> $with(plots):$
> $with(Physics[Vectors]):$
> $Setup(mathematicalnotation = true):$
>

Συμβολισμοί:

$f(x)$: Συνάρτηση πυκνότητας της κανονικής κατανομής $N(\mu, \sigma^2)$

$\varphi(z)$: Συνάρτηση πυκνότητας της Τυποποιημένης κανονικής κατανομής $N(0, 1)$

>

> $\sigma := 1$

$$\sigma := 1 \quad (1)$$

> $\mu := 0$

$$\mu := 0 \quad (2)$$

> $f := x \rightarrow \frac{1}{\sigma \cdot \sqrt{2 \cdot \pi}} \cdot e^{-\frac{(x-\mu)^2}{2 \cdot \sigma^2}}$

$$f := x \mapsto \frac{e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}} \quad (3)$$

> $\varphi := z \rightarrow \left(\frac{1}{\sqrt{2 \cdot \pi}}\right) \cdot e^{-\frac{z^2}{2}}$

$$\varphi := z \mapsto \frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}} \quad (4)$$

> *G1* := *plot*(*f*(*x*), *x* = -3 ..3, *labels* = [*x*, *z*], *labelfont* = [*arial*, *bold*, 14], *thickness* = 3, *color* = *blue*, *title* = "Καμπύλη Gauss στο OXZ συντεταγμένο επίπεδο", *titlefont* = [*arial*, *bold*, 14], *gridlines*) :

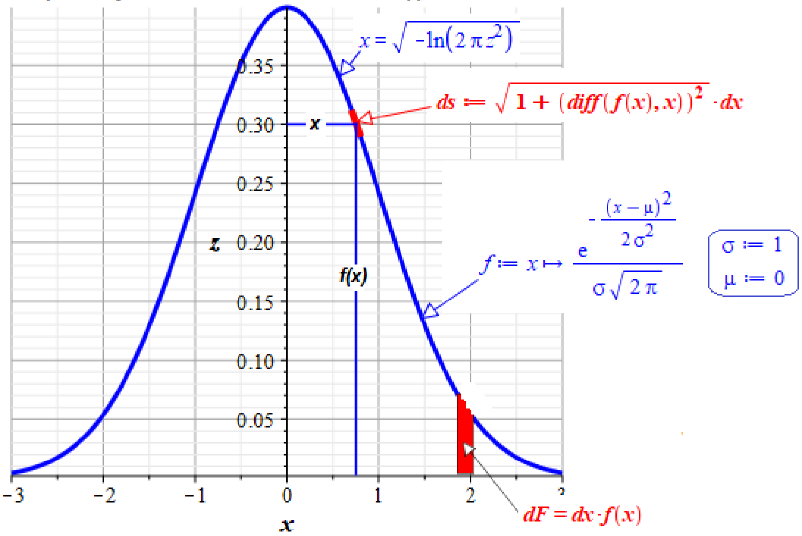
> *G2* := *plot*(*f*(*x*), *x* = 0.7 ..0.8, *labels* = [*x*, *z*], *labelfont* = [*arial*, *bold*, 14], *thickness* = 5, *color* = *red*, *title* = "Καμπύλη Gauss στο OXZ συντεταγμένο επίπεδο", *titlefont* = [*arial*, *bold*, 14], *gridlines*) :

> *display*(*G1*, *G2*) :

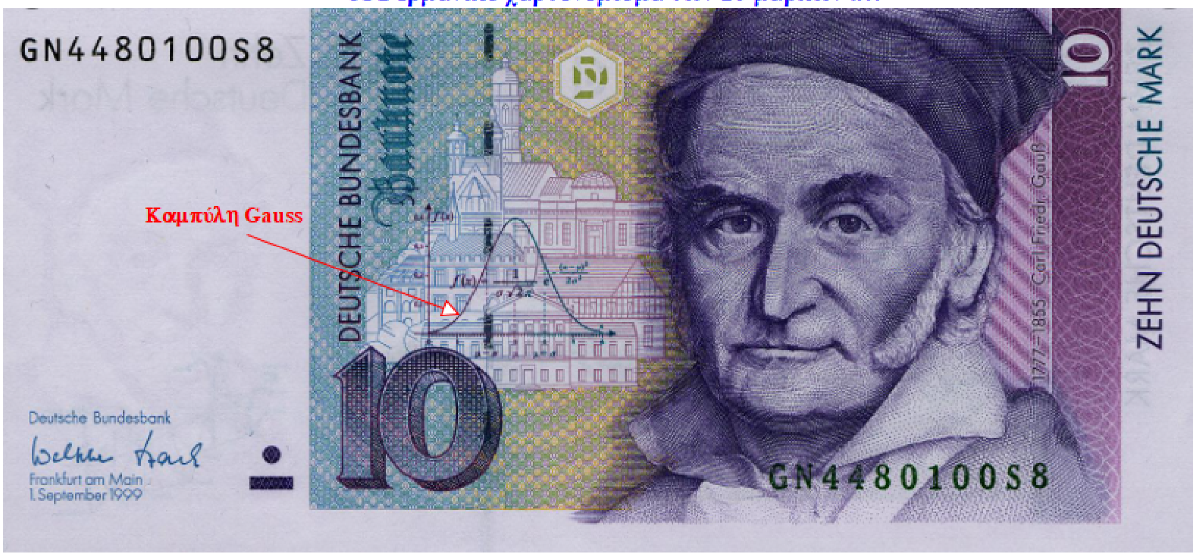
>

ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ
Νίκης 9 Βέροια

Καμπύλη Gauss στο OXZ συντεταγμένο επίπεδο



Καμπύλη-Gauss με την Εξίσωσή της και την φωτογραφία του C.F.Gauss σε Γερμανικό χαρτονόμισμα των 10 μάρκων .!!!



> $evalf(f(3))$

$$0.004431848411 \quad (5)$$

> $evalf(f(-3))$

$$0.004431848411 \quad (6)$$

> $evalf(f(0))$

$$0.3989422802 \quad (7)$$

> $g := x \to \frac{x}{3} \cdot (f(3) - f(0)) + f(0)$

$$g := x \mapsto \frac{x\,(f(3) - f(0))}{3} + f(0) \quad (8)$$

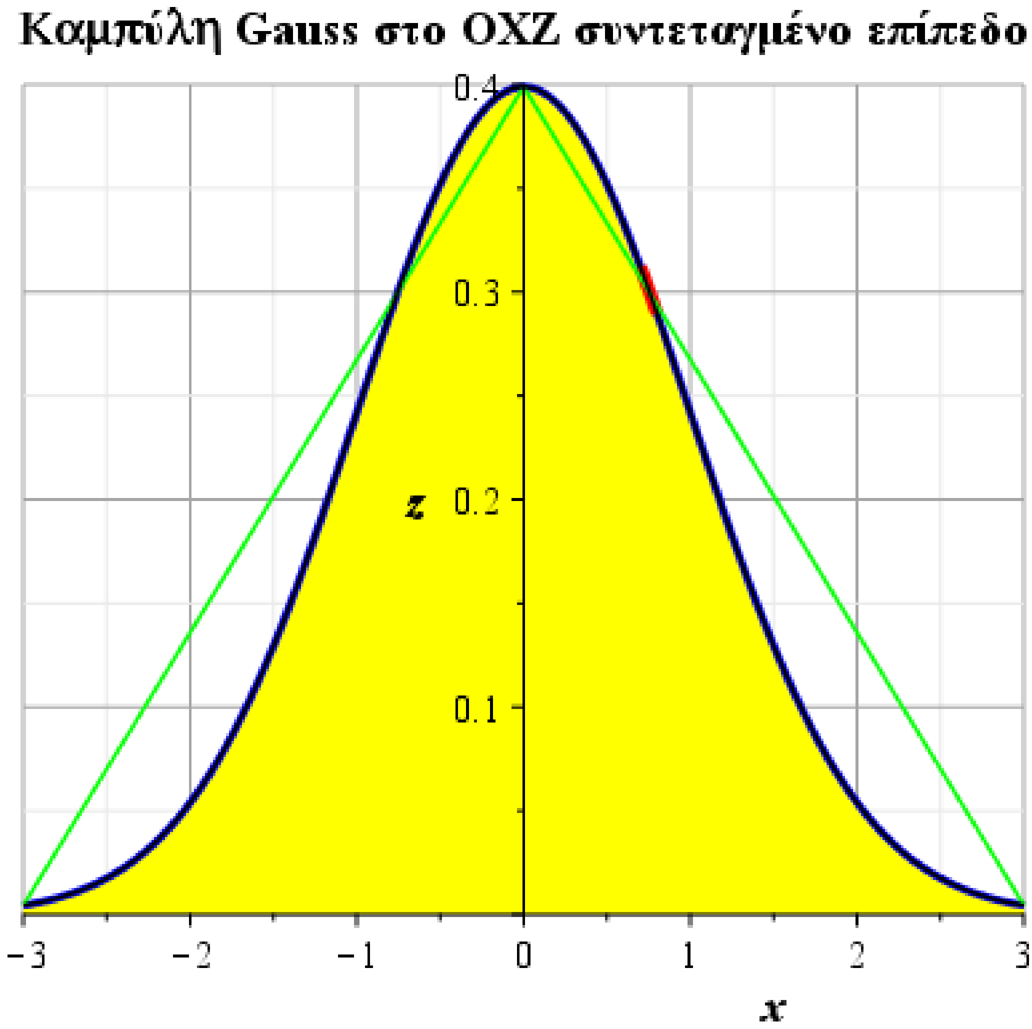
>

> $LIN1 := plot(g(x), x = 0..3, color = green, thickness = 1)$:

> $LIN2 := plot(g(-x), x = -3..0, color = green, thickness = 1)$:

> $ineq := inequal\left(y - \left(\frac{1}{\sqrt{2 \cdot \pi}}\right) \cdot e^{-\frac{x^2}{2}} \le 0, x = -3..3, y = 0..0.4, color = yellow\right)$:

> $display(G1, G2, LIN1, LIN2, ineq)$:



>

>

Καμπύλη Gauss στο OXZ συντεταγμένο επίπεδο ,
Περιστροφή γύρω από τον άξονα OX κατά γωνία *a*.

1. Παραμετρικές εξισώσεις της αντίστοιχης εκ περιστροφής επιφάνειας γύρω από τον άξονα OX:

> $[x, -f(x)\cdot\sin(a), f(x)\cdot\cos(a)]$

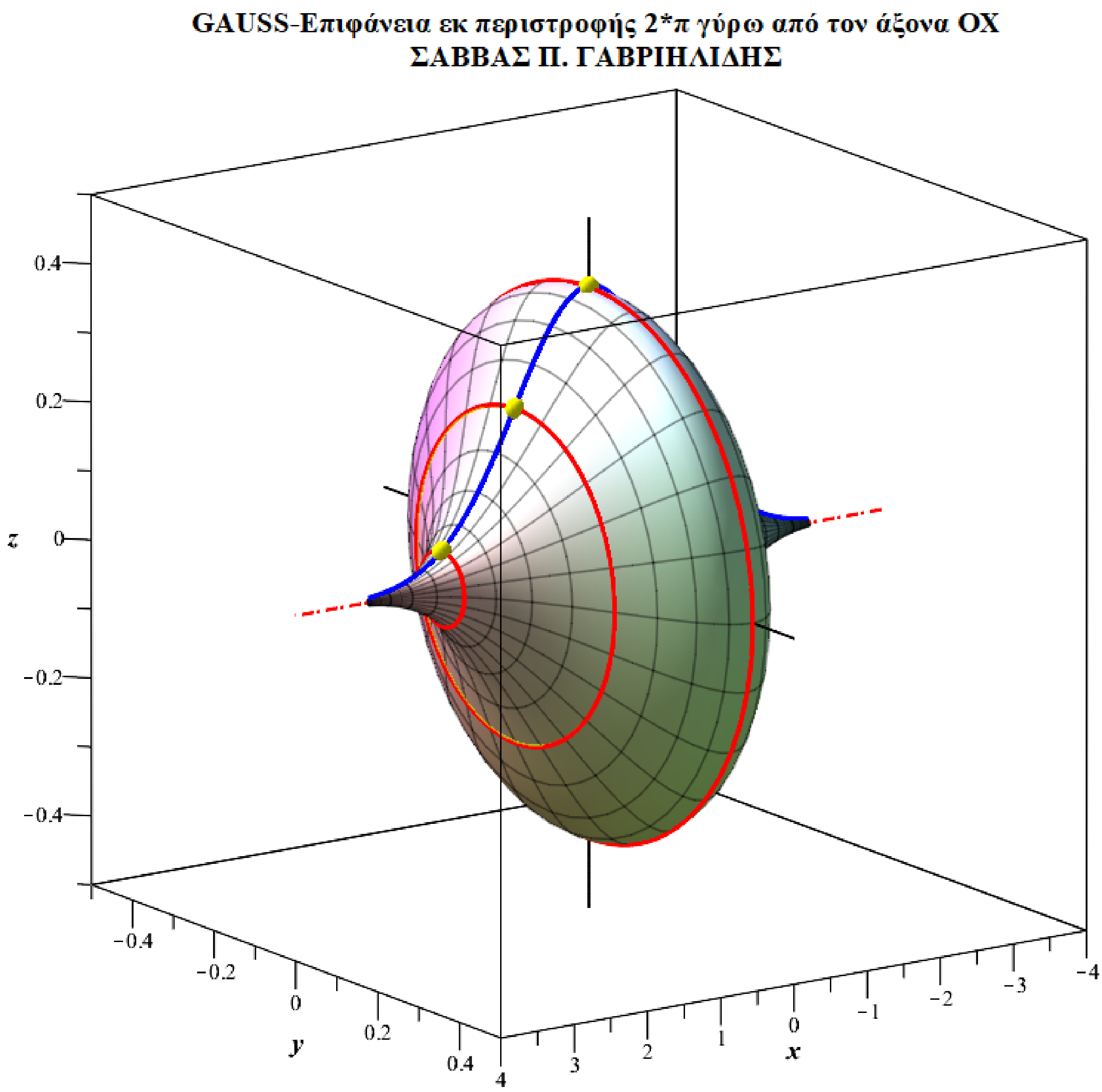
$$\left[x, -\frac{\sqrt{2}\,\mathrm{e}^{-\frac{x^2}{2}}\sin(a)}{2\sqrt{\pi}}, \frac{\sqrt{2}\,\mathrm{e}^{-\frac{x^2}{2}}\cos(a)}{2\sqrt{\pi}}\right] \quad (9)$$

> *p1* := *plot3d*($[x, -f(x)\cdot\sin(a), f(x)\cdot\cos(a)]$, *x* = -3 ..3, ***a* = 0 ..2 · Pi,** *labels* = [*x*, *y*, *z*], *labelfont* = [*arial*, *bold*, 14], *title* = "GAUSS-Επιφάνεια εκ περιστροφής 2 · π γύρω από τον άξονα OX\nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ" , *titlefont* = [*arial*, *bold*, 16], *transparency* = 0.00) :

> *ARXH* := *pointplot3d*([0, 0, 0,], *symbol* = *solidcircle*, *symbolsize* = 10) :

> *p2* := *spacecurve*([*x*, 0, *f*(*x*)], *x* = -3 ..3, *thickness* = 5, *color* = *blue*) :

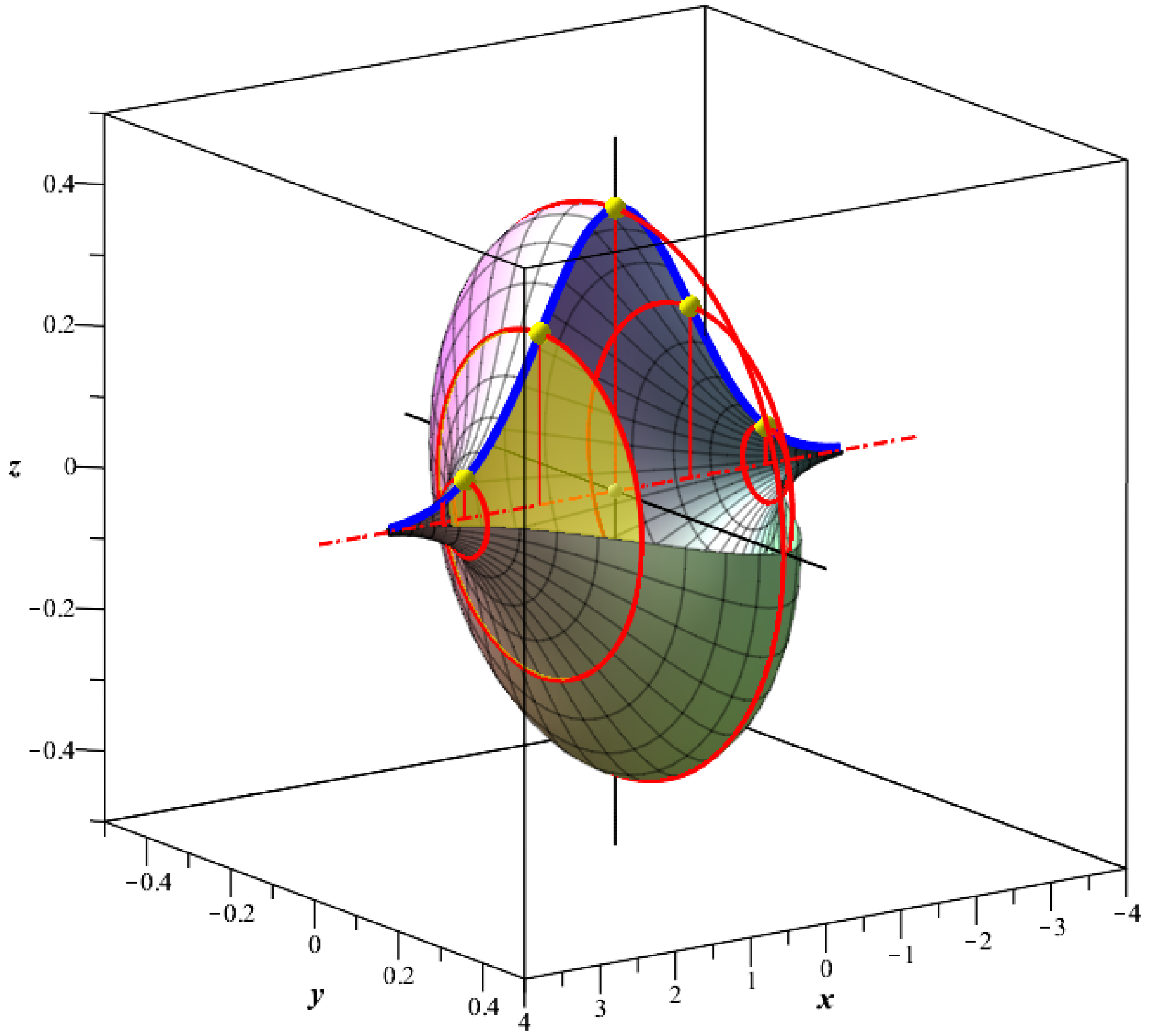
```
> paxX := spacecurve([x, 0, 0], x = -4 ..4, color = red, thickness = 2, linestyle = 4) :
> paxXa := spacecurve([x, 0, 0], x = -4 ..4, color = black, thickness = 1, linestyle = 1) :
> paxY := spacecurve([0, y, 0], y = -0.5 ..0.5, color = black, thickness = 1, linestyle = 1) :
> paxYa := spacecurve([0, y, 0], y = -4 ..4, color = black, thickness = 1, linestyle = 1) :
> paxZ := spacecurve([0, 0, z], z = -0.5 ..0.5, color = black, thickness = 1, linestyle = 1) :
> paxZa := spacecurve([0, 0, z], z = -0.05 ..0.5, color = red, thickness = 2, linestyle = 4) :
> DISK1 := spacecurve([1, f(1)·sin(a), f(1)·cos(a)], a = 0 ..2·Pi, color = red, thickness
    = 3) :
> pointA := pointplot3d([1, f(1)·sin(0), f(1)·cos(0)], symbol = solidcircle, symbolsize = 15,
    color = yellow) :
> rayonA := spacecurve([1 + λ·(1 − 1), 0 + λ·(f(1)·sin(0) − 0), 0 + λ·(f(1)·cos(0)
    − 0)], λ = 0 ..1, color = red) :
> TOMH1 := plot3d([1, r·sin(a), r·cos(a)], r = 0 ..f(1), a = 0 ..2·Pi, style = surface, color
    = yellow, transparency = 0.50) :
> DISK1SYM := spacecurve([-1, f(-1)·sin(a), f(-1)·cos(a)], a = 0 ..2·Pi, color = red,
    thickness = 3) :
> pointASYM := pointplot3d([-1, f(-1)·sin(0), f(-1)·cos(0)], symbol = solidcircle,
    symbolsize = 15, color = yellow) :
> rayonASYM := spacecurve([-1 + λ·(-1 − (-1)), 0 + λ·(f(-1)·sin(0) − 0), 0 + λ·(f(
    -1)·cos(0) − 0)], λ = 0 ..1, color = red) :
> DISK2 := spacecurve([2, f(2)·sin(a), f(2)·cos(a)], a = 0 ..2·Pi, color = red, thickness
    = 3) :
> pointB := pointplot3d([2, f(2)·sin(0), f(2)·cos(0)], symbol = solidcircle, symbolsize = 15,
    color = yellow) :
> rayonB := spacecurve([2 + λ·(2 − 2), 0 + λ·(f(2)·sin(0) − 0), 0 + λ·(f(2)·cos(0)
    − 0)], λ = 0 ..1, color = red) :
> DISK2SYM := spacecurve([-2, f(-2)·sin(a), f(-2)·cos(a)], a = 0 ..2·Pi, color = red,
    thickness = 3) :
> pointBSYM := pointplot3d([-2, f(-2)·sin(0), f(-2)·cos(0)], symbol = solidcircle,
    symbolsize = 15, color = yellow) :
> rayonBSYM := spacecurve([-2 + λ·(-2 − (-2)), 0 + λ·(f(-2)·sin(0) − 0), 0 + λ·(f(
    -2)·cos(0) − 0)], λ = 0 ..1, color = red) :
> DISKO := spacecurve([0, f(0)·sin(a), f(0)·cos(a)], a = 0 ..2·Pi, color = red, thickness
    = 3) :
> pointO := pointplot3d([0, f(0)·sin(0), f(0)·cos(0)], symbol = solidcircle, symbolsize
    = 15, color = yellow) :
> rayonO := spacecurve([0 + λ·(0 − 0), 0 + λ·(f(0)·sin(0) − 0), 0 + λ·(f(0)·cos(0)
    − 0)], λ = 0 ..1, color = red) :
>
> display(TOMH1, ARXH, p1, p2, paxX, paxY, paxZ, DISK1, pointA, DISK2, rayonA, pointB,
    rayonB, DISKO, pointO, rayonO, DISK1SYM, pointASYM, rayonASYM, DISK2SYM,
    pointBSYM, rayonBSYM, orientation = [55, 75, 0]) :
```



> *p1a* := *plot3d*([**x**, −**f**(**x**) · **sin**(**a**), **f**(**x**) · **cos**(**a**)], *x* = −3 ..3, ***a*** = **0** .. $\frac{3}{2}$ · **Pi**, *labels* = [*x*, *y*, *z*], *labelfont* = [*arial*, *bold*, 14], *title* = "GAUSS-Επιφάνεια εκ περιστροφής 3 · π/2 γύρω από τον άξονα ΟΧ\nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ" , *titlefont* = [*arial*, *bold*, 16], *transparency* = 0.00) :

> *display*(*TOMH1*, *ARXH*, *p1a*, *p2*, *paxX*, *paxY*, *paxZ*, *DISK1*, *pointA*, *DISK2*, *rayonA*, *pointB*, *rayonB*, *DISKO*, *pointO*, *rayonO*, *DISK1SYM*, *pointASYM*, *rayonASYM*, *DISK2SYM*, *pointBSYM*, *rayonBSYM*, *orientation* = [55, 75, 0]) :

GAUSS-Επιφάνεια εκ περιστροφής 3*π/2 γύρω από τον άξονα ΟΧ
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ



$$> animX := animate\left(plot3d, [[x, -f(x)\cdot\sin(a), f(x)\cdot\cos(a)], x=-3..3, a=0..X], X=0..\frac{3}{2}\cdot\mathrm{Pi}, frames=80\right):$$

$$> animpointA := animate\left(pointplot3d, [[1, -f(1)\cdot\sin(a), f(1)\cdot\cos(a)], symbol=solidcircle, symbolsize=15, color=yellow], a=0..\frac{3}{2}\cdot\mathrm{Pi}, frames=80\right):$$

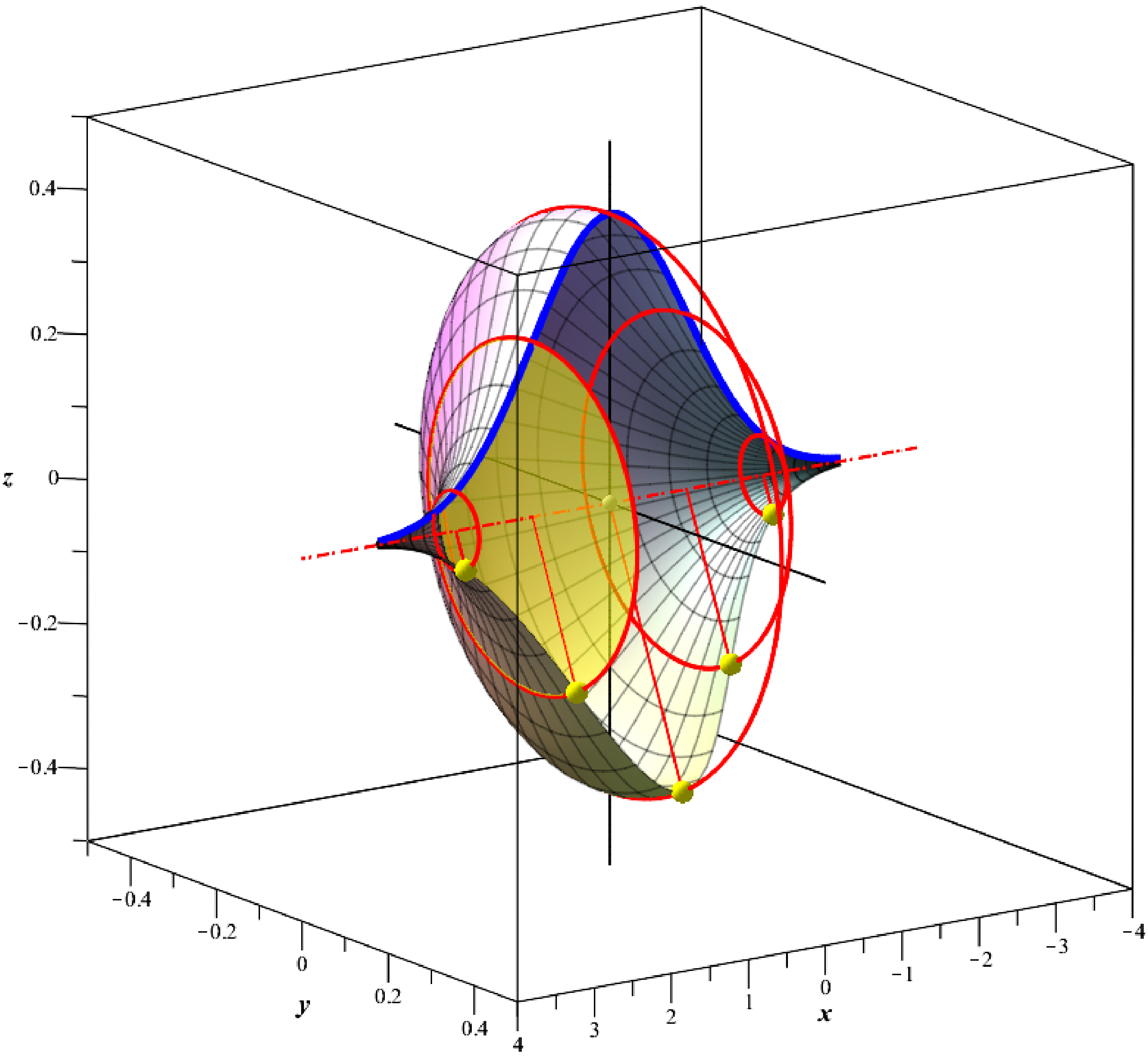
$$> animrayonA := animate\left(spacecurve, [[1+\lambda\cdot(1-1), 0+\lambda\cdot(-f(1)\cdot\sin(a)-0), 0+\lambda\cdot(f(1)\cdot\cos(a)-0)], \lambda=0..1, color=red], a=0..\frac{3}{2}\cdot\mathrm{Pi}, frames=80\right):$$

$$> animpointASYM := animate\left(pointplot3d, [[-1, -f(-1)\cdot\sin(a), f(-1)\cdot\cos(a)], symbol=solidcircle, symbolsize=15, color=yellow], a=0..\frac{3}{2}\cdot\mathrm{Pi}, frames=80\right):$$

$$> animrayonASYM := animate\left(spacecurve, [[-1+\lambda\cdot(-1-(-1)), 0+\lambda\cdot(-f(-1)\cdot\sin(a)\right.$$

```
−0), 0 + λ·(f(−1)·cos(a) − 0)], λ = 0..1, color = red], a = 0..3/2·Pi, frames = 80) :

> animpointB := animate(pointplot3d, [[2, −f(2)·sin(a), f(2)·cos(a)], symbol = solidcircle,
      symbolsize = 15, color = yellow], a = 0..3/2·Pi, frames = 80) :

> animrayonB := animate(spacecurve, [[2 + λ·(2 − 2), 0 + λ·(−f(2)·sin(a) − 0), 0 + λ
      ·(f(2)·cos(a) − 0)], λ = 0..1, color = red], a = 0..3/2·Pi, frames = 80) :

> animpointBSYM := animate(pointplot3d, [[−2, −f(−2)·sin(a), f(−2)·cos(a)], symbol
      = solidcircle, symbolsize = 15, color = yellow], a = 0..3/2·Pi, frames = 80) :

> animrayonBSYM := animate(spacecurve, [[−2 + λ·(−2 − (−2)), 0 + λ·(−f(2)·sin(a)
      − 0), 0 + λ·(f(2)·cos(a) − 0)], λ = 0..1, color = red], a = 0..3/2·Pi, frames = 80) :

> animpointO := animate(pointplot3d, [[0, −f(0)·sin(a), f(0)·cos(a)], symbol
      = solidcircle, symbolsize = 15, color = yellow], a = 0..3/2·Pi, frames = 80) :

> animrayonO := animate(spacecurve, [[0 + λ·(0 − 0), 0 + λ·(−f(0)·sin(a) − 0), 0 + λ
      ·(f(0)·cos(a) − 0)], λ = 0..1, color = red], a = 0..3/2·Pi, frames = 80) :

>
> display(TOMH1, ARXH, p2, paxX, paxY, paxZ, DISK1, DISK2, DISK1SYM, DISK2SYM,
      DISKO, animX, animpointA, animrayonA, animpointB, animrayonB, animpointO,
      animrayonO, animpointASYM, animrayonASYM, animpointBSYM, animrayonBSYM, title
      = "ANIMATE-GAUSS\n Επιφάνεια εκ περιστροφής 3·π/2 γύρω από τον άξονα
      OX\nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ", titlefont = [arial, bold, 16], labels = [x, y, z], labelfont
      = [arial, bold, 14]) :
```

ANIMATE-GAUSS
Επιφάνεια εκ περιστροφής 3*π/2 γύρω από τον άξονα OX
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ



>

Εμβαδόν περιβάλλουσας επιφάνειας .

>

> $S1 := Int\left(2 \cdot Pi \cdot f(x) \cdot \operatorname{sqrt}\left(1 + (diff(f(x), x))^2\right), x = -3..3\right) = evalf\left(int\left(2 \cdot Pi \cdot f(x) \cdot \operatorname{sqrt}\left(1 + (diff(f(x), x))^2\right), x = -3..3\right)\right)$

$$S1 := \int_{-3}^{3} \frac{\sqrt{\pi}\,\sqrt{2}\,e^{-\frac{x^2}{2}}\sqrt{4 + \frac{2\,x^2\left(e^{-\frac{x^2}{2}}\right)^2}{\pi}}}{2}\,dx = 6.361404180 \qquad \textbf{(10)}$$

> $S1LIN1 := Int\left(2 \cdot Pi \cdot g(x) \cdot \operatorname{sqrt}\left(1 + (diff(g(x), x))^2\right), x = 0..3\right) = evalf\left(int\left(2 \cdot Pi \cdot g(x) \cdot \operatorname{sqrt}\left(1 + (diff(g(x), x))^2\right), x = 0..3\right)\right)$

$$SILIN1 := \int_{0}^{3} 2\pi \left(\frac{x \left(\frac{\sqrt{2}\, e^{-\frac{9}{2}}}{2\sqrt{\pi}} - \frac{\sqrt{2}}{2\sqrt{\pi}} \right)}{3} + \frac{\sqrt{2}}{2\sqrt{\pi}} \right) \sqrt{1 + \left(\frac{\sqrt{2}\, e^{-\frac{9}{2}}}{6\sqrt{\pi}} - \frac{\sqrt{2}}{6\sqrt{\pi}} \right)^2}\, dx = 3.834442514 \quad \textbf{(11)}$$

> *S1LIN2* := *Int*(2·Pi·g(−x)· sqrt(1 + (*diff*(g(−x), x))^2), x = −3 ..0) = *evalf*(*int*(2·Pi·g(−x)· sqrt(1 + (*diff*(g(−x), x))^2), x = −3 ..0))

$$SILIN2 := \int_{-3}^{0} 2\pi \left(-\frac{x \left(\frac{\sqrt{2}\, e^{-\frac{9}{2}}}{2\sqrt{\pi}} - \frac{\sqrt{2}}{2\sqrt{\pi}} \right)}{3} + \frac{\sqrt{2}}{2\sqrt{\pi}} \right) \sqrt{1 + \left(-\frac{\sqrt{2}\, e^{-\frac{9}{2}}}{6\sqrt{\pi}} + \frac{\sqrt{2}}{6\sqrt{\pi}} \right)^2}\, dx = 3.834442514 \quad \textbf{(12)}$$

> *S1LIN1* + *S1LIN2*

$$\int_{0}^{3} 2\pi \left(\frac{x \left(\frac{\sqrt{2}\, e^{-\frac{9}{2}}}{2\sqrt{\pi}} - \frac{\sqrt{2}}{2\sqrt{\pi}} \right)}{3} + \frac{\sqrt{2}}{2\sqrt{\pi}} \right) \sqrt{1 + \left(\frac{\sqrt{2}\, e^{-\frac{9}{2}}}{6\sqrt{\pi}} - \frac{\sqrt{2}}{6\sqrt{\pi}} \right)^2}\, dx + \int_{-3}^{0} 2\pi \left(-\frac{x \left(\frac{\sqrt{2}\, e^{-\frac{9}{2}}}{2\sqrt{\pi}} - \frac{\sqrt{2}}{2\sqrt{\pi}} \right)}{3} + \frac{\sqrt{2}}{2\sqrt{\pi}} \right) \sqrt{1 + \left(-\frac{\sqrt{2}\, e^{-\frac{9}{2}}}{6\sqrt{\pi}} + \frac{\sqrt{2}}{6\sqrt{\pi}} \right)^2}\, dx = 7.668885028 \quad \textbf{(13)}$$

>

Όγκος περικλειόμενος .

>

> *V1* := *Int*(π· (f(x))^2, x = −3 ..3) − Pi·f(3)^2·2·3 = *evalf*(*int*(π· (f(x))^2, x = −3 ..3) − Pi·f(3)^2·2·3)

(14)

$$V1 := \int_{-3}^{3} \frac{\left(e^{-\frac{x^2}{2}}\right)^2}{2}\, dx - 3\left(e^{-\frac{9}{2}}\right)^2 = 0.8858371191 \quad (14)$$

> V1LIN1 := Int(π· (g(x))^2, x = 0..3) - Pi·f(3)^2·3 = evalf(int(π·(g(x))^2, x = 0..3) - Pi·f(3)^2·3)

$$V1LIN1 := \int_{0}^{3} \pi \left(\frac{x\left(\frac{\sqrt{2}\, e^{-\frac{9}{2}}}{2\sqrt{\pi}} - \frac{\sqrt{2}}{2\sqrt{\pi}}\right)}{3} + \frac{\sqrt{2}}{2\sqrt{\pi}} \right)^2 dx - \frac{3\left(e^{-\frac{9}{2}}\right)^2}{2} = 0.5054310893 \quad (15)$$

> V1LIN2 := Int(π··(g(-x))^2, x =-3..0) − Pi·f(3)^2·3 = evalf(int(π·(g(-x))^2, x =-3..0) − Pi·f(3)^2·3)

$$V1LIN2 := \int_{-3}^{0} \pi^{\left(-\frac{x\left(\frac{\sqrt{2}\, e^{-\frac{9}{2}}}{2\sqrt{\pi}} - \frac{\sqrt{2}}{2\sqrt{\pi}}\right)}{3} + \frac{\sqrt{2}}{2\sqrt{\pi}} \right)^2} dx - \frac{3\left(e^{-\frac{9}{2}}\right)^2}{2} = 0.5054310893 \quad (16)$$

> V1LIN1 + V1LIN2

$$\int_{0}^{3} \pi \left(\frac{x\left(\frac{\sqrt{2}\, e^{-\frac{9}{2}}}{2\sqrt{\pi}} - \frac{\sqrt{2}}{2\sqrt{\pi}}\right)}{3} + \frac{\sqrt{2}}{2\sqrt{\pi}} \right)^2 dx - 3\left(e^{-\frac{9}{2}}\right)^2 + \int_{-3}^{0} \pi^{\left(-\frac{x\left(\frac{\sqrt{2}\, e^{-\frac{9}{2}}}{2\sqrt{\pi}} - \frac{\sqrt{2}}{2\sqrt{\pi}}\right)}{3} + \frac{\sqrt{2}}{2\sqrt{\pi}} \right)^2} dx = 1.010862179 \quad (17)$$

>

>

Καμπύλη Gauss στο OXZ συντεταγμένο επίπεδο , Περιστροφή γύρω από τον άξονα OZ κατά γωνία *a*.

>

2. Παραμετρικές εξισώσεις της αντίστοιχης εκ περιστροφής

επιφάνειας γύρω από τον άξονα OZ:

> $[x\cdot\cos(a), x\cdot\sin(a), f(x)]$

$$\left[x\cos(a), x\sin(a), \frac{\sqrt{2}\,e^{-\frac{x^2}{2}}}{2\sqrt{\pi}}\right] \quad \textbf{(18)}$$

> p3 := plot3d([x·cos(a), x·sin(a), f(x)], x = 0 ..3, a = 0 .. 2· Pi, labels = [x, y, z], labelfont = [arial, bold, 14], title = "GAUSS-Επιφάνεια εκ περιστροφής 2·π γύρω από τον άξονα OZ\nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ", titlefont = [arial, bold, 16]) :

>

> DISK3 := spacecurve([1·cos(a), 1·sin(a), f(1)], a = 0 ..2·Pi, color = red, thickness = 3) :

> pointC := pointplot3d([1·cos(0), 1·sin(0), f(1)], symbol = solidcircle, symbolsize = 15, color = yellow) :

> rayonC := spacecurve([0 + λ·(1·cos(0) − 0), 0 + λ·(1·sin(0) − 0), f(1) + λ·(f(1) − f(1))], λ = 0 ..1, color = red) :

> **TOMH2 := plot3d([r·cos(a), r·sin(a), f(1)], r = 0 ..1, a = 0 ..2·Pi, style = surface, color = yellow, transparency = 0.50) :**

> DISK4 := spacecurve([3·cos(a), 3·sin(a), f(3)], a = 0 ..2·Pi, color = red, thickness = 3) :

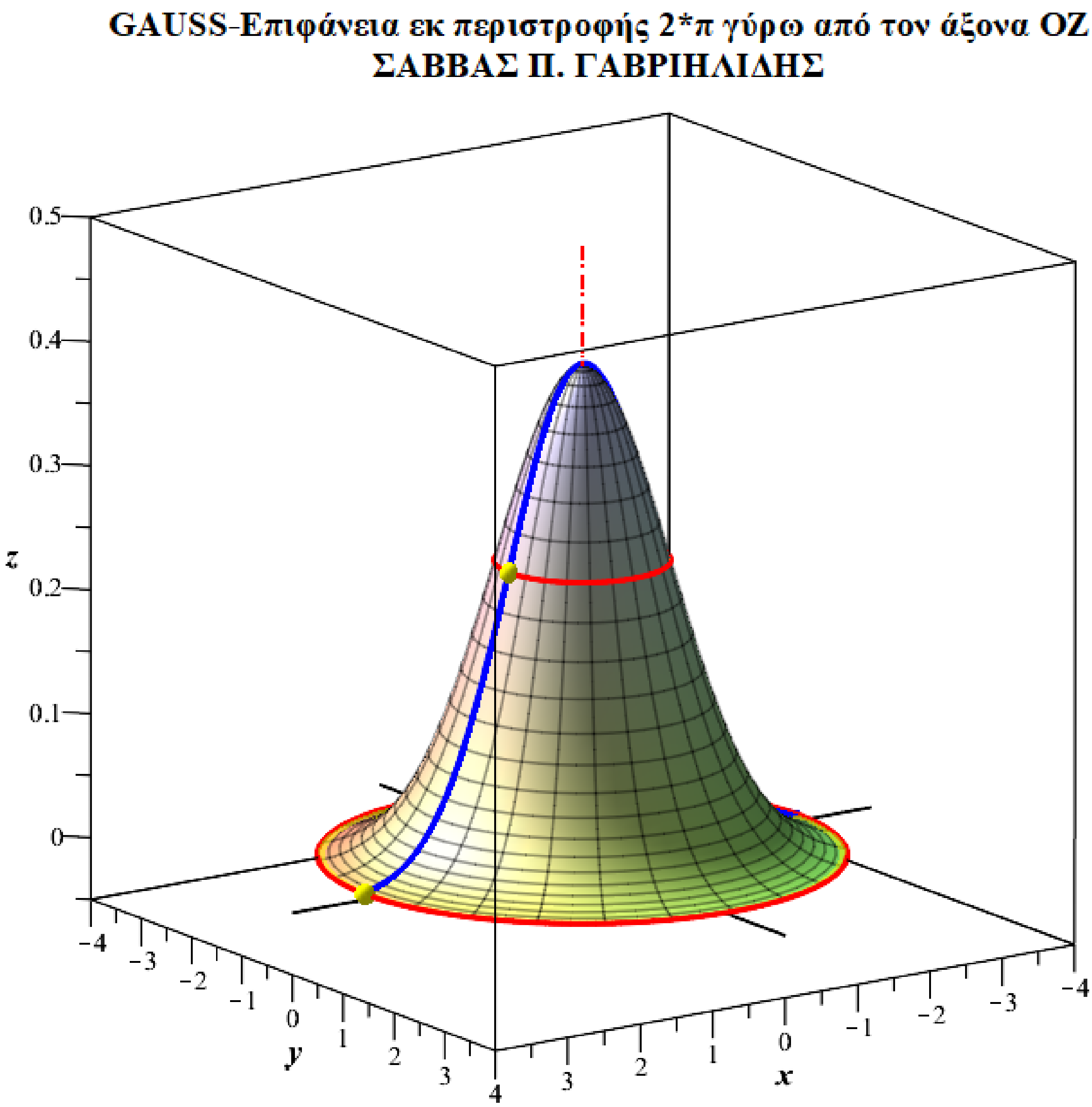
> pointD := pointplot3d([3·cos(0), 3·sin(0), f(3)], symbol = solidcircle, symbolsize = 15, color = yellow) :

> rayonD := spacecurve([0 + λ·(3·cos(0) − 0), 0 + λ·(3·sin(0) − 0), f(3) + λ·(f(3) − f(3))], λ = 0 ..1, color = red) :

> **TOMH3 := plot3d([r·cos(a), r·sin(a), f(3)], r = 0 ..3, a = 0 ..2·Pi, style = surface, color = yellow, transparency = 0.50) :**

>

> display(TOMH2, TOMH3, ARXH, p2, p3, paxXa, paxYa, paxZa, DISK3, pointC, rayonC, DISK4, pointD, rayonD) :



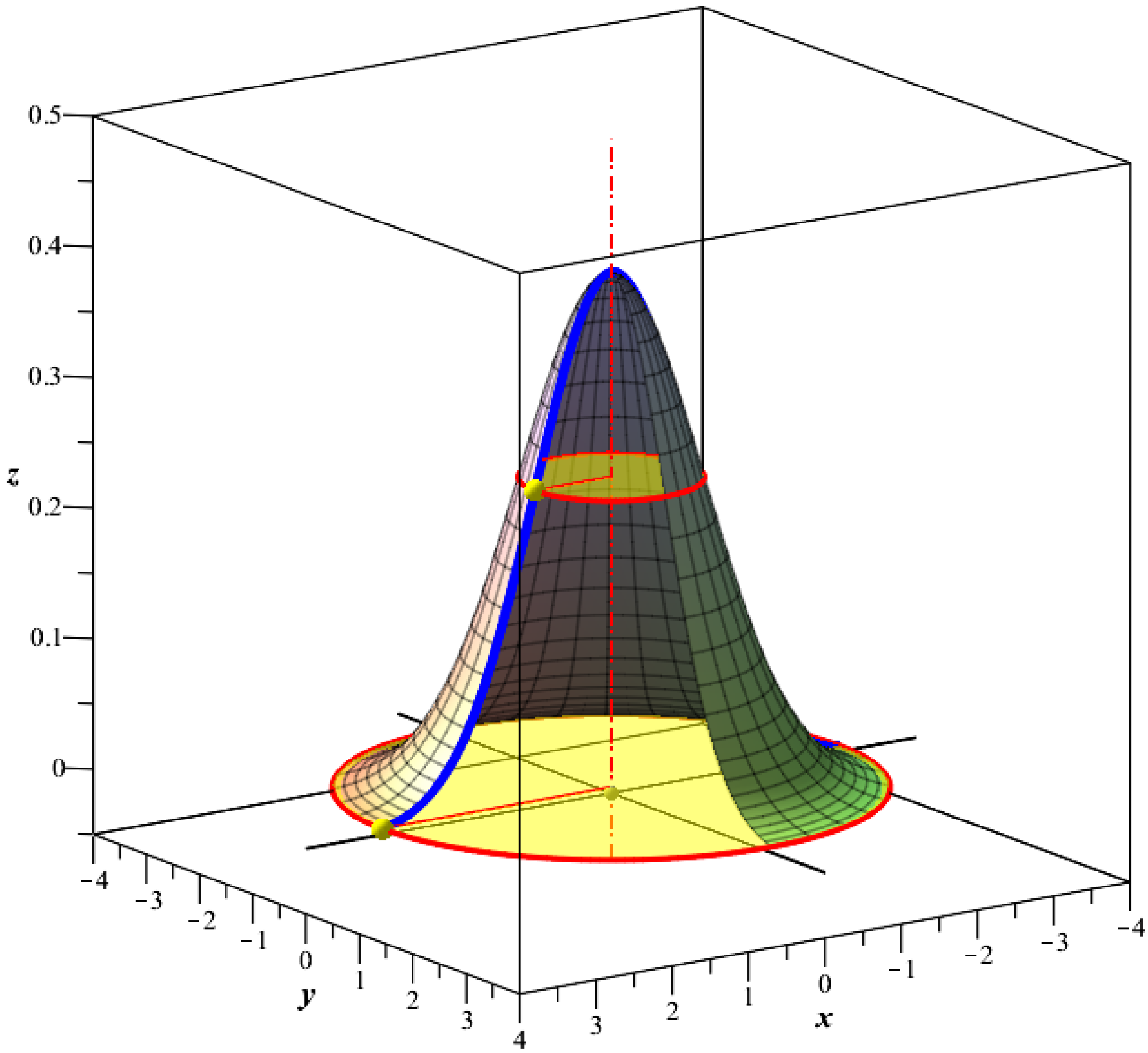
> $p3a := plot3d\left(\left[\boldsymbol{x \cdot \cos(a), x \cdot \sin(a), f(x)} \right], x = 0..3, \boldsymbol{a = 0..-\frac{3}{2} \cdot \mathrm{Pi}}, \boldsymbol{labels} = [x, y, z], labelfont = [arial, bold, 14], title \right.$

= "GAUSS-Επιφάνεια εκ περιστροφής -3·π/2 γύρω από τον άξονα ΟΖ\nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ" , $titlefont = [arial, bold, 16]\big)$:

>

> *display*(*TOMH2*, *TOMH3*, *ARXH*, *p2*, *p3a*, *paxXa*, *paxYa*, *paxZa*, *DISK3*, *pointC*, *rayonC*, *DISK4*, *pointD*, *rayonD*) :

GAUSS-Επιφάνεια εκ περιστροφής -3*π/2 γύρω από τον άξονα OZ
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ



> $animZ := animate\left(plot3d, [[x\cdot\cos(a), x\cdot\sin(a), f(x)], x=0..3, a=0..X], X=0..-\frac{3}{2}\cdot\text{Pi}, frames=80\right):$

> $animpointC := animate\left(pointplot3d, [[1\cdot\cos(a), 1\cdot\sin(a), f(1)], symbol=solidcircle, symbolsize=15, color=yellow], a=0..-\frac{3}{2}\cdot\text{Pi}, frames=80\right):$

> $animrayonC := animate\left(spacecurve, [[0+\lambda\cdot(1\cdot\cos(a)-0), 0+\lambda\cdot(1\cdot\sin(a)-0), f(1)+\lambda\cdot(f(1)-f(1))], \lambda=0..1, color=red], a=0..-\frac{3}{2}\cdot\text{Pi}, frames=80\right):$

> $animpointD := animate\left(pointplot3d, [[3\cdot\cos(a), 3\cdot\sin(a), f(3)], symbol=solidcircle, symbolsize=15, color=yellow], a=0..-\frac{3}{2}\cdot\text{Pi}, frames=80\right):$

> $animrayonD := animate\left(spacecurve, [[0+\lambda\cdot(3\cdot\cos(a)-0), 0+\lambda\cdot(3\cdot\sin(a)-0), f(3)\right.$

$+ \lambda \cdot (f(3) - f(3))], \lambda = 0..1, color = red], a = 0..-\frac{3}{2} \cdot \text{Pi}, frames = 80\Big)$:

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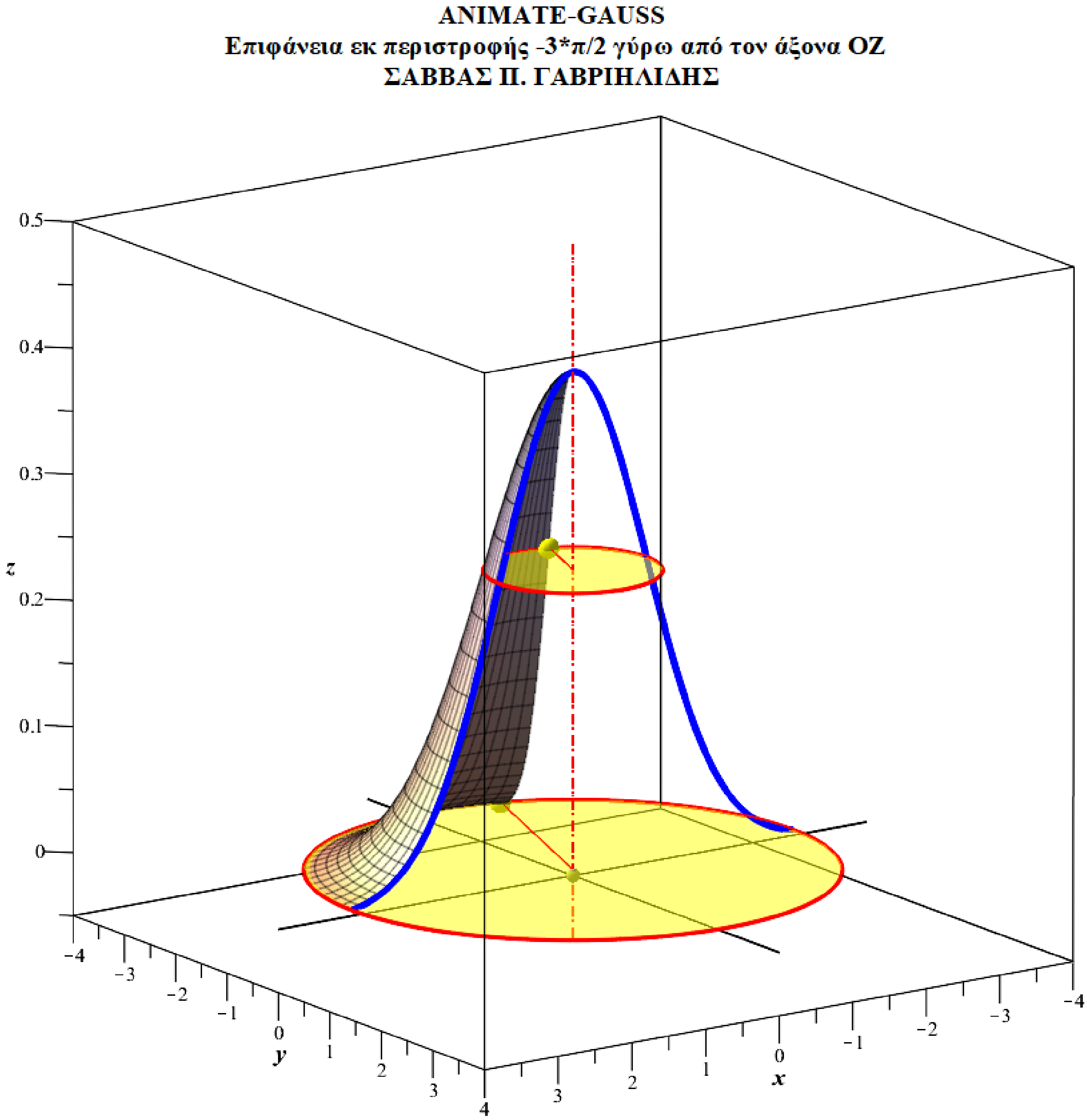
> *display*(*TOMH2*, *TOMH3*, *ARXH*, *p2*, *paxXa*, *paxYa*, *paxZa*, *DISK3*, *animZ*, *animpointC*, *animrayonC*, *DISK4*, *animpointD*, *animrayonD*, *title* = "ANIMATE-GAUSS\n Επιφάνεια εκ περιστροφής -3·π/2 γύρω από τον άξονα ΟΖ\nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ" , *titlefont* = [*arial*, *bold*, 16], *labels* = [*x*, *y*, *z*], *labelfont* = [*arial*, *bold*, 14]) :



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Εμβαδόν περιβάλλουσας επιφάνειας .

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> $S2 := Int\left(2 \cdot \text{Pi} \cdot x \cdot \text{sqrt}\left(1 + (diff(f(x), x))^2\right), x = 0..3\right) = evalf\left(int\left(2 \cdot \text{Pi} \cdot x \cdot \text{sqrt}\left(1 + (diff(f(x), x))^2\right), x = 0..3\right)\right)$

(19)

$$S2 := \int_0^3 \pi x \sqrt{4 + \frac{2 x^2 \left(e^{-\frac{x^2}{2}}\right)^2}{\pi}}\, dx = 28.52159542 \quad \textbf{(19)}$$

> $S2LIN1 := Int\left(2 \cdot Pi \cdot x \cdot sqrt\left(1 + (diff(g(x), x))^2\right), x = 0 ..3\right) = evalf\left(int\left(2 \cdot Pi \cdot x \cdot sqrt\left(1 + (diff(g(x), x))^2\right), x = 0 ..3\right)\right)$

$$S2LIN1 := \int_0^3 2 \pi x \sqrt{1 + \left(\frac{\sqrt{2}\, e^{-\frac{9}{2}}}{6\sqrt{\pi}} - \frac{\sqrt{2}}{6\sqrt{\pi}}\right)^2}\, dx = 28.51776236 \quad \textbf{(20)}$$

> $test1 := evalf\left(Pi \cdot 3 \cdot sqrt\left(3^2 + (f(0) - f(3))^2\right)\right)$

$$test1 := 28.51776234 \quad \textbf{(21)}$$

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Όγκος περικλειόμενος .

> $V2 := Int(2 \cdot Pi \cdot x \cdot f(x), x = 0 ..3) - Pi \cdot 3^2 \cdot f(3) = evalf\left(int(2 \cdot Pi \cdot x \cdot f(x), x = 0 ..3) - Pi \cdot 3^2 \cdot f(3)\right)$

$$V2 := \int_0^3 \sqrt{\pi}\, x \sqrt{2}\, e^{-\frac{x^2}{2}}\, dx - \frac{9\sqrt{\pi}\sqrt{2}\, e^{-\frac{9}{2}}}{2} = 2.353474588 \quad \textbf{(22)}$$

> $V2LIN1 := Int(2 \cdot Pi \cdot x \cdot g(x), x = 0 ..3) - Pi \cdot 3^2 \cdot f(3) = evalf\left(int(2 \cdot Pi \cdot x \cdot g(x), x = 0 ..3) - Pi \cdot 3^2 \cdot f(3)\right)$

$$V2LIN1 := \int_0^3 2 \pi x \left(\frac{x\left(\frac{\sqrt{2}\, e^{-\frac{9}{2}}}{2\sqrt{\pi}} - \frac{\sqrt{2}}{2\sqrt{\pi}}\right)}{3} + \frac{\sqrt{2}}{2\sqrt{\pi}}\right) dx - \frac{9\sqrt{\pi}\sqrt{2}\, e^{-\frac{9}{2}}}{2}$$

$$= 3.718173219 \quad \textbf{(23)}$$

> $test2 := evalf\left(\frac{1}{3} \cdot Pi \cdot 3^2 \cdot (f(0) - f(3))\right)$

$$test2 := 3.718173222 \quad \textbf{(24)}$$

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ΔΙΣΚΟΣ ΚΑΘΕΤΟΣ ΣΤΟΝ Ζ . Λύνοντας ως προς x.

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> $allvalues\left(evalf\left(isolate\left(z = \frac{1}{\sqrt{2 \cdot \pi}} \cdot e^{-\frac{x^2}{2}}, x\right)\right)\right)$

$$x = \sqrt{-\ln(2\pi z^2)},\ x = -\sqrt{-\ln(2\pi z^2)} \quad \textbf{(25)}$$

> $Int\left(Pi \cdot (rhs((\mathbf{25})[1]))^2, z = f(3) ..f(0)\right) = evalf\left(int\left(Pi \cdot (rhs((\mathbf{25})[1]))^2, z = f(3) ..f(0)\right)\right)$

(26)

$$\int_{\frac{\sqrt{2}\,e^{-\frac{9}{2}}}{2\sqrt{\pi}}}^{\frac{\sqrt{2}}{2\sqrt{\pi}}} -\pi \ln\left(2\,\pi\,z^2\right)\,dz = 2.353474588 \qquad (26)$$

ΣΩΣΤΑ!!!!!!

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> $evalf\left(isolate\left(z=\frac{x}{3}\cdot(f(3)-f(0))+f(0), x\right)\right)$

$$x = -7.604361653\,z + 3.033701377 \qquad (27)$$

> $Int\left(\mathrm{Pi}\cdot(rhs((\mathbf{27})))^2, z=f(3)..f(0)\right) = int\left(\mathrm{Pi}\cdot(rhs((\mathbf{27})))^2, z=f(3)..f(0)\right)$

$$\int_{\frac{\sqrt{2}\,e^{-\frac{9}{2}}}{2\sqrt{\pi}}}^{\frac{\sqrt{2}}{2\sqrt{\pi}}} \pi\,(-7.604361653\,z + 3.033701377)^2\,dz = 3.718173220 \qquad (28)$$

ΣΩΣΤΑ!!!!!!

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