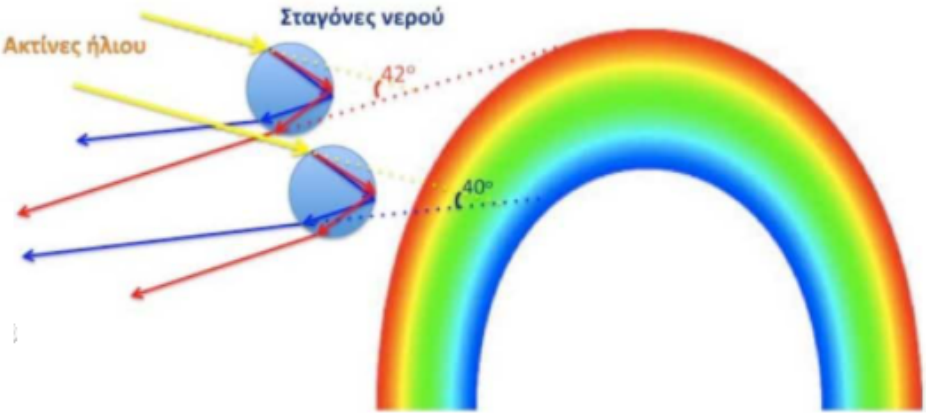
```
> with(plots):
>
```

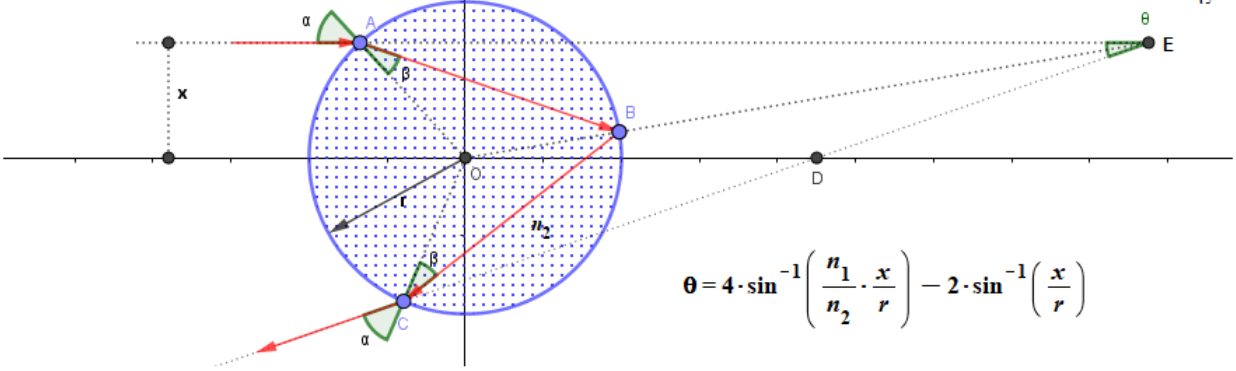
ΟΥΡΑΝΙΟ ΤΟΞΟ
ΜΑΘΗΜΑΤΙΚΗ ΚΑΤΑΝΟΗΣΗ ΤΗΣ ΦΥΣΗΣ



" Στα βραχύτερα μήκη κύματος, κοντά στα ιώδη και μπλε χρώματα, η ανάλυση είναι ισχυρότερη απ' ό,τι στα μακρύτερα μήκη κύματος, όπως στα κόκκινα και τα κίτρινα. Η γωνία κυμαίνεται από 40,6º για το βιολετί φως έως 42,3º για το κόκκινο, ενώ τα υπόλοιπα χρώματα κατανέμονται μέσα στο φάσμα. "

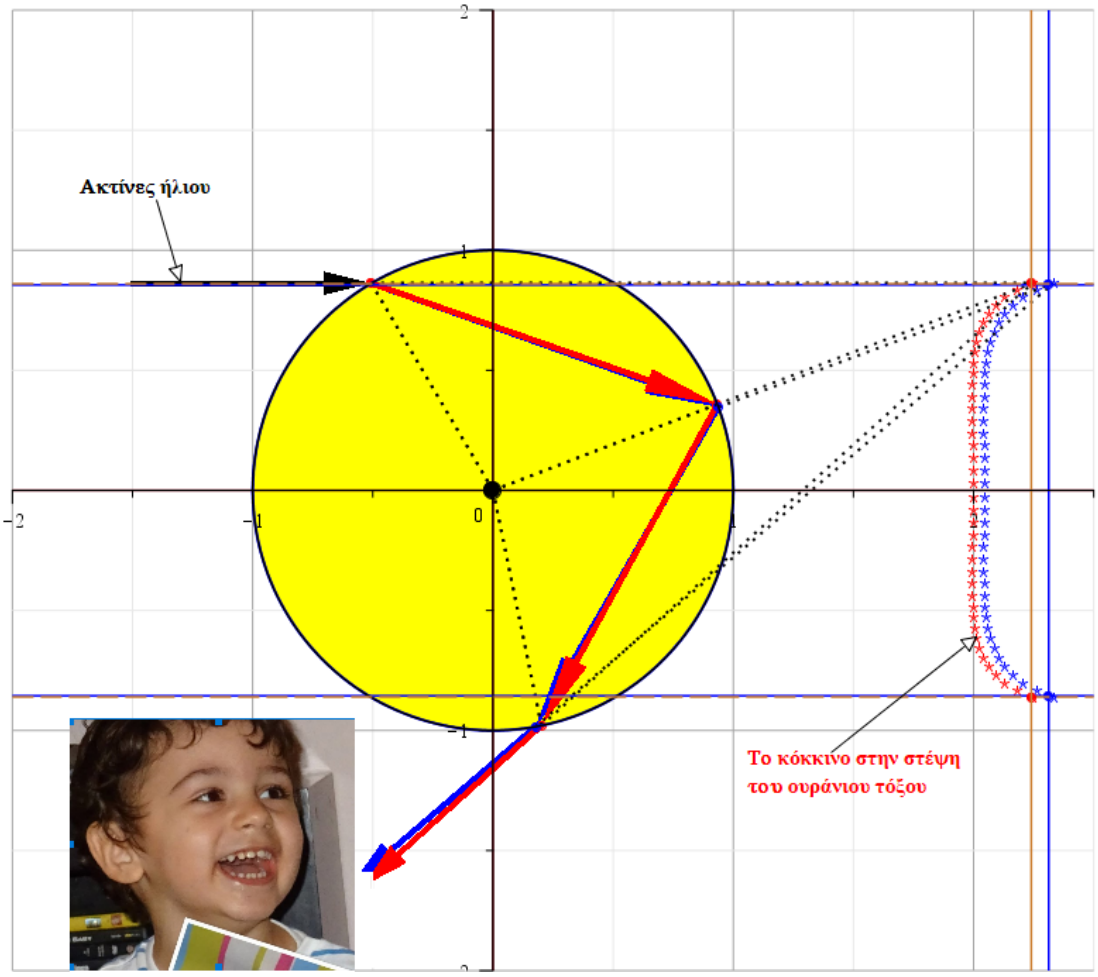
$$BOD + (\pi - 2\cdot\beta) + \alpha = \pi$$
$$\Rightarrow BOD = 2\cdot\beta - \alpha$$
$$\theta = 2\cdot BOD = 4\cdot\beta - 2\cdot\alpha$$

$$\sin(\alpha) = \frac{x}{r} \Rightarrow \alpha = \sin^{-1}\left(\frac{x}{r}\right)$$
$$n_1\cdot\sin(\alpha) = n_2\cdot\sin(\beta)$$
$$\sin(\beta) = \frac{n_1}{n_2}\cdot\sin(\alpha) = \frac{n_1}{n_2}\cdot\frac{x}{r}$$
$$\beta = \sin^{-1}\left(\frac{n_1}{n_2}\cdot\frac{x}{r}\right)$$



$$\theta = 4\cdot\sin^{-1}\left(\frac{n_1}{n_2}\cdot\frac{x}{r}\right) - 2\cdot\sin^{-1}\left(\frac{x}{r}\right)$$

ΟΥΡΑΝΙΟ ΤΟΞΟ
Μία σταγόνα νερού διαθλά -ανακλά -διαθλά το φώς
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ



1. ΚΟΚΚΙΝΟ
ΦΑΣΜΑΤΟΣ

> $n[2] := \frac{4}{3}$

$$n_2 := \frac{4}{3} \quad (1)$$

> $n[1] := 1$

$$n_1 := 1 \quad (2)$$

> $r := 1$

$$r := 1 \quad (3)$$

> $CriticalAngle := evalf\left(\sin^{-1}\left(\frac{n[1]}{n[2]}\right)\right)$

$$CriticalAngle := 0.8480620790 \quad (4)$$

> $convert((4), units, rad, deg)$

$$48.59037788 \quad (5)$$

> $b := \sin^{-1}\left(\frac{n[1]}{n[2]} \cdot \sin(a)\right)$

$$b := \arcsin\left(\frac{3\sin(a)}{4}\right) \quad (6)$$

> $A := [r\cdot\cos(\pi - a), r\cdot\sin(\pi - a)]$

$$A := [-\cos(a), \sin(a)] \tag{7}$$

> $B := expand([r\cdot\cos(2\cdot b - a), r\cdot\sin(2\cdot b - a)])$

$$B := \left[\cos(a) - \frac{9\cos(a)\sin(a)^2}{8} + \frac{3\sin(a)^2\sqrt{16 - 9\sin(a)^2}}{8}, \frac{3\cos(a)\sin(a)\sqrt{16 - 9\sin(a)^2}}{8} - \sin(a) + \frac{9\sin(a)^3}{8}\right] \tag{8}$$

> $C := expand([r\cdot\cos(\pi + 4\cdot b - a), r\cdot\sin(\pi + 4\cdot b - a)])$

$$C := \left[-\cos(a) + \frac{9\cos(a)\sin(a)^2}{2} - \frac{81\cos(a)\sin(a)^4}{32} - \frac{3\sin(a)^2\sqrt{16 - 9\sin(a)^2}}{4} + \frac{27\sqrt{16 - 9\sin(a)^2}\sin(a)^4}{32}, -\frac{3\cos(a)\sin(a)\sqrt{16 - 9\sin(a)^2}}{4} + \frac{27\cos(a)\sqrt{16 - 9\sin(a)^2}\sin(a)^3}{32} + \sin(a) - \frac{9\sin(a)^3}{2} + \frac{81\sin(a)^5}{32}\right] \tag{9}$$

> $E := simplify\left(expand\left(\left[\frac{r\cdot\sin(a)}{\tan(2\cdot b - a)}, r\cdot\sin(a)\right]\right)\right)$

$$E := \left[\frac{(9\cos(a)^3 - \cos(a))\sqrt{9\cos(a)^2 + 7} - 27\cos(a)^4 + 6\cos(a)^2 + 21}{27\cos(a)^3 - 9\sqrt{9\cos(a)^2 + 7}\cos(a)^2 + 21\cos(a) + \sqrt{9\cos(a)^2 + 7}}, \sin(a)\right] \tag{10}$$

> $4\cdot\sin^{-1}\left(\frac{n[1]\cdot x}{n[2]\cdot r}\right) - 2\cdot\sin^{-1}\left(\frac{x}{r}\right)$

$$4\arcsin\left(\frac{3x}{4}\right) - 2\arcsin(x) \tag{11}$$

> $\theta := x \rightarrow$ **(11)**

$$\theta := x \mapsto 4\arcsin\left(\frac{3x}{4}\right) - 2\arcsin(x) \tag{12}$$

> $diff(\theta(x), x) = 0$

$$\frac{12}{\sqrt{-9x^2 + 16}} - \frac{2}{\sqrt{-x^2 + 1}} = 0 \tag{13}$$

> $evalf(solve(\textbf{(13)}, x))$

$$-0.8606629657, 0.8606629657 \tag{14}$$

> $\theta max := evalf(subs(x = \textbf{(14)}[2], \theta(x)))$

$$\theta max := 0.733555931 \tag{15}$$

> $\theta max := convert(evalf(subs(x = \textbf{(14)}[2], \theta(x))), units, rad, deg)$

$$\theta max := 42.02965888 \tag{16}$$

> $diff(\theta(x), x\$2)$

$$\frac{108x}{(-9x^2 + 16)^{3/2}} - \frac{2x}{(-x^2 + 1)^{3/2}} \tag{17}$$

> $evalf(subs(x = \textbf{(14)}[2], \textbf{(17)}))$

$$-9.779642065 \tag{18}$$

Συντεταγμένες του Ε

$[2.240370339, -0.8606629657]$
$[2.240370339, 0.8606629657]$

για το οποίο η γωνία απόκλισης θ γίνεται max .

> $\sin(a) = \mathbf{(14)}[1]$

$$\sin(a) = -0.8606629657 \quad \mathbf{(19)}$$

> $a = evalf\left(\sin^{-1}(\mathbf{(14)}[1])\right)$

$$a = -1.036570282 \quad \mathbf{(20)}$$

> $\sin(a) = \mathbf{(14)}[2]$

$$\sin(a) = 0.8606629657 \quad \mathbf{(21)}$$

> $a = evalf\left(\sin^{-1}(\mathbf{(14)}[2])\right)$

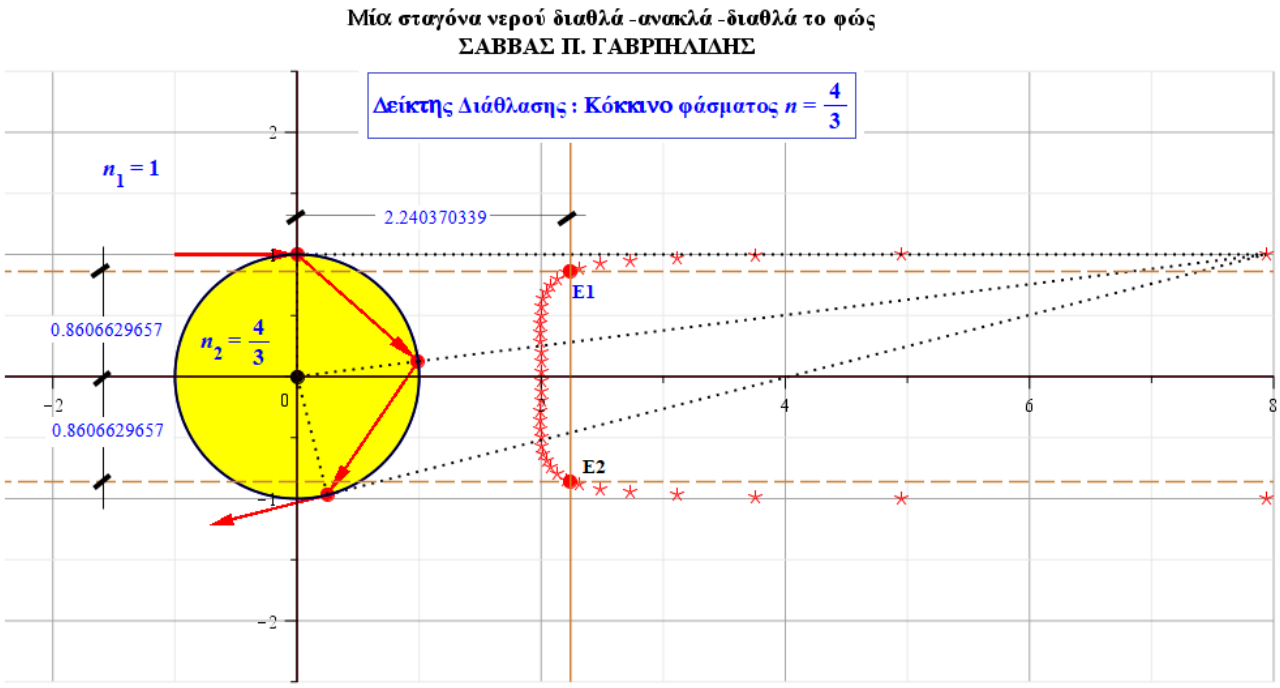
$$a = 1.036570282 \quad \mathbf{(22)}$$

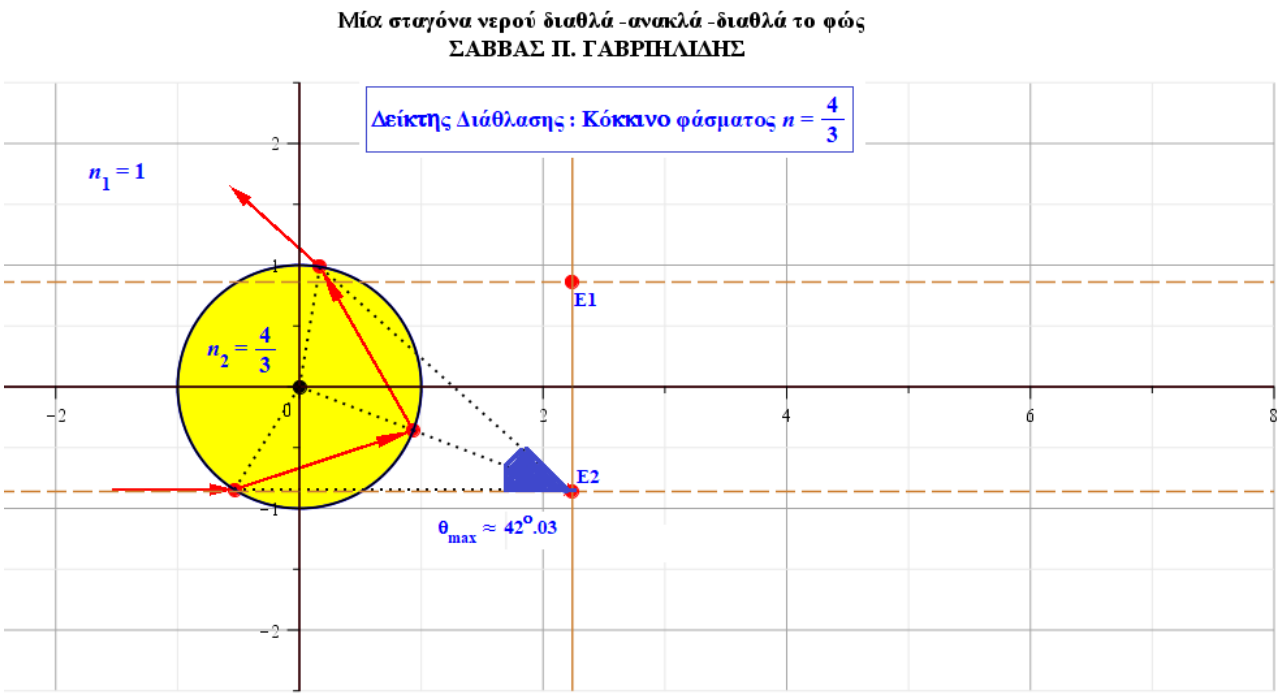
> $evalf(subs(\mathbf{(20)}, E))$

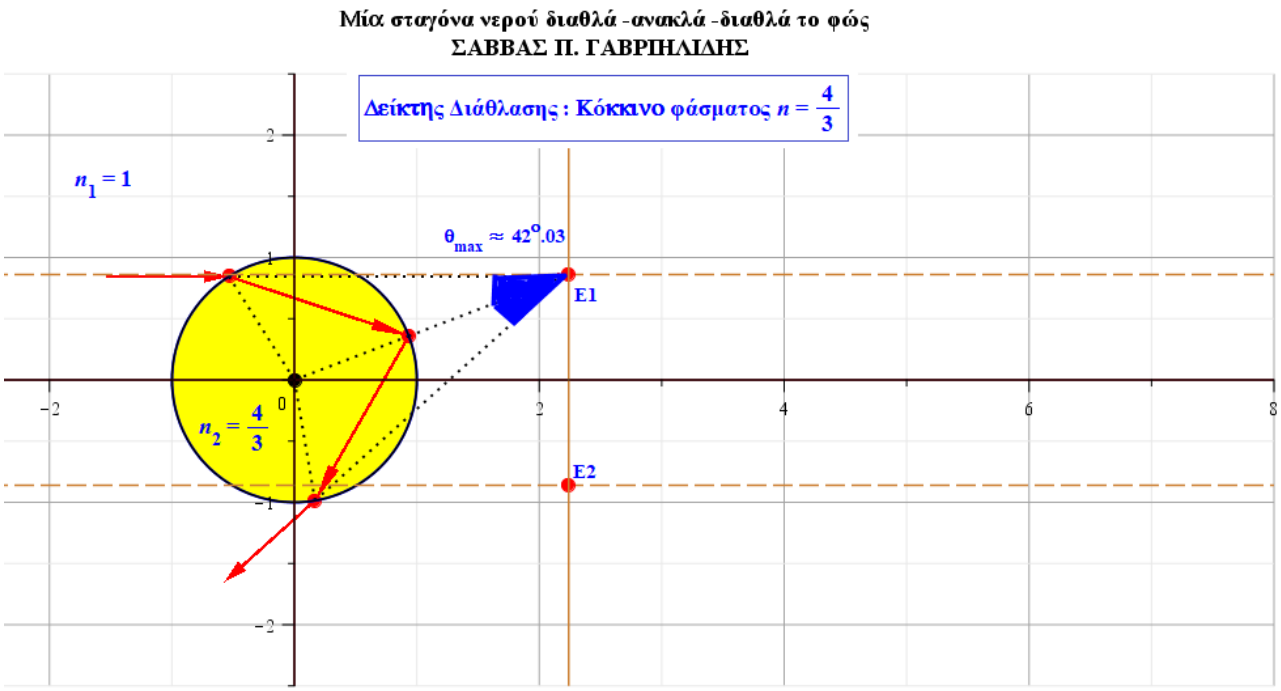
$$[2.240370339, -0.8606629657] \quad \mathbf{(23)}$$

> $evalf(subs(\mathbf{(22)}, E))$

$$[2.240370339, 0.8606629657] \quad \mathbf{(24)}$$







>

2. ΜΠΛΕ ΦΑΣΜΑΤΟΣ

> $n[3] := \frac{4.03}{3}$

$$n_3 := 1.343333333 \quad (25)$$

> $n[1] := 1$

$$n_1 := 1 \quad (26)$$

> $r := 1$

$$r := 1 \quad (27)$$

> $CriticalAngle := evalf\left(\sin^{-1}\left(\frac{n[1]}{n[3]}\right)\right)$

$$CriticalAngle := 0.8396610993 \quad (28)$$

> $convert(\mathbf{(28)}, units, rad, deg)$

$$48.10903721 \quad (29)$$

> $\mathbf{bb := \sin^{-1}\left(\frac{n[1]}{n[3]}\cdot\sin(a)\right)}$

$$bb := \arcsin(0.7444168736\sin(a)) \quad (30)$$

> $\mathbf{A := [r\cdot\cos(\pi - a), r\cdot\sin(\pi - a)]}$

$$A := [-\cos(a), \sin(a)] \quad (31)$$

> $\mathbf{Bb := simplify(expand([r\cdot\cos(2\cdot bb - a), r\cdot\sin(2\cdot bb - a)]))}$

$$Bb := \left[\left(-1.488833747\cos(a)^2 + 1.488833747\right)\sqrt{0.5541564817\cos(a)^2 + 0.4458435183} - 0.108312963\cos(a) + 1.108312963\cos(a)^3,\ \sin(a)\left(1.488833747\cos(a)\sqrt{0.5541564817\cos(a)^2 + 0.4458435183} + 0.108312963 - 1.108312963\cos(a)^2\right)\right] \quad (32)$$

> $\mathbf{Cb := simplify(expand([r\cdot\cos(\pi + 4\cdot bb - a), r\cdot\sin(\pi + 4\cdot bb - a)]))}$

$$Cb := \left[\left(-3.622707475\cos(a)^2 + 0.322519991 + 3.300187484\cos(a)^4\right)\sqrt{0.5541564817\cos(a)^2 + 0.4458435183} + 0.976536605\cos(a) + 0.480178643\cos(a)^3 - 2.456715248\cos(a)^5,\ \sin(a)\left(-0.976536605 + 2.456715248\cos(a)^4 - 3.300187484\sqrt{0.5541564817\cos(a)^2 + 0.4458435183}\cos(a)^3 - 0.480178643\cos(a)^2 + 0.322519991\cos(a)\sqrt{0.5541564817\cos(a)^2 + 0.4458435183}\right)\right] \quad (33)$$

> $\mathbf{Eb := simplify\left(expand\left(\left[\frac{r\cdot\sin(a)}{\tan(2\cdot bb - a)}, r\cdot\sin(a)\right]\right)\right)}$

$$Eb := \left[\left(\left(-0.108312963\cos(a) + 1.108312963\cos(a)^3\right)\sqrt{0.5541564817\cos(a)^2 + 0.4458435183} - 0.8250468711\left(\cos(a) + 0.999999999932833\right)\left(\cos(a) - 0.999999999932833\right)\left(\cos(a)^2 + 0.804544443764958\right)\right) \Big/ \left(\left(0.108312963 - 1.108312963\cos(a)^2\right)\sqrt{0.5541564817\cos(a)^2 + 0.4458435183} + 0.6637868759\cos(a) + 0.8250468711\cos(a)^3\right),\ \sin(a)\right] \quad (34)$$

> $4\cdot\sin^{-1}\left(\frac{n[1]\cdot x}{n[3]\cdot r}\right) - 2\cdot\sin^{-1}\left(\frac{x}{r}\right)$

$$4\arcsin(0.7444168736\,x) - 2\arcsin(x) \quad (35)$$

> $\theta := x \to \mathbf{(35)}$

$$\theta := x \mapsto 4\arcsin(x\,0.7444168736) - 2\arcsin(x) \quad (36)$$

> $diff(\theta(x), x) = 0$

$$\frac{2.977667494}{\sqrt{-0.5541564817\,x^2+1}}-\frac{2}{\sqrt{-x^2+1}}=0 \quad (37)$$

> $solve((37), x)$

$$-0.8554639201,\ 0.8554639201 \quad (38)$$

> $\theta bmax := evalf(subs(x=(38)[2], \theta(x)))$

$$\theta bmax := 0.708584222 \quad (39)$$

> $\theta bmax := convert(evalf(subs(x=(38)[2], \theta(x))), units, rad, deg)$

$$\theta bmax := 40.59888536 \quad (40)$$

> $diff(\theta(x), x\$2)$

$$\frac{1.650093742\,x}{(-0.5541564817\,x^2+1)^{3/2}}-\frac{2\,x}{(-x^2+1)^{3/2}} \quad (41)$$

> $evalf(subs(x=(38)[2], (41)))$

$$-9.239527846 \quad (42)$$

Συντεταγμένες του Ε $[2.31268861521228, -0.8554639199]$ $[2.31268861521228, 0.8554639199]$ **για το οποίο η γωνία απόκλισης θ γίνεται max .**

> $\sin(a) = (38)[1]$

$$\sin(a) = -0.8554639201 \quad (43)$$

> $a = evalf(\sin^{-1}((38)[1]))$

$$a = -1.026446014 \quad (44)$$

> $\sin(a) = (38)[2]$

$$\sin(a) = 0.8554639201 \quad (45)$$

> $a = evalf(\sin^{-1}((38)[2]))$

$$a = 1.026446014 \quad (46)$$

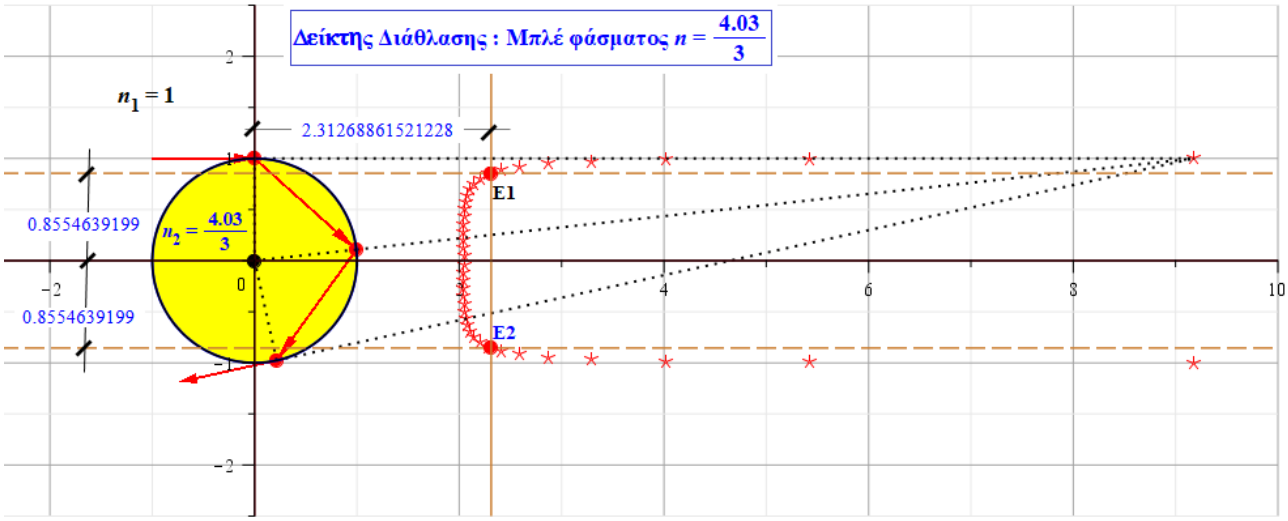
> $evalf(subs((44), Eb))$

$$[2.31268861521228,\ -0.8554639199] \quad (47)$$

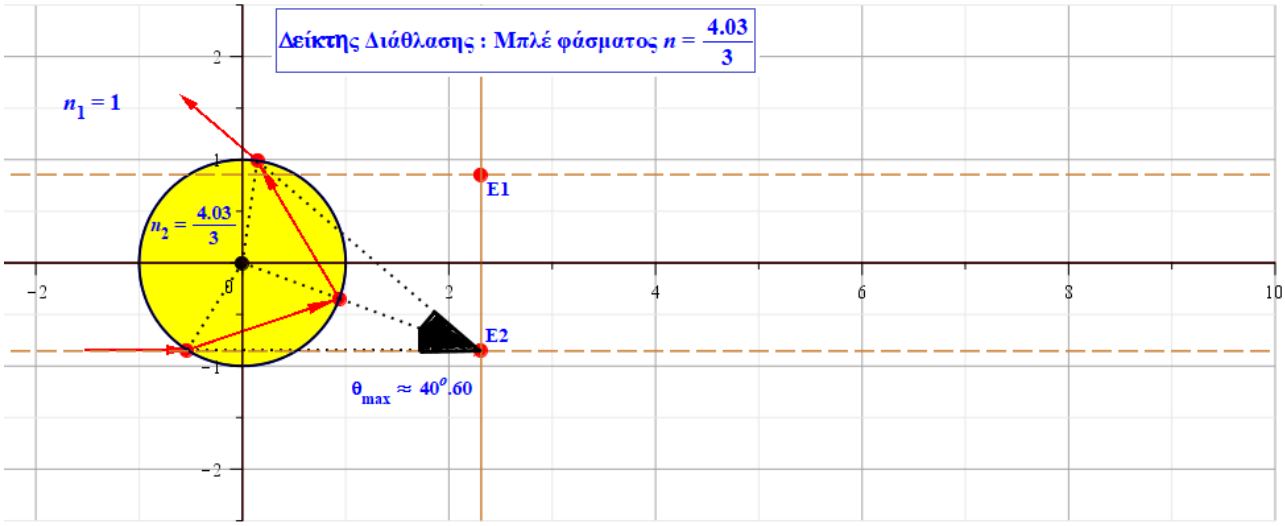
> $evalf(subs((46), Eb))$

$$[2.31268861521228,\ 0.8554639199] \quad (48)$$

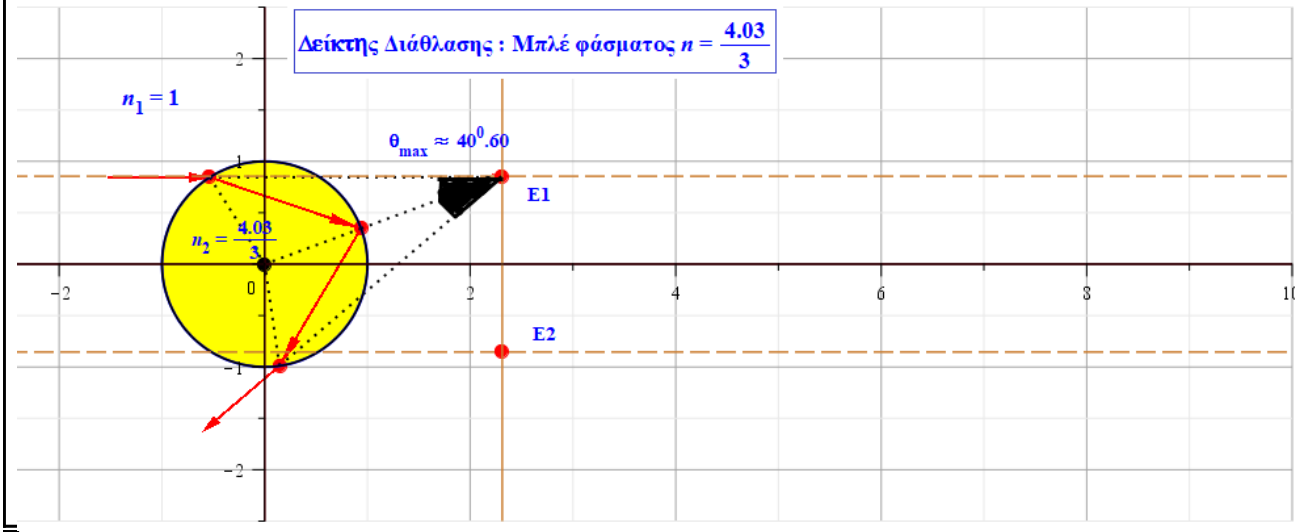
Μία σταγόνα νερού διαθλά -ανακλά -διαθλά το φώς
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ



Μία σταγόνα νερού διαθλά -ανακλά -διαθλά το φώς
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ



Μία σταγόνα νερού διαθλά -ανακλά -διαθλά το φώς
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ



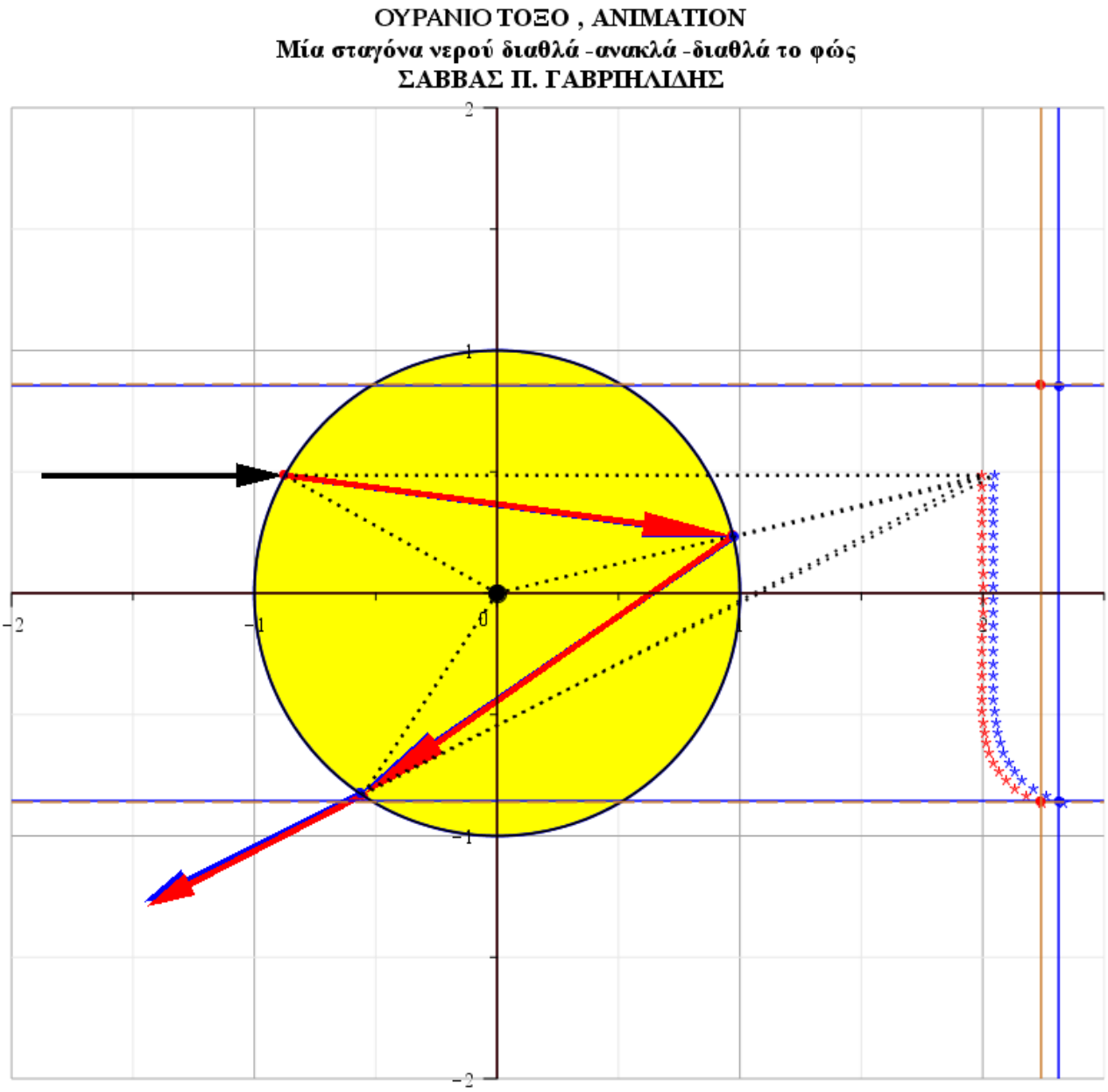
ANIMATION

(ΠΡΟΣΟΜΟΙΩΣΗ)

```
> S1 := plot([r·cos(φ), r·sin(φ), φ = 0..2·Pi], color = blue, thickness = 2, gridlines):
> S2 := inequal(X^2 + Y^2 ≤ 1, X = -1..1, Y = -1..1, style = surface, color = yellow):
> S3 := pointplot([0, 0], symbol = solidcircle, symbolsize = 10):
> Ymax := plot([x, (14)[2], x = -2..2.5], color = gold, linestyle = 3):
> Ymin := plot([x, (14)[1], x = -2..2.5], color = gold, linestyle = 3):
> E1 := pointplot((23), symbol = solidcircle, symbolsize = 5, color = red):
> E2 := pointplot((24), symbol = solidcircle, symbolsize = 5, color = red):
> E1E2 := plot([(23)[1], y, y = -2.0..2], color = gold, linestyle = 1):
> animA := animate(pointplot, [A, symbol = solidcircle, symbolsize = 5, color = red], a = -1.04
      ..1.04, frames = 40):
> animB := animate(pointplot, [B, symbol = solidcircle, symbolsize = 5, color = red], a = -1.04
      ..1.04, frames = 40):
> animC := animate(pointplot, [C, symbol = solidcircle, symbolsize = 5, color = red], a = -1.04
      ..1.04, frames = 40):
> animE := animate(pointplot, [E, symbol = asterisk, symbolsize = 5, color = red], a = -1.04
      ..1.04, frames = 40, trace = 40):
> animAO := animate(plot, [[A[1] + λ·(0 - A[1]), A[2] + λ·(0 - A[2]), λ = 0..1],
      thickness = 2, linestyle = 2, color = black], a = -1.04..1.04, frames = 40):
> animBO := animate(plot, [[B[1] + λ·(0 - B[1]), B[2] + λ·(0 - B[2]), λ = 0..1],
      thickness = 2, linestyle = 2, color = black], a = -1.04..1.04, frames = 40):
> animCO := animate(plot, [[C[1] + λ·(0 - C[1]), C[2] + λ·(0 - C[2]), λ = 0..1],
      thickness = 2, linestyle = 2, color = black], a = -1.04..1.04, frames = 40):
> animAB := animate(plot, [[A[1] + λ·(B[1]-A[1]), A[2] + λ·(B[2]-A[2]), λ = 0..1],
      thickness = 1, linestyle = 1, color = black], a = -1.04..1.04, frames = 40):
> animBC := animate(plot, [[B[1] + λ·(C[1] - B[1]), B[2] + λ·(C[2] - B[2]), λ = 0
      ..1], thickness = 1, linestyle = 1, color = black], a = -1.04..1.04, frames = 40):
> animAE := animate(plot, [[A[1] + λ·(E[1]-A[1]), A[2] + λ·(E[2]-A[2]), λ = 0..1],
      thickness = 2, linestyle = 2, color = black], a = -1.04..1.04, frames = 40):
> animCE := animate(plot, [[C[1] + λ·(E[1]-C[1]), C[2] + λ·(E[2]-C[2]), λ = 0..1],
      thickness = 2, linestyle = 2, color = black], a = -1.04..1.04, frames = 40):
> animBE := animate(plot, [[B[1] + λ·(E[1]-B[1]), B[2] + λ·(E[2]-B[2]), λ = 0..1],
      thickness = 2, linestyle = 2, color = black], a = -1.04..1.04, frames = 40):
> anarAE := animate(arrow, [[A[1] - 1, A[2]], [1, 0], color = black, width = 0.025,
      head_width = 0.1, border = false], a = -1.04..1.04, frames = 40):
> anarAB := animate(arrow, [[A[1], A[2]], [B[1] - A[1], B[2] - A[2]], color = red, width
      = 0.025, head_width = 0.1, border = false], a = -1.04..1.04, frames = 40):
> anarBC := animate(arrow, [[B[1], B[2]], [C[1] - B[1], C[2] - B[2]], color = red, width
      = 0.025, head_width = 0.1, border = false], a = -1.04..1.04, frames = 40):
> anarEC := animate(arrow, [[C[1], C[2]], [-1·cos(tan^(-1)((E[2] - C[2])/(E[1] - C[1]))), -1
      ·sin(tan^(-1)((E[2] - C[2])/(E[1] - C[1])))], color = red, width = 0.025, head_width = 0.1, border
      = false], a = -1.04..1.04, frames = 40):
> RED := display(S1, S2, S3, Ymax, Ymin, E1, E2, E1E2, animA, animB, animC, animE,
```

```
    animAO, animBO, animCO, animAE, animCE, animBE, anarAE, anarAB, anarBC,
    anarEC, scaling = constrained, title
    = "Μία σταγόνα νερού διαθλά -ανακλά -διαθλά το φώς\nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ",
    titlefont = [arial, bold, 14]) :
> XX := plot([x, 0, x = -2..2.5]) :
> YY := plot([0, y, y = -2.0..2]) :
> Ybmax := plot([x, (38)[2], x = -2..2.5], color = blue, linestyle = 1) :
> Ybmin := plot([x, (38)[1], x = -2..2.5], color = blue, linestyle = 1) :
> E1b := pointplot((48), symbol = solidcircle, symbolsize = 5, color = blue) :
> E2b := pointplot((47), symbol = solidcircle, symbolsize = 5, color = blue) :
> E1bE2b := plot([(47)[1], y, y = -2.0..2], color = blue, linestyle = 1) :
> animBb := animate(pointplot, [Bb, symbol = solidcircle, symbolsize = 5, color = blue], a =
    -1.04..1.04, frames = 40) :
> animCb := animate(pointplot, [Cb, symbol = solidcircle, symbolsize = 5, color = blue], a =
    -1.04..1.04, frames = 40) :
> animEb := animate(pointplot, [Eb, symbol = asterisk, symbolsize = 5, color = blue], a = -1.04
    ..1.04, frames = 40, trace = 40) :
> animAO := animate(plot, [[A[1] + λ·(0 − A[1]), A[2] + λ·(0 − A[2]), λ = 0..1],
    thickness = 2, linestyle = 2, color = black], a = -1.04..1.04, frames = 40) :
> animBbO := animate(plot, [[Bb[1] + λ·(0 − Bb[1]), Bb[2] + λ·(0 − Bb[2]), λ = 0..1],
    thickness = 2, linestyle = 2, color = black], a = -1.04..1.04, frames = 40) :
> animCbO := animate(plot, [[Cb[1] + λ·(0 − Cb[1]), Cb[2] + λ·(0 − Cb[2]), λ = 0..1],
    thickness = 2, linestyle = 2, color = black], a = -1.04..1.04, frames = 40) :
> animABb := animate(plot, [[A[1] + λ·(Bb[1] - A[1]), A[2] + λ·(Bb[2] - A[2]), λ = 0
    ..1], thickness = 1, linestyle = 1, color = black], a = -1.04..1.04, frames = 40) :
> animBbCb := animate(plot, [[Bb[1] + λ·(Cb[1] − Bb[1]), Bb[2] + λ·(Cb[2]
    − Bb[2]), λ = 0..1], thickness = 1, linestyle = 1, color = black], a = -1.04..1.04, frames
    = 40) :
> animAEb := animate(plot, [[A[1] + λ·(Eb[1] - A[1]), A[2] + λ·(Eb[2] - A[2]), λ = 0
    ..1], thickness = 2, linestyle = 2, color = black], a = -1.04..1.04, frames = 40) :
> animCbEb := animate(plot, [[Cb[1] + λ·(Eb[1] − Cb[1]), Cb[2] + λ·(Eb[2]
    − Cb[2]), λ = 0..1], thickness = 2, linestyle = 2, color = black], a = -1.04..1.04, frames
    = 40) :
> animBbEb := animate(plot, [[Bb[1] + λ·(Eb[1] − Bb[1]), Bb[2] + λ·(Eb[2] − Bb[2]),
    λ = 0..1], thickness = 2, linestyle = 2, color = black], a = -1.04..1.04, frames = 40) :
> anarAEb := animate(arrow, [[A[1] − 1, A[2]], [1, 0], color = black, width = 0.025,
    head_width = 0.1, border = false], a = -1.04..1.04, frames = 40) :
> anarABb := animate(arrow, [[A[1], A[2]], [Bb[1] − A[1], Bb[2] − A[2]], color = blue,
    width = 0.025, head_width = 0.1, border = false], a = -1.04..1.04, frames = 40) :
> anarBbCb := animate(arrow, [[Bb[1], Bb[2]], [Cb[1] − Bb[1], Cb[2] − Bb[2]], color
    = blue, width = 0.025, head_width = 0.1, border = false], a = -1.04..1.04, frames = 40) :
> anarEbCb := animate(arrow, [[Cb[1], Cb[2]], [-1·cos(tan^-1((Eb[2] − Cb[2])/(Eb[1] − Cb[1]))), -1
    ·sin(tan^-1((Eb[2] − Cb[2])/(Eb[1] − Cb[1])))], color = blue, width = 0.025, head_width = 0.1, border
    = false], a = -1.04..1.04, frames = 40) :
> BLUE := display(XX, YY, Ybmax, Ybmin, E1b, E2b, E1bE2b, animBb, animCb, animEb,
```

```
    animAO, animBO, animCO, animAE, animCbEb, animBbEb, anarAEb, anarABb,
    anarBbCb, anarEbCb, scaling = constrained, title
     = "ΟΥΡΑΝΙΟ ΤΟΞΟ , ANIMATION\nΜία σταγόνα νερού διαθλά -ανακλά -διαθλά το
    φώς\nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ", titlefont = [arial, bold, 14]) :
> display(S1, S2, S3, Ymax, Ymin, E1, E2, E1E2, animA, animB, animC, animE, animAO,
    animBO, animCO, animAE, animCE, animBE, anarAE, anarAB, anarBC, anarEC,
    BLUE) :
```




```
>
```