EQ2R, EQ3R :$

> $solR := dsolve(\{sysR, icsR\}, numeric, output = listprocedure, maxfun = 0)$

$solR := [t = \mathbf{proc}(t)\ \ldots\ \mathbf{end\ proc}, \psi(t) = \mathbf{proc}(t)\ \ldots\ \mathbf{end\ proc}, \dot{\psi}(t) = \mathbf{proc}(t)$ **(37)**

...

$\mathbf{end\ proc}, \theta(t) = \mathbf{proc}(t)\ \ldots\ \mathbf{end\ proc}, \dot{\theta}(t) = \mathbf{proc}(t)\ \ldots\ \mathbf{end\ proc}, \varphi(t) = \mathbf{proc}(t)$

...

$\mathbf{end\ proc}, \dot{\varphi}(t) = \mathbf{proc}(t)\ \ldots\ \mathbf{end\ proc}]$

>

> $solR(100)$

$[t(100) = 100., \psi(t)(100) = 62824.8221174724, (\dot{\psi}(t))(100) = 628.232064540726,$ **(38)**
$\theta(t)(100) = 0.786575857921940, (\dot{\theta}(t))(100) = -0.0687207325484587, \varphi(t)(100)$
$= 10.4811868741577, (\dot{\varphi}(t))(100) = 0.122425626065090]$

>

Απαιτούμενος χρόνος γιά πλήρη μετάπτωση (precession) φ:

$$T[precession] := fsolve\left(rhs(solR[6](t)) = \frac{Pi}{6} + 2 \cdot Pi, t = 0..100\right)$$

$$T_{precession} := 63.12003130\ \ s$$

> $T[precession] := fsolve\left(rhs(solR[6](t)) = \frac{Pi}{6} + 2 \cdot Pi, t = 0..100\right)$

$$T_{precession} := 63.12003130 \quad (39)$$

ΣΥΝΤΕΤΑΓΜΕΝΕΣ ΣΗΜΕΙΩΝ ΣΤΟ ΑΔΡΑΝΕΙΑΚΟ ΣΥΣΤΗΜΑ ΣΥΝΤΕΤΑΓΜΕΝΩΝ .

ΠΡΟΣΟΧΗ ΣΤΟ ΜΗΤΡΩΟ

ΜΕΤΑΒΙΒΑΣΕΩΣ !!!!!!

>

> $RT := [[\cos(\varphi(t))\cdot\cos(\psi(t)) - \cos(\theta(t))\cdot\sin(\varphi(t))\cdot\sin(\psi(t)), -\cos(\theta(t))\cdot\cos(\psi(t))\cdot\sin(\varphi(t)) - \cos(\varphi(t))\cdot\sin(\psi(t)), \sin(\theta(t))\cdot\sin(\varphi(t))], [\cos(\psi(t))\cdot\sin(\varphi(t)) + \cos(\theta(t))\cdot\cos(\varphi(t))\cdot\sin(\psi(t)), \cos(\theta(t))\cdot\cos(\varphi(t))\cdot\cos(\psi(t)) - \sin(\varphi(t))\cdot\sin(\psi(t)), -\cos(\varphi(t))\cdot\sin(\theta(t))], [\sin(\theta(t))\cdot\sin(\psi(t)), \cos(\psi(t))\cdot\sin(\theta(t)), \cos(\theta(t))]]:$

>

> $\Phi := seq(rhs(solR[6](t)), t = 0..63):$

>

> $\Theta := seq(rhs(solR[4](t)), t = 0..63):$

>

Το Π είναι η γωνία Ψ :

> $\Pi := seq(rhs(solR[2](t)), t = 0..63):$

>

> $RR := seq(simplify(subs(\{\varphi(t) = \Phi[i], \theta(t) = \Theta[i], \psi(t) = \Pi[i]\}, RT)), i = 1..64):$

> $RR[1]$

$$\begin{bmatrix} 0.8660254038 & -0.3535533906 & 0.3535533906 \\ 0.5000000000 & 0.6123724357 & -0.6123724357 \\ 0. & 0.7071067812 & 0.7071067812 \end{bmatrix} \tag{40}$$

> $RR[64]$

$$\begin{bmatrix} -0.5767358832 & -0.7393819295 & 0.3474047831 \\ 0.4541538831 & -0.6436734344 & -0.6159778894 \\ 0.6790581503 & -0.1974813208 & 0.7070227411 \end{bmatrix} \tag{41}$$

>

ΠΑΡΑΒΟΛΟΕΙΔΕΣ

>

> $\begin{bmatrix} u\cos(v) \\ u\sin(v) \\ -u^2+2 \end{bmatrix}$

$$\begin{bmatrix} u\cos(v) \\ u\sin(v) \\ -u^2+2 \end{bmatrix} \tag{42}$$

> $RR[1].\mathbf{(42)}$

(43)

$$\begin{bmatrix} 0.8660254038\, u \cos(v) - 0.3535533906\, u \sin(v) - 0.3535533906\, u^2 + 0.7071067812 \\ 0.5000000000\, u \cos(v) + 0.6123724357\, u \sin(v) + 0.6123724357\, u^2 - 1.224744871 \\ 1.414213562 + 0.7071067812\, u \sin(v) - 0.7071067812\, u^2 \end{bmatrix} \quad \textbf{(43)}$$

> ***PARAB*** := [**(43)**[**1**][**1**], **(43)**[**2**][**1**], **(43)**[**3**][**1**]]

$$PARAB := [0.8660254038\, u \cos(v) - 0.3535533906\, u \sin(v) - 0.3535533906\, u^2 + 0.7071067812, 0.5000000000\, u \cos(v) + 0.6123724357\, u \sin(v) + 0.6123724357\, u^2 - 1.224744871, 1.414213562 + 0.7071067812\, u \sin(v) - 0.7071067812\, u^2] \quad \textbf{(44)}$$

>

> ***PRR*** := ***seq***(***RR***[***m***].(42), ***m*** = **1 ..64**) :

> *PRR*[1]

$$\begin{bmatrix} 0.8660254038\, u \cos(v) - 0.3535533906\, u \sin(v) - 0.3535533906\, u^2 + 0.7071067812 \\ 0.5000000000\, u \cos(v) + 0.6123724357\, u \sin(v) + 0.6123724357\, u^2 - 1.224744871 \\ 1.414213562 + 0.7071067812\, u \sin(v) - 0.7071067812\, u^2 \end{bmatrix} \quad \textbf{(45)}$$

> [*PRR*[1][1][1], *PRR*[1][2][1], *PRR*[1][3][1]]

$$[0.8660254038\, u \cos(v) - 0.3535533906\, u \sin(v) - 0.3535533906\, u^2 + 0.7071067812, 0.5000000000\, u \cos(v) + 0.6123724357\, u \sin(v) + 0.6123724357\, u^2 - 1.224744871, 1.414213562 + 0.7071067812\, u \sin(v) - 0.7071067812\, u^2] \quad \textbf{(46)}$$

>

> *PARABOLOEIDES* := *display*(*seq*(*plot3d*([*PRR*[*n*][1][1], *PRR*[*n*][2][1], *PRR*[*n*][3][1]], *u* = 0 ..0.7922869910, *v* = 0 ..2 Pi, *style* = *surface*, *color* = *gold*, *transparency* = 0.00), *n* = 1 ..64), *insequence* = *true*) :

>

ΚΩΝΟΣ

>

$$> \begin{bmatrix} u \cos(v) \\ u \sin(v) \\ u\sqrt{3} \end{bmatrix}$$

$$\begin{bmatrix} u \cos(v) \\ u \sin(v) \\ u\sqrt{3} \end{bmatrix} \quad \textbf{(47)}$$

> *RR*[1].**(47)**

$$\begin{bmatrix} 0.8660254038\, u \cos(v) - 0.3535533906\, u \sin(v) + 0.3535533906\, u\sqrt{3} \\ 0.5000000000\, u \cos(v) + 0.6123724357\, u \sin(v) - 0.6123724357\, u\sqrt{3} \\ 0.7071067812\, u \sin(v) + 0.7071067812\, u\sqrt{3} \end{bmatrix} \quad \textbf{(48)}$$

> ***KRR*** := ***seq***(***RR***[***m***].(47), ***m*** = **1 ..64**) :

> [*KRR*[1][1][1], *KRR*[1][2][1], *KRR*[1][3][1]]

$$[0.8660254038\,u\cos(v) - 0.3535533906\,u\sin(v) + 0.3535533906\,u\sqrt{3},\; 0.5000000000\,u\cos(v) + 0.6123724357\,u\sin(v) - 0.6123724357\,u\sqrt{3},\; 0.7071067812\,u\sin(v) + 0.7071067812\,u\sqrt{3}] \quad (49)$$

>

> *KONOS* := *display*(*seq*(*plot3d*([*KRR*[*n*][1][1], *KRR*[*n*][2][1], *KRR*[*n*][3][1]], *u* = 0 ..0.7922869910, *v* = 0 ..2 Pi, *style* = *surface*, *color* = *gold*, *transparency* = 0.00), *n* = 1 ..64), *insequence* = *true*) :

>

> *KNS* := [**(48)**[1][1], **(48)**[2][1], **(48)**[3][1]]

$$KNS := [0.8660254038\,u\cos(v) - 0.3535533906\,u\sin(v) + 0.3535533906\,u\sqrt{3},\; 0.5000000000\,u\cos(v) + 0.6123724357\,u\sin(v) - 0.6123724357\,u\sqrt{3},\; 0.7071067812\,u\sin(v) + 0.7071067812\,u\sqrt{3}] \quad (50)$$

> $evalf\left(-\frac{\sqrt{3}}{2} + \frac{\sqrt{11}}{2}\right)$

$$0.7922869910 \quad (51)$$

>

> $\begin{bmatrix} 0.7922869910\cos(v) \\ 0.7922869910\sin(v) \\ 1.372281324 \end{bmatrix}$

$$\begin{bmatrix} 0.7922869910\cos(v) \\ 0.7922869910\sin(v) \\ 1.372281324 \end{bmatrix} \quad (52)$$

> ***KTRR* := *seq*(*RR*[*m*].(52), *m* = 1 ..64)** :

> *KTRR*[64]

$$\begin{bmatrix} -0.4569403375\cos(v) - 0.5858026841\sin(v) + 0.4767370957 \\ 0.3598202135\cos(v) - 0.5099740885\sin(v) - 0.8452949536 \\ 0.5380089386\cos(v) - 0.1564618814\sin(v) + 0.9702341033 \end{bmatrix} \quad (53)$$

>

> ***KYKLOSTOMHS* := *display*(*seq*(*spacecurve*([*KTRR*[*n*][1][1], *KTRR*[*n*][2][1], *KTRR*[*n*][3][1]], *v* = 0 ..2 · Pi, *color* = *green*, *thickness* = 3), *n* = 1 ..64), *insequence* = *true*)** :

>

ΚΕΝΤΡΟ ΚΥΚΛΟΥ ΤΟΜΗΣ

> $\begin{bmatrix} 0 \\ 0 \\ 1.372281324 \end{bmatrix}$

(54)

$$\begin{bmatrix} 0 \\ 0 \\ 1.372281324 \end{bmatrix} \quad (54)$$

> **KKTRR := seq(RR[m].(54), m = 1 ..64) :**

>

> *KENTROKYKLOYTOMHS* := *display*(*seq*(*pointplot3d*([*KKTRR*[*n*][1], *KKTRR*[*n*][2], *KKTRR*[*n*][3]], *symbol* = *solidcircle*, *symbolsize* = 10, *color* = *green*), *n* = 1 ..64), *insequence* = *true*) :

>

ΣΗΜΕΙΟ ΚΥΚΛΟΥ ΤΟΜΗΣ

>

> $evalf\left(\begin{bmatrix} 0.3961434955\sqrt{2} \\ 0.3961434955\sqrt{2} \\ 1.372281324 \end{bmatrix}\right)$

$$\begin{bmatrix} 0.5602315038 \\ 0.5602315038 \\ 1.372281324 \end{bmatrix} \quad (55)$$

> **SKTRR := seq(RR[m].(55), m = 1 ..64) :**

> *SHMEIOKYKLOYTOMHS* := *display*(*seq*(*pointplot3d*([*SKTRR*[*n*][1], *SKTRR*[*n*][2], *SKTRR*[*n*][3]], *symbol* = *solidcircle*, *symbolsize* = 10, *color* = *green*), *n* = 1 ..64), *insequence* = *true*) :

>

ΑΚΤΙΝΑ ΚΥΚΛΟΥ ΤΟΜΗΣ

> $evalf\left(\begin{bmatrix} 0.3961434955\, t\sqrt{2} \\ 0.3961434955\, t\sqrt{2} \\ 1.372281324 \end{bmatrix}\right)$

$$\begin{bmatrix} 0.5602315038\, t \\ 0.5602315038\, t \\ 1.372281324 \end{bmatrix} \quad (56)$$

> **AKTRR := seq(RR[m].(56), m = 1 ..64) :**

> *AKTRR*[1][1][1]

$$0.2871029666\, t + 0.4851747150 \quad (57)$$

> *AKTINAKYKLOY* := *display*(*seq*(*spacecurve*([*AKTRR*[*n*][1][1], *AKTRR*[*n*][2][1], *AKTRR*[*n*][3][1]], *t* = 0 ..1, *thickness* = 3, *color* = *green*), *n* = 1 ..64), *insequence* = *true*) :

>

ΚΥΡΙΟΣ ΑΞΟΝΑΣ ζ

> $\begin{bmatrix} 0 \\ 0 \\ z \end{bmatrix}$

$$\begin{bmatrix} 0 \\ 0 \\ z \end{bmatrix} \quad (58)$$

> **KAZRR := seq(RR[m].(58), m = 1 ..64) :**

> KAZ := display(seq(spacecurve([KAZRR[n][1][1], KAZRR[n][2][1], KAZRR[n][3][1]], z = 0 ..2.0, linestyle = 3, thickness = 2, color = blue), n = 1 ..64), insequence = true) :

>

ΚΟΡΥΦΗ ΠΑΡΑΒΟΛΟΕΙΔΟΥΣ

> $\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$

$$\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} \quad (59)$$

> **KPAR := seq(RR[m].(59), m = 1 ..64) :**

> KPAR[64][1][1]

0.694809566200000 **(60)**

> KPAR[64]

$$\begin{bmatrix} 0.694809566200000 \\ -1.23195577880000 \\ 1.41404548220000 \end{bmatrix} \quad (61)$$

> KORYFHPARABOL := display(seq(pointplot3d([KPAR[n][1][1], KPAR[n][2][1], KPAR[n][3][1]], symbol = solidcircle, symbolsize = 10, color = red), n = 1 ..64), insequence = true) :

>

ΚΕΝΤΡΟ ΜΑΖΑΣ

> $\begin{bmatrix} 0 \\ 0 \\ 1.253962274 \end{bmatrix}$

$$\begin{bmatrix} 0 \\ 0 \\ 1.253962274 \end{bmatrix} \quad (62)$$

> **KMAZ := seq(RR[m].(62), m = 1 ..64) :**

> KMAZ[64][1][1]

(63)

$$0.435632491814553 \quad (63)$$

> $KENTROMAZAS := display(seq(pointplot3d([KMAZ[n][1][1], KMAZ[n][2][1], KMAZ[n][3][1]], symbol = solidcircle, symbolsize = 10, color = yellow), n = 1..64), insequence = true):$

>

ΚΥΡΙΟΣ ΑΞΟΝΑΣ ξ

> $\begin{bmatrix} x \\ 0 \\ 1.253962274 \end{bmatrix}$

$$\begin{bmatrix} x \\ 0 \\ 1.253962274 \end{bmatrix} \quad (64)$$

> $\boldsymbol{KAXRR := seq(RR[m].(64), m = 1..64)}:$

>

> $KAXRR[64][1][1]$

$$-0.5767358832\, x + 0.4356324918 \quad (65)$$

> $KAX := display(seq(spacecurve([KAXRR[n][1][1], KAXRR[n][2][1], KAXRR[n][3][1]], x = -1.0..1.0, linestyle = 3, thickness = 2, color = blue), n = 1..64), insequence = true):$

>

ΚΥΡΙΟΣ ΑΞΟΝΑΣ η

> $\begin{bmatrix} 0 \\ y \\ 1.253962274 \end{bmatrix}$

$$\begin{bmatrix} 0 \\ y \\ 1.253962274 \end{bmatrix} \quad (66)$$

> $\boldsymbol{KAYRR := seq(RR[m].(66), m = 1..64)}:$

>

> $KAYRR[64][1][1]$

$$0.4356324918 - 0.7393819295\, y \quad (67)$$

> $KAY := display(seq(spacecurve([KAYRR[n][1][1], KAYRR[n][2][1], KAYRR[n][3][1]], y = -1.0..1.0, linestyle = 3, thickness = 2, color = blue), n = 1..64), insequence = true):$

>

ΒΕΛΟΣ ΚΥΡΙΟΥ ΑΞΟΝΑ ζ .!!!!!

```
> subs(z = 2.2, [0, 0, z])
```

$$\begin{bmatrix} 0 \\ 0 \\ 2.2 \end{bmatrix} \qquad \textbf{(68)}$$

```
> ZARROW := seq(RR[m].(68), m = 1 ..64) :
> ZARROW[64][1][1]
```

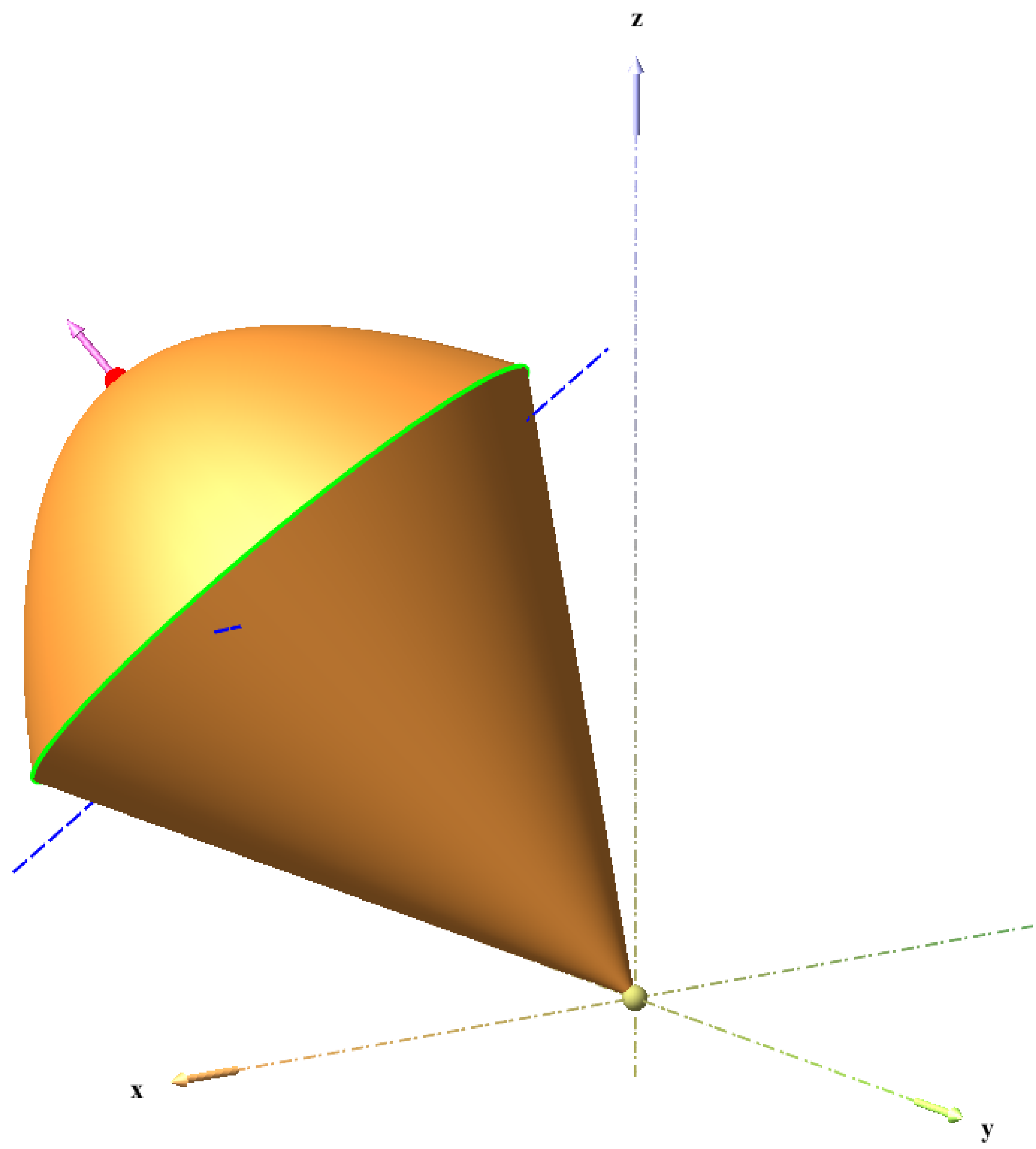
0.764290522820000 **(69)**

```
>
> BELOSZ := display(seq(arrow([ZARROW[n][1][1], ZARROW[n][2][1],
      ZARROW[n][3][1]], width = 0.02, head_length = 0.05, shape = cylindrical_arrow), n
      = 1 ..64), insequence = true) :
>
```

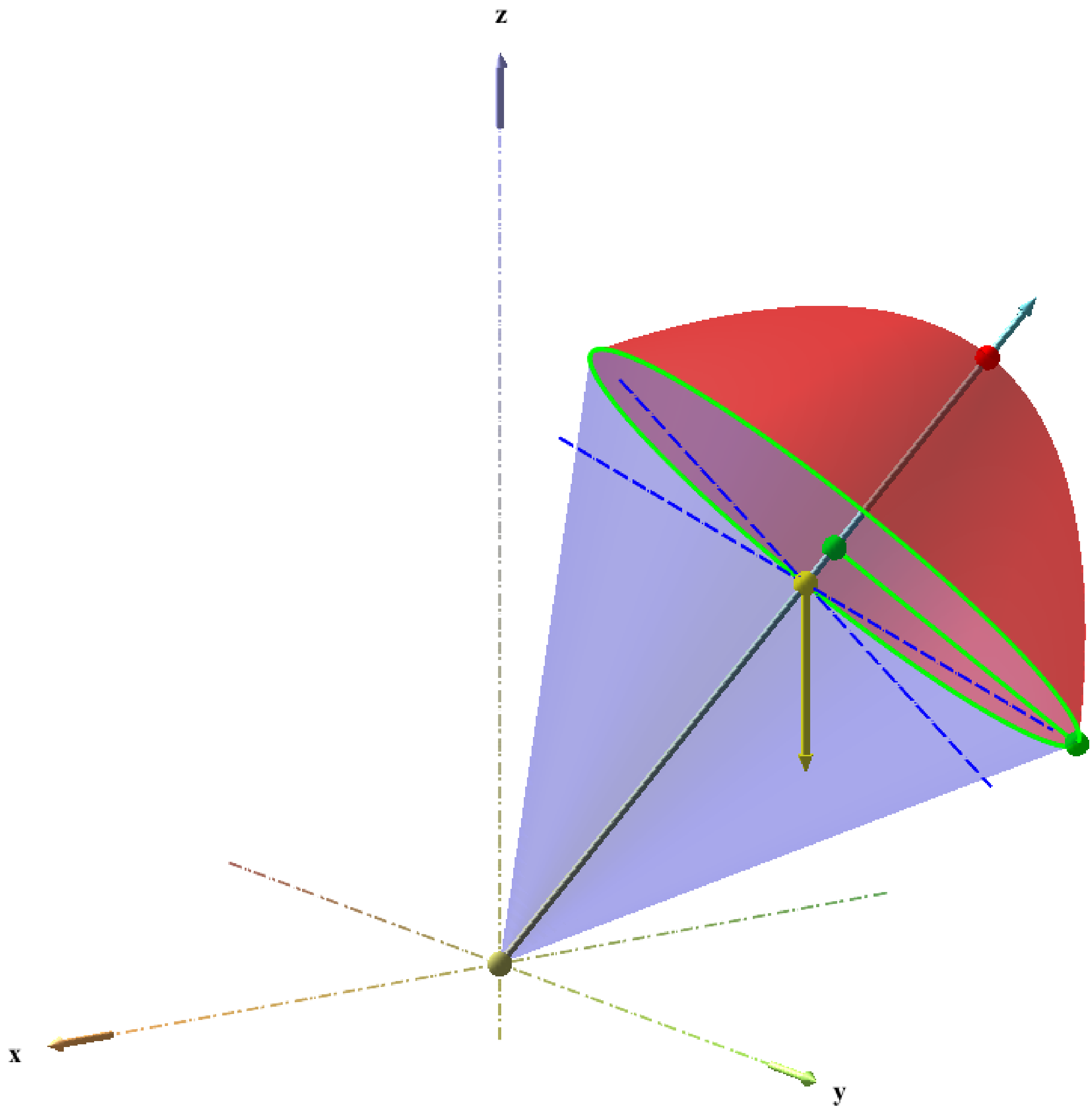
ΒΕΛΟΣ ΜΑΖΑΣ .!!!!!

```
> KENTROMAZAS := display(seq(pointplot3d([KMAZ[n][1][1], KMAZ[n][2][1],
      KMAZ[n][3][1]], symbol = solidcircle, symbolsize = 10, color = yellow), n = 1 ..64),
      insequence = true) :
> BELOSM := display(seq(arrow([KMAZ[n][1][1], KMAZ[n][2][1], KMAZ[n][3][1]],
      [0, 0, -0.5], color = yellow, width = 0.02, head_length = 0.05, shape = cylindrical_arrow)
      , n = 1 ..64), insequence = true) :
>
>
> PAR := plot3d(PARAB, u = 0 ..0.7922869910, v = 0 ..2·Pi, style = surface, color = red) :
> KN := plot3d(KNS, u = 0 ..0.7922869910, v = 0 ..2·Pi, style = surface, color = blue) :
> OO := pointplot3d([0, 0, 0], symbol = solidcircle, symbolsize = 10) :
> axX := spacecurve([x, 0, 0], x = -1.2 ..1.2, linestyle = 4, thickness = 1) :
> axY := spacecurve([0, y, 0], y = -1.2 ..1.2, linestyle = 4, thickness = 1) :
> axZ := spacecurve([0, 0, z], z = -0.2 ..2.2, linestyle = 4, thickness = 1) :
> ARaxX := arrow([1.2, 0, 0], [0.2, 0, 0], width = 0.02, length = 0.2, shape
      = cylindrical_arrow) :
> ARaxY := arrow([0, 1.2, 0], [0, 0.2, 0], width = 0.02, length = 0.2, shape
      = cylindrical_arrow) :
> ARaxZ := arrow([0, 0, 2.2], [0, 0, 0.2], width = 0.02, length = 0.2, shape
      = cylindrical_arrow) :
> tX := textplot3d([1.5, 0.0, 0, "x"], font = [arial, bold, 14]) :
> tY := textplot3d([0, 1.5, 0, "y"], font = [arial, bold, 14]) :
> tZ := textplot3d([0, 0, 2.5, "z"], font = [arial, bold, 14]) :
>
> display(PAR, KN, OO, axX, axY, axZ, ARaxX, ARaxY, ARaxZ, tX, tY, tZ, axes = none) :
> display(PARABOLOEIDES, KONOS, KYKLOSTOMHS, KENTROKYKLOYTOMHS,
      SHMEIOKYKLOYTOMHS, AKTINAKYKLOY, KAZ, KORYFHPARABOL,
      KENTROMAZAS, KAX, KAY, BELOSZ, BELOSM, OO, axX, axY, axZ, ARaxX, ARaxY,
      ARaxZ, tX, tY, tZ, scaling = constrained, axes = none, title
      = "ΜΕΤΑΠΤΩΣΗ (Precession φ) ΣΤΡΟΒΟΥ (ΣΒΟΥΡΑΣ)\nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ",
      titlefont = [arial, bold, 14], orientation = [55, 75, 0]) :
```

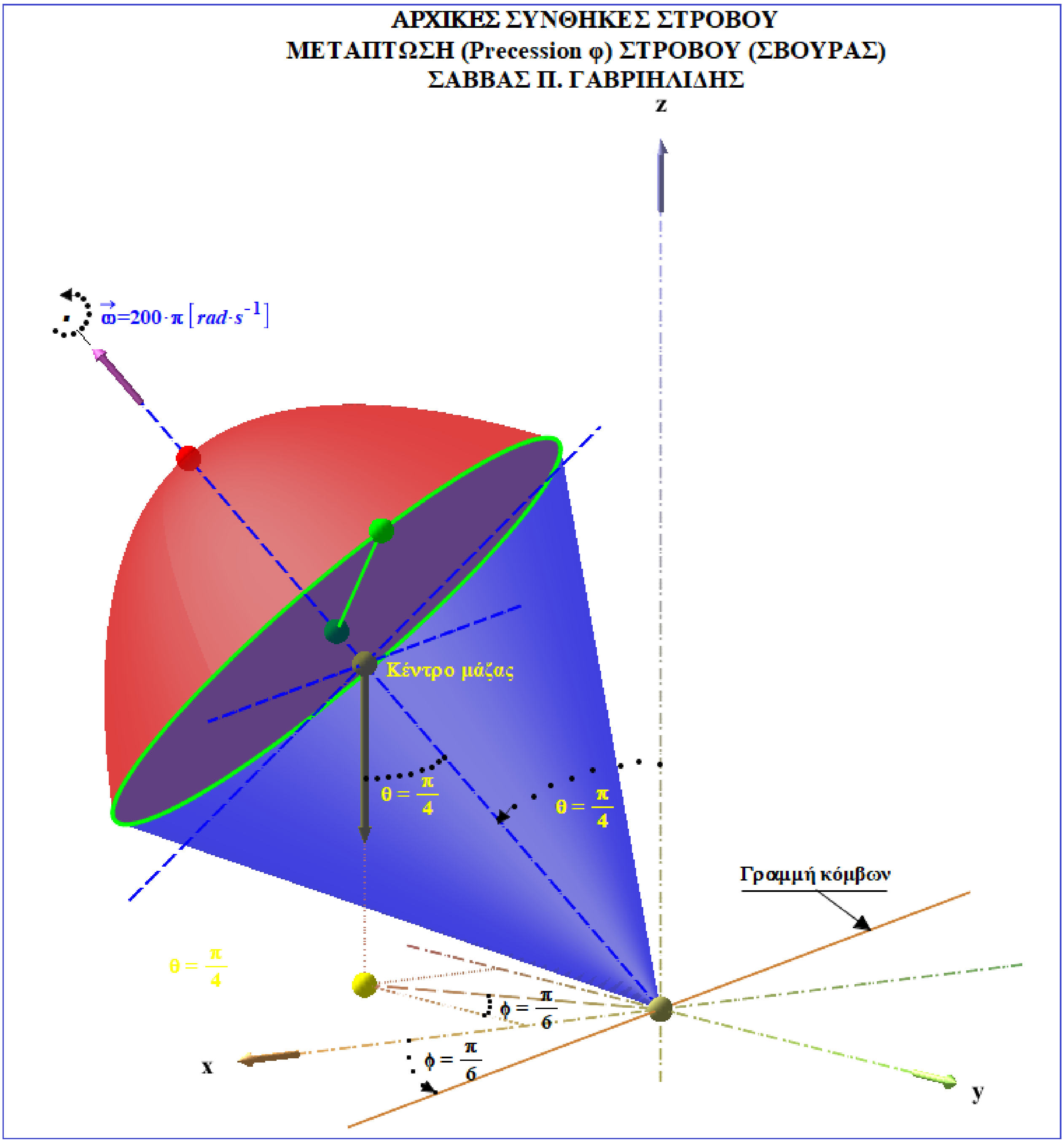
ΜΕΤΑΠΤΩΣΗ (Precession φ) ΣΤΡΟΒΟΥ (ΣΒΟΥΡΑΣ)
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ



ΜΕΤΑΠΤΩΣΗ (Precession φ) ΣΤΡΟΒΟΥ (ΣΒΟΥΡΑΣ)
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ



>
>

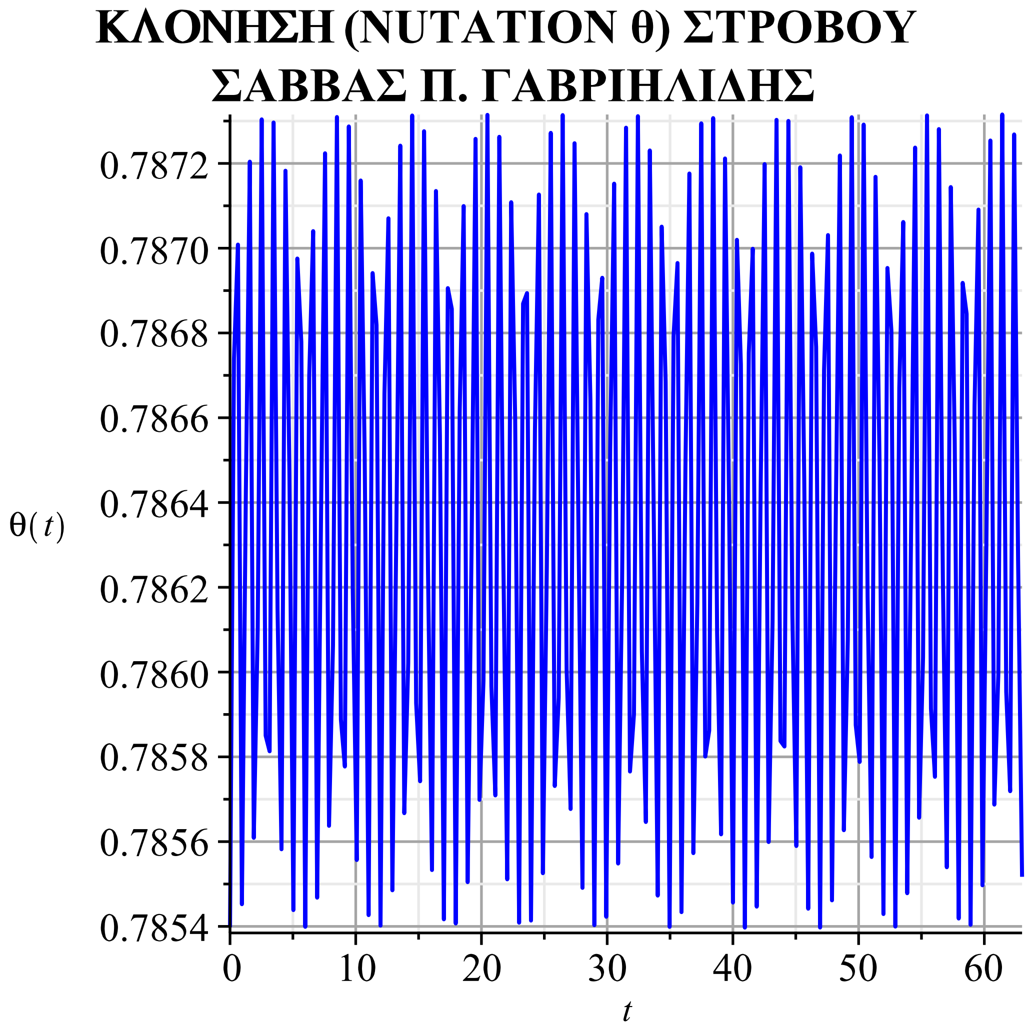


>

>

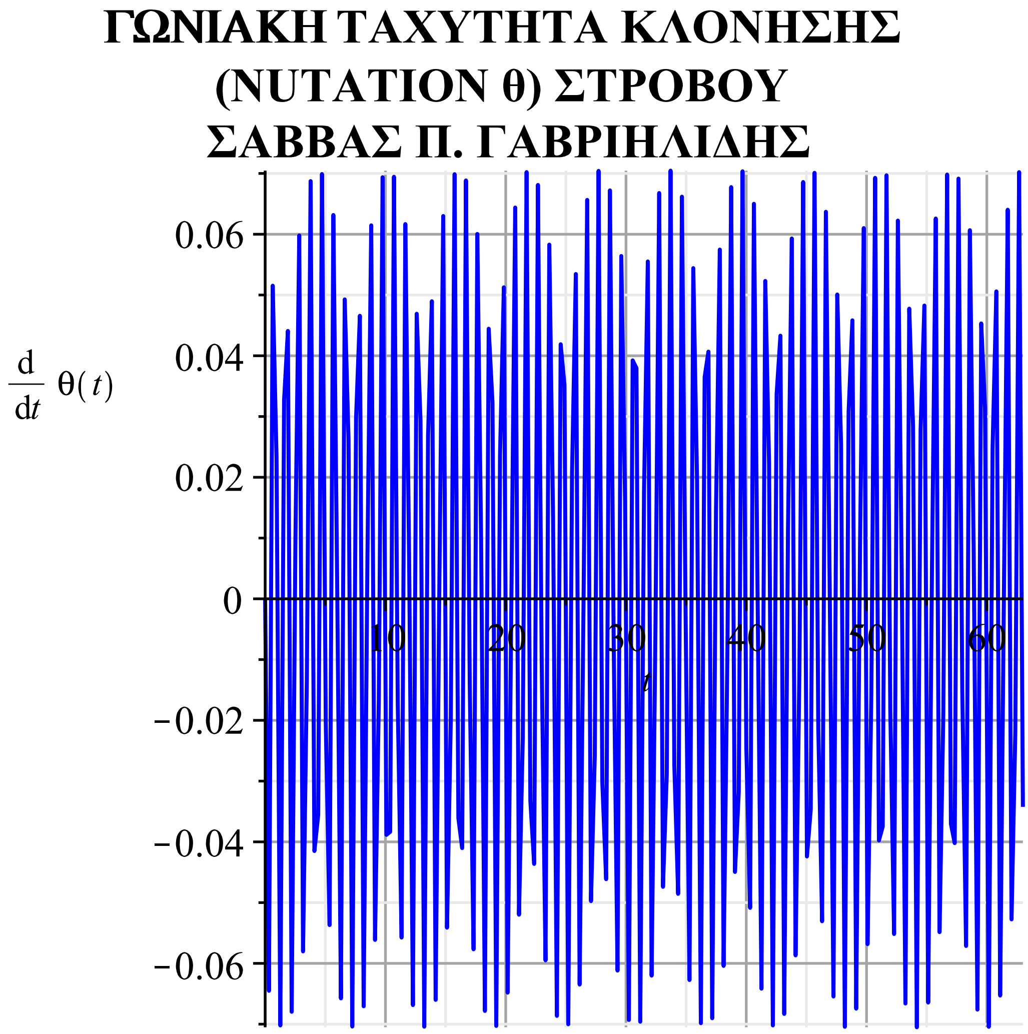
> *s1* := *odeplot*(*solR*, [*t*, θ(*t*)], 0 ..63, *color* = *blue*, *labels* = [*t*, θ(*t*)], *title* = "ΚΛΟΝΗΣΗ (NUTATION θ) ΣΤΡΟΒΟΥ \nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ", *titlefont* = [*arial*, *bold*, 14]) :

> *display*(*s1*, *gridlines*)



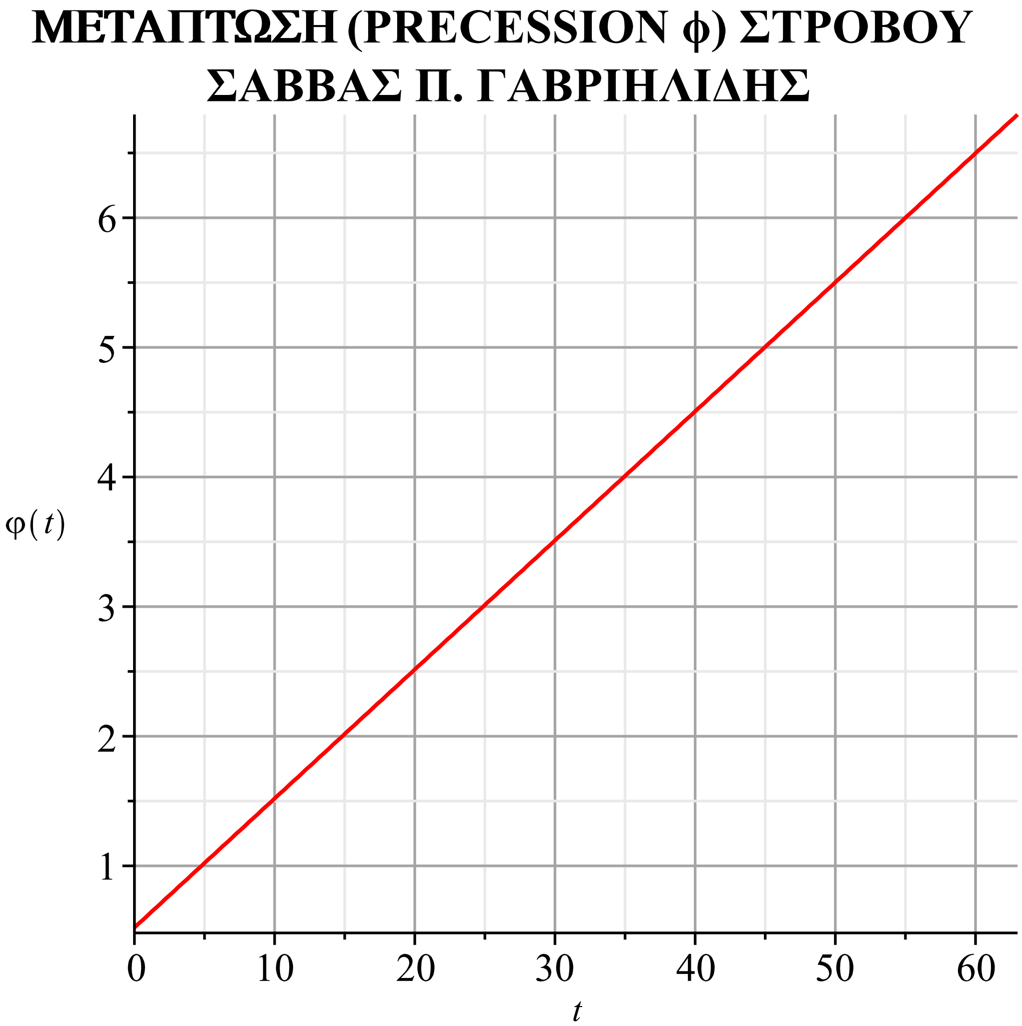
> *s1a* := *odeplot*(*solR*, [*t*, *diff*(θ(*t*), *t*)], 0 ..63, *color* = *blue*, *labels* = [*t*, $\dot{\theta}(t)$], *title* = "ΓΩΝΙΑΚΗ ΤΑΧΥΤΗΤΑ ΚΛΟΝΗΣΗΣ (NUTATION θ) ΣΤΡΟΒΟΥ \nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ", *titlefont* = [*arial*, *bold*, 14]) :

> *display*(*s1a*, *gridlines*)



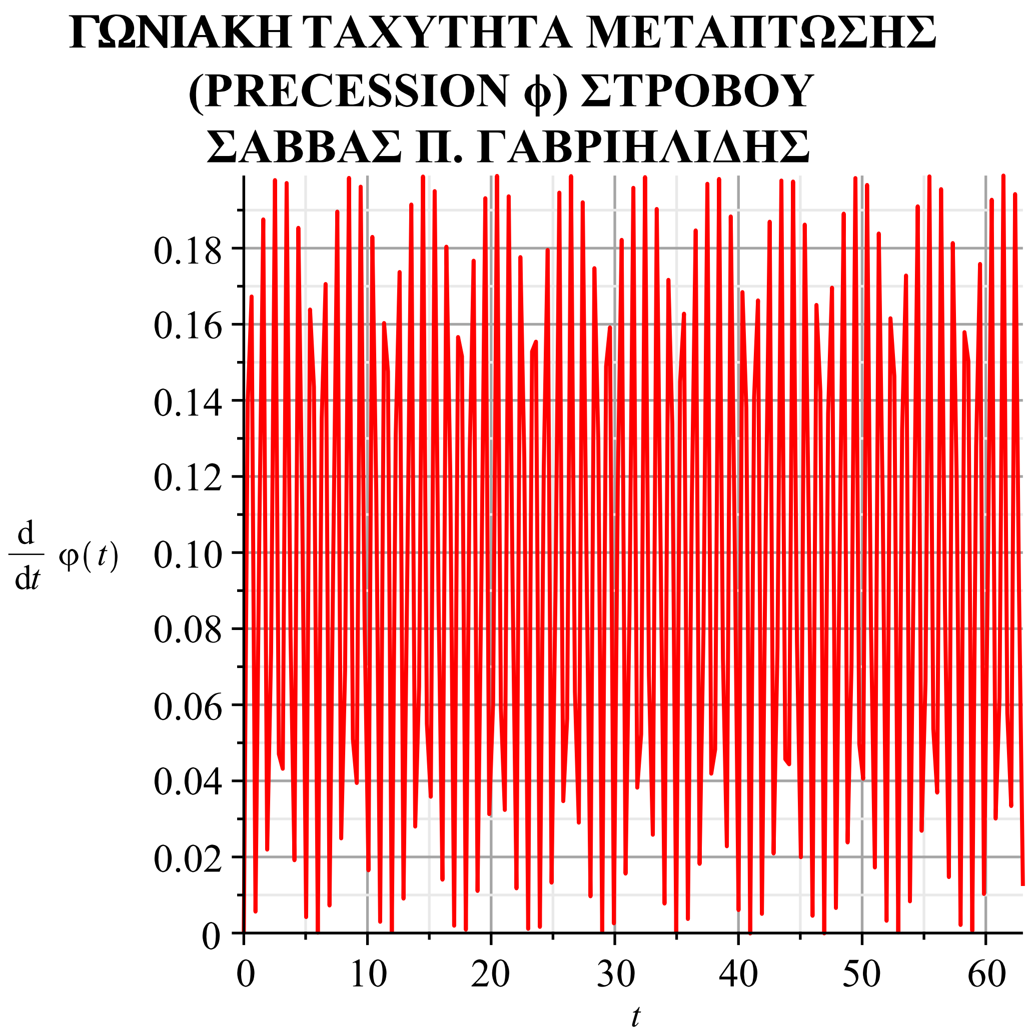
> *s2* := *odeplot*(*solR*, [*t*, φ(*t*)], 0 ..63, *color* = *red*, *labels* = [*t*, φ(*t*)], *title* = "ΜΕΤΑΠΤΩΣΗ (PRECESSION φ) ΣΤΡΟΒΟΥ \nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ", *titlefont* = [*arial*, *bold*, 14]) :

> *display*(*s2*, *gridlines*)



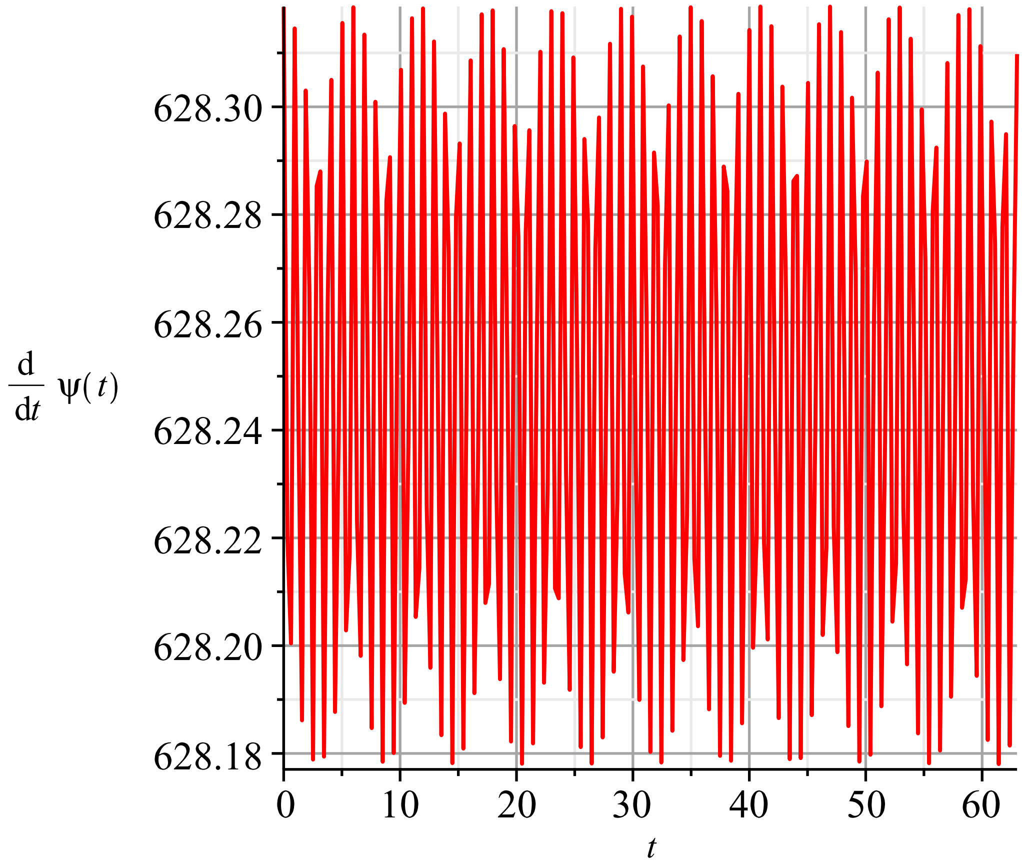
> *s2a* := *odeplot*(*solR*, [*t*, *diff*(φ(*t*), *t*)], 0 ..63, *color* = *red*, *labels* = [*t*, $\dot{\varphi}(t)$], *title* = "ΓΩΝΙΑΚΗ ΤΑΧΥΤΗΤΑ ΜΕΤΑΠΤΩΣΗΣ (PRECESSION φ) ΣΤΡΟΒΟΥ \nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ", *titlefont* = [*arial*, *bold*, 14]) :

> *display*(*s2a*, *gridlines*)




```
> s3 := odeplot(solR, [t, diff(ψ(t), t)], 0..63, color = red, labels = [t, ψ̇(t)], title
     = "ΓΩΝΙΑΚΗ ΤΑΧΥΤΗΤΑ ΙΔΙΟΠΕΡΙΣΤΡΟΦΗΣ  ΣΤΡΟΒΟΥ \nΣΑΒΒΑΣ Π.
    ΓΑΒΡΙΗΛΙΔΗΣ", titlefont = [arial, bold, 14]) :
> display(s3, gridlines)
```

ΓΩΝΙΑΚΗ ΤΑΧΥΤΗΤΑ ΙΔΙΟΠΕΡΙΣΤΡΟΦΗΣ ΣΤΡΟΒΟΥ
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ



>

ΣΤΗΝ ΠΕΡΙΠΤΩΣΗ ΜΕΓΑΛΗΣ ΣΤΡΟΦΟΡΜΗΣ Ο ΧΡΟΝΟΣ ΠΛΗΡΟΥΣ ΜΕΤΑΠΤΩΣΗΣ ΤΕΙΝΕΙ ΣΤΟ ΑΠΕΙΡΟ .

Ετσι γιά την Γη ,ο χρόνος πλήρους μετάπτωσης είναι ≈26.000 χρόνια .!

Αλλά και η κλόνηση είναι αμελητέα .

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