

> with(plots) :
> with(plottools) :

ΘΕΜΑ

ΥΠΟΛΟΓΙΣΜΟΣ ΔΙΑΡΚΕΙΑΣ ΗΜΕΡΑΣ-ΝΥΧΤΑΣ

Μεγαλύτερη ημέρα του χρόνου .!!!
Κατά το Θερινό Ηλιοστάσιο .

ΠΑΡΑΔΟΧΕΣ - ΔΕΔΟΜΕΝΑ

ω : ΚΑΙΣΗ ΑΞΟΝΑ ΠΕΡΙΣΤΡΟΦΗΣ της ΓΗΣ ως προς την κατακόρυφο : $23,439247^\circ = 0.40909 \text{ rad}$.

R_e : Μέση ακτίνα ΓΗΣ : 6371 km .

Αφήλιο : 152.098.232 km , Περιήλιο : 147.098.290 km , Εκκεντρότητα : 0.01671123 .

Μεγάλος Ημιάξονας Ελλειπτικής : $a=149.598.261 \text{ km} \Rightarrow c=2.499.971 \text{ km}$

Μικρός Ημιάξονας Ελλειπτικής : $b=149.577.371 \text{ km}$

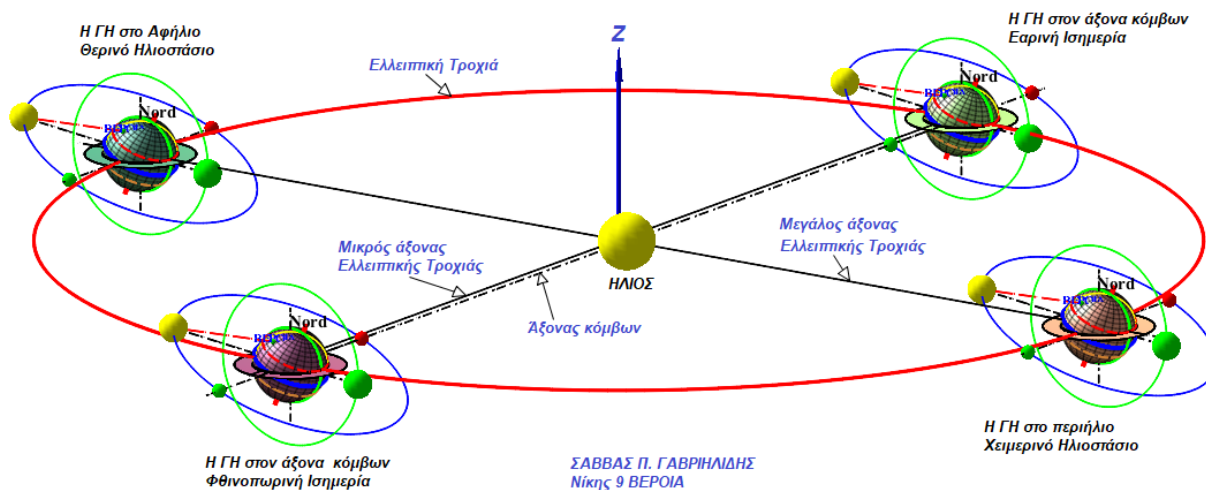
ω_1 : Γεωγραφικό πλάτος : Βέροιας: $0.7071969411 \text{ N(rad)}$,

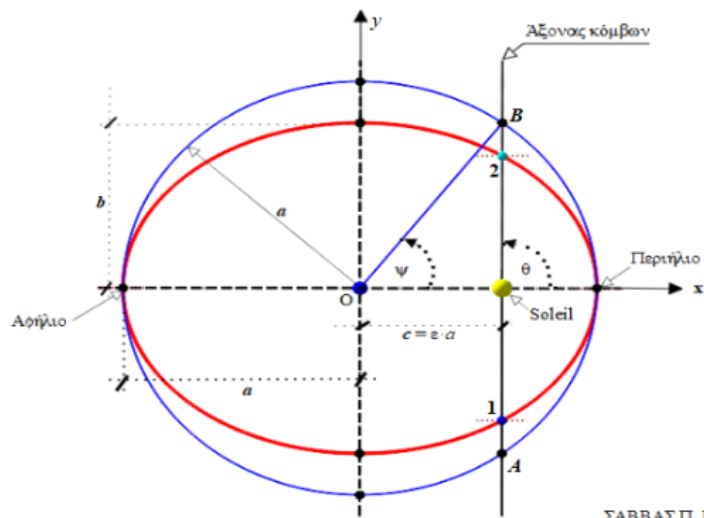
ΓΚΡΙΝΟΥΙΤΣ : $0.8984413939 \text{ N(rad)}$

ΑΡΚΤΙΚΟΥ ΚΥΚΛΟΥ : $1.161704293 \text{ N(rad)}$

ΣΗΜΕΡΙΝΟΥ : 0 (rad)

ΔΙΑΡΚΕΙΑ ΠΕΡΙΣΤΡΟΦΗΣ ΤΗΣ ΓΗΣ ΠΕΡΙ ΤΟΝ ΑΞΟΝΑ ΤΗΣ : 24 h(ώρες)

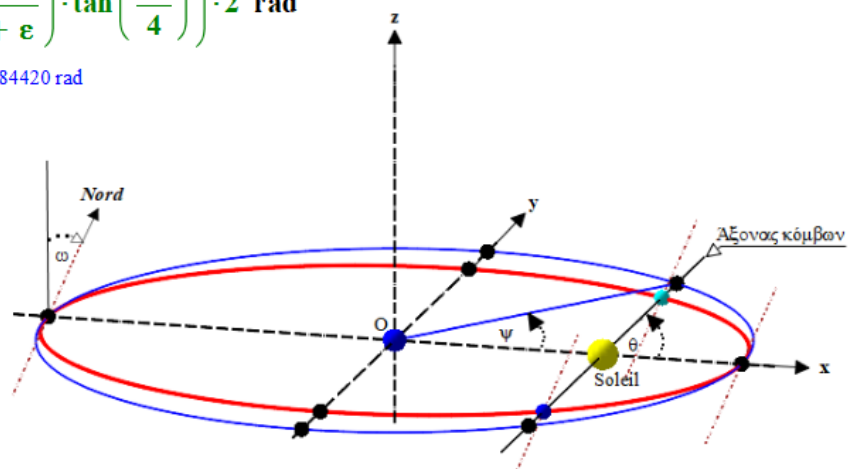




ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ
Νίκης 9 - ΒΕΡΟΙΑ

$$\psi_{nodes} := \tan^{-1} \left(\sqrt{\frac{1-\epsilon}{1+\epsilon}} \cdot \tan \left(\frac{\text{Pi}}{4} \right) \right) \cdot 2 \text{ rad}$$

$$\psi_{nodes} := 1.554084420 \text{ rad}$$



Με πόλο την Εστία-ΗΛΙΟΣ
Εξίσωση Ελλειπτικής Τροχιάς σε Πολικές Συντεταγμένες

$$r(\theta) = \frac{a \cdot (1 - \epsilon^2)}{1 + \epsilon \cdot \cos(\theta)}$$

$$> \omega := \frac{23.439247 \cdot \text{Pi}}{180} ;$$

$$> a := \frac{149598261}{1500}$$

$$a := \frac{49866087}{500}$$

(1)

$$> b := \frac{149577371}{1500}$$

$$b := \frac{149577371}{1500}$$

(2)

$$> \epsilon := \text{evalf}\left(\sqrt{1 - \left(\frac{b}{a}\right)^2}\right)$$

$$\epsilon := 0.01671112868$$

(3)

$$> c := a \cdot \epsilon$$

$$c := 1666.637193$$

(4)

$$r(\theta) = \frac{a \cdot (1 - \epsilon^2)}{1 + \epsilon \cdot \cos(\theta)} \quad \tan\left(\frac{\psi}{2}\right) = \sqrt{\frac{1 - \epsilon I}{1 + \epsilon I}} \cdot \tan\left(\frac{\theta}{2}\right)$$

ψ :=Εκκεντρική Ανωμαλία

θ :=Αληθής Ανωμαλία

Εξίσωση Έλλειψης σε καρτεσιανές Συντεταγμένες με Αρχή τον πόλο Εκκεντρικής Ανωμαλίας

$$[a \cdot \cos(\psi), b \cdot \sin(\psi), 0]$$

a ο μεγάλος ημιάξονας

b ο μικρός ημιάξονας της Έλλειψης

Συντεταγμένες της Σφαίρας με κεκλιμένο κατά γωνία ω άξονα .

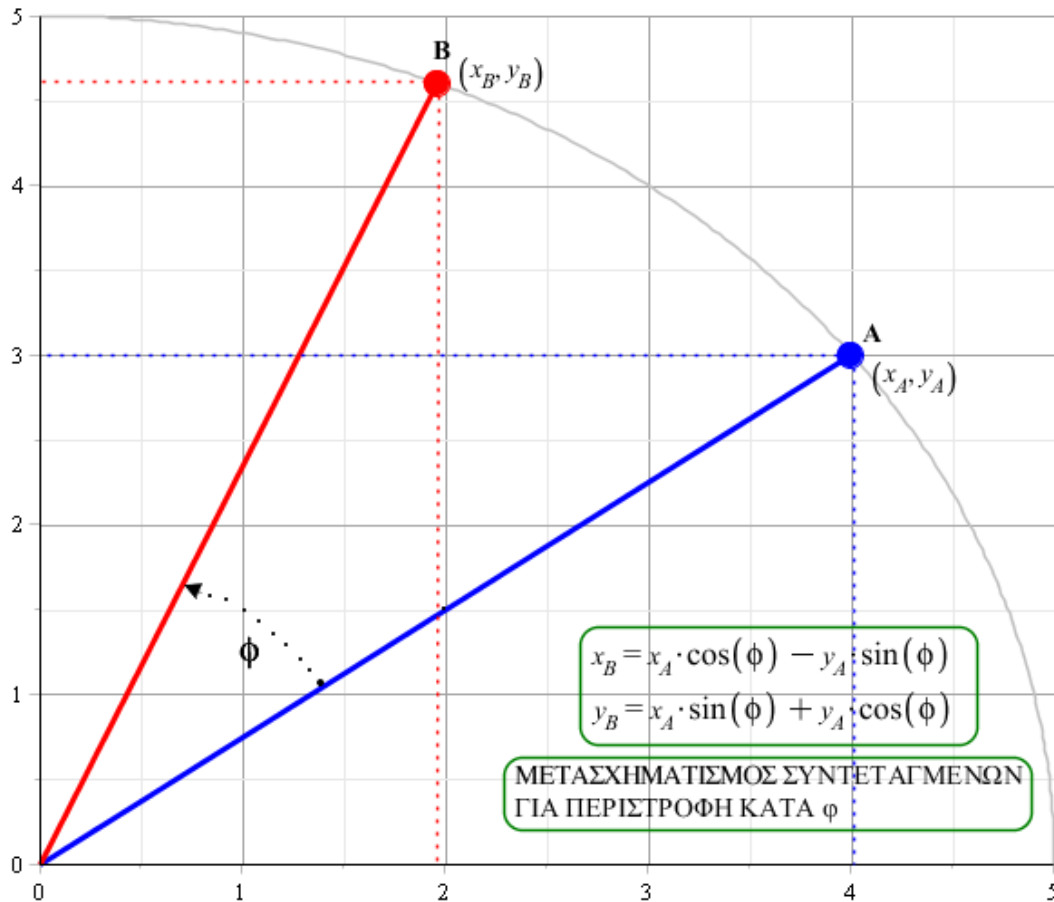
Περιστροφή περί τον άξονα y - y

$$x[B] := z[A] \cdot \sin(\omega) + x[A] \cdot \cos(\omega)$$

$$y[B] := y[A]$$

$$z[B] := z[A] \cdot \cos(\omega) - x[A] \cdot \sin(\omega)$$

ΠΕΡΙ ΠΕΡΙΣΤΡΟΦΗΣ (rotate)



$$> \omega := \frac{23.439247 \cdot \text{Pi}}{180}$$

$$\omega := 0.4090920344$$

(5)

$$> R[e] := 6371$$

$$R_e := 6371$$

(6)

> *Hlios* := *pointplot3d*([*c*, 0, 0], *symbol* = *solidcircle*, *symbolsize* = 5, *color* = *yellow*) :

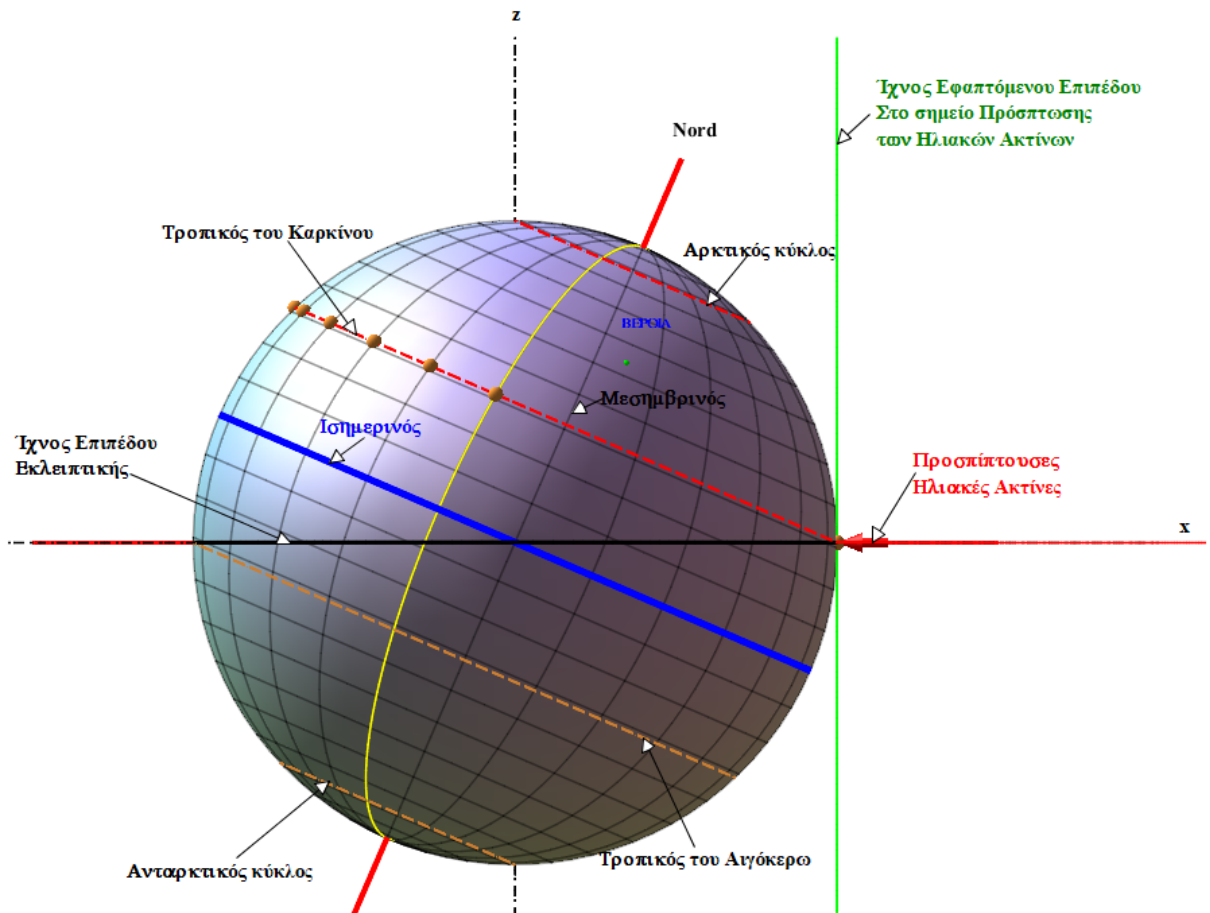
> *textHlios* := *textplot3d*([3 · *c*, − 3 · *c*, 0, "ΗΛΙΟΣ"], *font* = [*arial*, *bold*, 12]) :

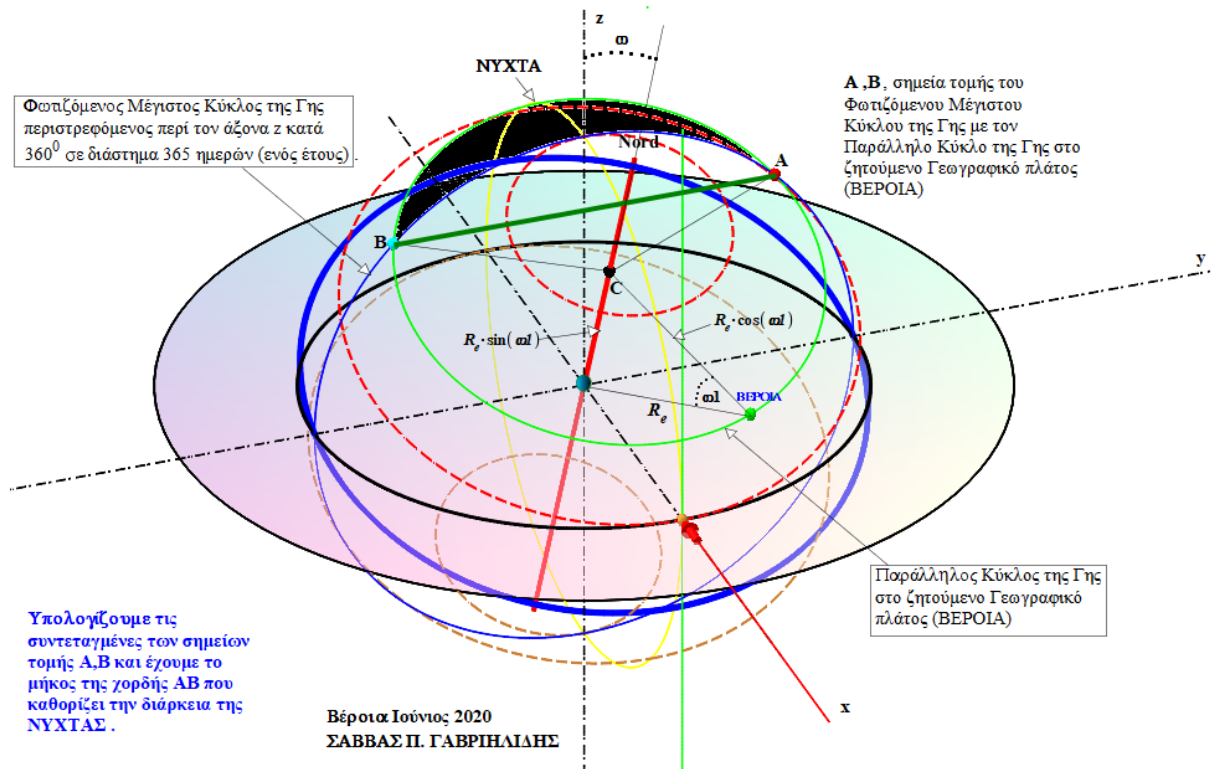
>

**Η ΓΗ ΣΤΟ ΑΦΗΛΙΟ (ΚΑΛΟΚΑΙΡΙ ΣΤΟ ΒΟΡΕΙΟ
ΗΜΙΣΦΑΙΡΙΟ)**

$$\mu=0$$

ΠΡΟΒΟΛΗ ΣΤΟ ΕΠΙΠΕΔΟ $x-z$
ΚΑΛΟΚΑΙΡΙ ΣΤΟ ΒΟΡΕΙΟ ΗΜΙΣΦΑΙΡΙΟ
Η ΓΗ ΣΤΟ ΑΦΗΛΙΟ
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ





$$> \psi_{af} := \text{Pi}$$

$$\psi_{af} := \pi \quad (7)$$

$$> xEL := a \cdot \cos(\psi)$$

$$xEL := \frac{49866087 \cos(\psi)}{500} \quad (8)$$

$$> yEL := b \cdot \sin(\psi)$$

$$yEL := \frac{149577371 \sin(\psi)}{1500} \quad (9)$$

$$> zEL := 0$$

$$zEL := 0 \quad (10)$$

$$> X := xEL + R[e] \cdot \sin(\theta) \cdot \cos(\phi + \tau) \cdot \cos(\omega) + R[e] \cdot \cos(\theta) \cdot \sin(\omega)$$

$$X := \frac{49866087 \cos(\psi)}{500} + 5845.280168 \sin(\theta) \cos(\phi + \tau) + 2534.233760 \cos(\theta) \quad (11)$$

$$> Y := yEL + R[e] \cdot \sin(\theta) \cdot \sin(\phi + \tau)$$

$$Y := \frac{149577371 \sin(\psi)}{1500} + 6371 \sin(\theta) \sin(\phi + \tau) \quad (12)$$

$$> Z := zEL - R[e] \cdot \sin(\theta) \cdot \cos(\phi + \tau) \cdot \sin(\omega) + R[e] \cdot \cos(\theta) \cdot \cos(\omega)$$

$$Z := -2534.233760 \sin(\theta) \cos(\phi + \tau) + 5845.280168 \cos(\theta) \quad (13)$$

$$> \text{evalf}(\text{subs}(\psi = \psi_{af}, X))$$

$$-99732.17400 + 5845.280168 \sin(\theta) \cos(\phi + \tau) + 2534.233760 \cos(\theta) \quad (14)$$

$$> \text{evalf}(\text{subs}(\psi = \psi_{af}, Y))$$

$$-0.00004090509931 + 6371 \cdot \sin(\theta) \sin(\phi + \tau) \quad (15)$$

$$> \text{evalf}(\text{subs}(\psi = \psi_{af}, Z))$$

$$-2534.233760 \sin(\theta) \cos(\phi + \tau) + 5845.280168 \cos(\theta) \quad (16)$$

> *anSFaf* := *animate*(*plot3d*, [(14), (15), (16)], $\theta = 0 \dots \text{Pi}$, $\phi = 0 \dots 2 \cdot \text{Pi}$, $\tau = 0 \dots 2 \cdot \text{Pi}$, *frames* = 49) :

Συντεταγμένες του ΙΣΗΜΕΡΙΝΟΥ της ΓΗΣ με κεκλιμένο κατά γωνία ω άξονα :

$$\begin{aligned} > XISHM := xEL + R[e] \cdot \sin\left(\frac{\text{Pi}}{2}\right) \cdot \cos(\phi) \cdot \cos(\omega) + R[e] \cdot \cos\left(\frac{\text{Pi}}{2}\right) \cdot \sin(\omega) \\ XISHM &:= \frac{49866087 \cos(\psi)}{500} + 5845.280168 \cos(\phi) \end{aligned} \quad (17)$$

$$\begin{aligned} > YISHM := yEL + R[e] \cdot \sin\left(\frac{\text{Pi}}{2}\right) \cdot \sin(\phi) \\ YISHM &:= \frac{149577371 \sin(\psi)}{1500} + 6371 \sin(\phi) \end{aligned} \quad (18)$$

$$\begin{aligned} > ZISHM := zEL - R[e] \cdot \sin\left(\frac{\text{Pi}}{2}\right) \cdot \cos(\phi) \cdot \sin(\omega) + R[e] \cdot \cos\left(\frac{\text{Pi}}{2}\right) \cdot \cos(\omega) \\ ZISHM &:= -2534.233760 \cos(\phi) \end{aligned} \quad (19)$$

$$\begin{aligned} > evalf(\text{subs}(\psi = \psi_{af}, XISHM)) \\ &-99732.17400 + 5845.280168 \cos(\phi) \end{aligned} \quad (20)$$

$$\begin{aligned} > evalf(\text{subs}(\psi = \psi_{af}, YISHM)) \\ &-0.00004090509931 + 6371 \cdot \sin(\phi) \end{aligned} \quad (21)$$

$$\begin{aligned} > evalf(\text{subs}(\psi = \psi_{af}, ZISHM)) \\ &-2534.233760 \cos(\phi) \end{aligned} \quad (22)$$

> *ISHMERINOSaf* := *spacecurve*((20), (21), (22)], $\phi = 0 \dots 2 \cdot \text{Pi}$, *color* = blue, *thickness* = 5) :

Συντεταγμένες του ΤΡΟΠΙΚΟΥ ΤΟΥ ΚΑΡΚΙΝΟΥ της ΓΗΣ με κεκλιμένο κατά γωνία ω άξονα :

$$\begin{aligned} > Xcancer := xEL + R[e] \cdot \sin\left(\frac{\text{Pi}}{2} - \omega\right) \cdot \cos(\phi) \cdot \cos(\omega) + R[e] \cdot \cos\left(\frac{\text{Pi}}{2} - \omega\right) \cdot \sin(\omega) \\ Xcancer &:= \frac{49866087 \cos(\psi)}{500} + 5362.941494 \cos(\phi) + 1008.058506 \end{aligned} \quad (23)$$

$$\begin{aligned} > Ycancer := yEL + R[e] \cdot \sin\left(\frac{\text{Pi}}{2} - \omega\right) \cdot \sin(\phi) \\ Ycancer &:= \frac{149577371 \sin(\psi)}{1500} + 5845.280170 \sin(\phi) \end{aligned} \quad (24)$$

$$\begin{aligned} > Zcancer := zEL - R[e] \cdot \sin\left(\frac{\text{Pi}}{2} - \omega\right) \cdot \cos(\phi) \cdot \sin(\omega) + R[e] \cdot \cos\left(\frac{\text{Pi}}{2} - \omega\right) \cdot \cos(\omega) \\ Zcancer &:= 2325.114789 - 2325.114793 \cos(\phi) \end{aligned} \quad (25)$$

$$\begin{aligned} > evalf(\text{subs}(\psi = \psi_{af}, Xcancer)) \\ &-98724.11549 + 5362.941494 \cos(\phi) \end{aligned} \quad (26)$$

$$\begin{aligned} > evalf(\text{subs}(\psi = \psi_{af}, Ycancer)) \\ &-0.00004090509931 + 5845.280170 \sin(\phi) \end{aligned} \quad (27)$$

$$\begin{aligned} > evalf(\text{subs}(\psi = \psi_{af}, Zcancer)) \\ &2325.114789 - 2325.114793 \cos(\phi) \end{aligned} \quad (28)$$

> *CANCERaf* := *spacecurve*([(26), (27), (28)], $\phi = 0 \dots 2 \cdot \text{Pi}$, *color* = red, *linestyle* = 3, *thickness* = 2) :

> *animCANCER* := *animate*(*pointplot3d*, [(26), (27), (28)], *symbol* = solidcircle, *symbolsize* = 5, *color* = gold, $\phi = 0 \dots 2 \cdot \text{Pi}$, *frames* = 49, *trace* = 24) :

>

Συντεταγμένες του ΑΡΚΤΙΚΟΥ ΚΥΚΛΟΥ της ΓΗΣ με κεκλιμένο κατά γωνία ω άξονα :

> $X_{arkt} := xEL + R[e] \cdot \sin(\omega) \cdot \cos(\phi) \cdot \cos(\omega) + R[e] \cdot \cos(\omega) \cdot \sin(\omega)$

$$X_{arkt} := \frac{49866087 \cos(\psi)}{500} + 2325.114792 \cos(\phi) + 2325.114792 \quad (29)$$

> $Y_{arkt} := yEL + R[e] \cdot \sin(\omega) \cdot \sin(\phi)$

$$Y_{arkt} := \frac{149577371 \sin(\psi)}{1500} + 2534.233760 \sin(\phi) \quad (30)$$

> $Z_{arkt} := zEL - R[e] \cdot \sin(\omega) \cdot \cos(\phi) \cdot \sin(\omega) + R[e] \cdot \cos(\omega) \cdot \cos(\omega)$

$$Z_{arkt} := 5362.941492 - 1008.058508 \cos(\phi) \quad (31)$$

> *evalf*(*subs*($\psi = \psi_{af}$, X_{arkt}))

$$-97407.05921 + 2325.114792 \cos(\phi) \quad (32)$$

> *evalf*(*subs*($\psi = \psi_{af}$, Y_{arkt}))

$$-0.00004090509931 + 2534.233760 \sin(\phi) \quad (33)$$

> *evalf*(*subs*($\psi = \psi_{af}$, Z_{arkt}))

$$5362.941492 - 1008.058508 \cos(\phi) \quad (34)$$

> *ARKTIKOSaf* := *spacecurve*([(32), (33), (34)], $\phi = 0 \dots 2 \cdot \text{Pi}$, *color* = red, *linestyle* = 3, *thickness* = 2) :

>

Συντεταγμένες του ΤΡΟΠΙΚΟΥ ΤΟΥ ΑΙΓΟΚΕΡΩ της ΓΗΣ με κεκλιμένο κατά γωνία ω άξονα :

> $X_{capricorne} := xEL + R[e] \cdot \sin\left(\frac{\text{Pi}}{2} + \omega\right) \cdot \cos(\phi) \cdot \cos(\omega) + R[e] \cdot \cos\left(\frac{\text{Pi}}{2} + \omega\right) \cdot \sin(\omega)$

$$X_{capricorne} := \frac{49866087 \cos(\psi)}{500} + 5362.941493 \cos(\phi) - 1008.058508 \quad (35)$$

> $Y_{capricorne} := yEL + R[e] \cdot \sin\left(\frac{\text{Pi}}{2} + \omega\right) \cdot \sin(\phi)$

$$Y_{capricorne} := \frac{149577371 \sin(\psi)}{1500} + 5845.280169 \sin(\phi) \quad (36)$$

> $Z_{capricorne} := zEL - R[e] \cdot \sin\left(\frac{\text{Pi}}{2} + \omega\right) \cdot \cos(\phi) \cdot \sin(\omega) + R[e] \cdot \cos\left(\frac{\text{Pi}}{2} + \omega\right) \cdot \cos(\omega)$

$$Z_{capricorne} := -2325.114792 - 2325.114793 \cos(\phi) \quad (37)$$

> *evalf*(*subs*($\psi = \psi_{af}$, $X_{capricorne}$))

$$-100740.2325 + 5362.941493 \cos(\phi) \quad (38)$$

> *evalf*(*subs*($\psi = \psi_{af}$, $Y_{capricorne}$))

$$-0.00004090509931 + 5845.280169 \sin(\phi) \quad (39)$$

> *evalf*(*subs*($\psi = \psi_{af}$, $Z_{capricorne}$))

$$-2325.114792 - 2325.114793 \cos(\phi) \quad (40)$$

> CAPRICORNEaf := spacecurve([(38), (39), (40)], $\phi = 0 \dots 2 \cdot \text{Pi}$, color = gold, linestyle = 3, thickness = 2) :

>

Συντεταγμένες του ΑΝΤΑΡΚΤΙΚΟΥ ΚΥΚΛΟΥ της ΓΗΣ με κεκλιμένο κατά γωνία ω άξονα :

$$\begin{aligned} Xantarkt &:= xEL + R[e] \cdot \sin(\text{Pi} - \omega) \cdot \cos(\phi) \cdot \cos(\omega) + R[e] \cdot \cos(\text{Pi} - \omega) \cdot \sin(\omega) \\ Xantarkt &:= \frac{49866087 \cos(\psi)}{500} + 2325.114788 \cos(\phi) - 2325.114793 \end{aligned} \quad (41)$$

$$\begin{aligned} Yantarkt &:= yEL + R[e] \cdot \sin(\text{Pi} - \omega) \cdot \sin(\phi) \\ Yantarkt &:= \frac{149577371 \sin(\psi)}{1500} + 2534.233756 \sin(\phi) \end{aligned} \quad (42)$$

$$\begin{aligned} Zantarkt &:= zEL - R[e] \cdot \sin(\text{Pi} - \omega) \cdot \cos(\phi) \cdot \sin(\omega) + R[e] \cdot \cos(\text{Pi} - \omega) \cdot \cos(\omega) \\ Zantarkt &:= -5362.941494 - 1008.058506 \cos(\phi) \end{aligned} \quad (43)$$

$$\begin{aligned} &evalf(subs(\psi = \psi af, Xantarkt)) \\ &\quad -102057.2888 + 2325.114788 \cos(\phi) \end{aligned} \quad (44)$$

$$\begin{aligned} &evalf(subs(\psi = \psi af, Yantarkt)) \\ &\quad -0.00004090509931 + 2534.233756 \sin(\phi) \end{aligned} \quad (45)$$

$$\begin{aligned} &evalf(subs(\psi = \psi af, Zantarkt)) \\ &\quad -5362.941494 - 1008.058506 \cos(\phi) \end{aligned} \quad (46)$$

> ANTARKTIKOSaf := spacecurve([(44), (45), (46)], $\phi = 0 \dots 2 \cdot \text{Pi}$, color = gold, linestyle = 3, thickness = 2) :

>

Συντεταγμένες του 1-ΜΕΣΗΜΒΡΙΝΟΥ της ΓΗΣ με κεκλιμένο κατά γωνία ω άξονα :

$$\begin{aligned} X &:= xEL + R[e] \cdot \sin(\theta) \cdot \cos(0) \cdot \cos(\omega) + R[e] \cdot \cos(\theta) \cdot \sin(\omega) \\ X &:= \frac{49866087 \cos(\psi)}{500} + 5845.280168 \sin(\theta) + 2534.233760 \cos(\theta) \end{aligned} \quad (47)$$

$$\begin{aligned} Y &:= yEL + R[e] \cdot \sin(\theta) \cdot \sin(0) \\ Y &:= \frac{149577371 \sin(\psi)}{1500} \end{aligned} \quad (48)$$

$$\begin{aligned} Z &:= zEL - R[e] \cdot \sin(\theta) \cdot \cos(0) \cdot \sin(\omega) + R[e] \cdot \cos(\theta) \cdot \cos(\omega) \\ Z &:= -2534.233760 \sin(\theta) + 5845.280168 \cos(\theta) \end{aligned} \quad (49)$$

$$\begin{aligned} &evalf(subs(\psi = \psi af, X)) \\ &\quad -99732.17400 + 5845.280168 \sin(\theta) + 2534.233760 \cos(\theta) \end{aligned} \quad (50)$$

$$\begin{aligned} &evalf(subs(\psi = \psi af, Y)) \\ &\quad -0.00004090509931 \end{aligned} \quad (51)$$

$$\begin{aligned} &evalf(subs(\psi = \psi af, Z)) \\ &\quad -2534.233760 \sin(\theta) + 5845.280168 \cos(\theta) \end{aligned} \quad (52)$$

> MESHMBRINOSaf := spacecurve([(50), (51), (52)], $\theta = 0 \dots 2 \cdot \text{Pi}$, color = green, thickness = 3) :

> *animMESHMBRINOSaf* := *animate*(*spacecurve*, [[*subs*($\psi = \psi_{af}$, $xEL + R[e] \cdot \sin(\theta) \cdot \cos(\phi) \cdot \cos(\omega)$) + $R[e] \cdot \cos(\theta) \cdot \sin(\omega)$, *subs*($\psi = \psi_{af}$, $yEL + R[e] \cdot \sin(\theta) \cdot \sin(\phi)$), *subs*($\psi = \psi_{af}$, $zEL - R[e] \cdot \sin(\theta) \cdot \cos(\phi) \cdot \sin(\omega) + R[e] \cdot \cos(\theta) \cdot \cos(\omega)$)] , $\theta = 0 \dots 2 \cdot \text{Pi}$, *color*=yellow, *thickness*=1], $\phi = 0 \dots 2 \cdot \text{Pi}$, *frames*=49) :

Συντεταγμένες του κεκλιμένου κατά γωνία ω άξονα [$r \cdot \sin(\omega)$, 0, $r \cdot \cos(\omega)$] της Σφαίρας με : $\lambda = -1.3 \dots 1.3$

> *Xaxon* := *evalf*(*subs*($\psi = \psi_{af}$, $xEL + \lambda \cdot R[e] \cdot \sin(\omega)$))
Xaxon := -99732.17400 + 2534.233760 λ (53)

> *Yaxon* := *evalf*(*subs*($\psi = \psi_{af}$, $yEL + \lambda \cdot 0$))
Yaxon := -0.00004090509931 (54)

> *Zaxon* := *evalf*(*subs*($\psi = \psi_{af}$, $zEL + \lambda \cdot R[e] \cdot \cos(\omega)$))
Zaxon := 5845.280168 λ (55)

> *axonSaf* := *spacecurve*([(53), (54), (55)], $\lambda = -1.3 \dots 1.3$, *color*=red, *thickness*=4) :

> *TEXTaxonaf* := *textplot3d*([*subs*($\lambda = 1.4$, (53)), *subs*($\lambda = 1.4$, (54)), *subs*($\lambda = 1.4$, (55)), "Nord"], *font* = [arial, bold, 12]) :

Συντεταγμένες του Επίγειου Σταθμού (ΒΕΡΟΙΑ) ,της , με κεκλιμένο κατά γωνία ω άξονα , ΓΗΣ .

ΒΕΡΟΙΑ

40.5194° N, 22.2052° E

ΑΘΗΝΑΙ

37.9838° N, 23.7275° E

ΠΑΡΙΣΙ

48.8566° N, 2.3522° E

> *xBER* := $xEL + R[e] \cdot \sin\left(\frac{(90 - 40.5194)}{180} \cdot \text{Pi}\right) \cdot \cos\left(\frac{22.2052}{180} \cdot \text{Pi} + \phi\right) \cdot \cos(\omega) + R[e] \cdot \cos\left(\frac{(90 - 40.5194)}{180} \cdot \text{Pi}\right) \cdot \sin(\omega)$

xBER := $\frac{49866087 \cos(\psi)}{500} + 4443.500284 \cos(0.3875538510 + \phi) + 1646.505561$ (56)

> *yBER* := $yEL + R[e] \cdot \sin\left(\frac{(90 - 40.5194)}{180} \cdot \text{Pi}\right) \cdot \sin\left(\frac{22.2052}{180} \cdot \text{Pi} + \phi\right)$

yBER := $\frac{149577371 \sin(\psi)}{1500} + 4843.145152 \sin(0.3875538510 + \phi)$ (57)

$$\begin{aligned} &> zBER := zEL - R[e] \cdot \sin\left(\frac{(90 - 40.5194)}{180} \cdot \text{Pi}\right) \cdot \cos\left(\frac{22.2052}{180} \cdot \text{Pi} + \phi\right) \cdot \sin(\omega) + R[e] \\ &\quad \cdot \cos\left(\frac{(90 - 40.5194)}{180} \cdot \text{Pi}\right) \cdot \cos(\omega) \\ &\quad zBER := 3797.710557 - 1926.489083 \cos(0.3875538510 + \phi) \end{aligned} \quad (58)$$

$$\begin{aligned} &> evalf(subs(\psi = \psi_{af}, xBER)) \\ &\quad -98085.66844 + 4443.500284 \cos(0.3875538510 + \phi) \end{aligned} \quad (59)$$

$$\begin{aligned} &> evalf(subs(\psi = \psi_{af}, yBER)) \\ &\quad -0.00004090509931 + 4843.145152 \sin(0.3875538510 + \phi) \end{aligned} \quad (60)$$

$$\begin{aligned} &> evalf(subs(\psi = \psi_{af}, zBER)) \\ &\quad 3797.710557 - 1926.489083 \cos(0.3875538510 + \phi) \end{aligned} \quad (61)$$

$$\begin{aligned} &> xBERtxt := xEL + 7000 \cdot \sin\left(\frac{(90 - 45)}{180} \cdot \text{Pi}\right) \cdot \cos\left(\frac{22.2052}{180} \cdot \text{Pi} + \phi\right) \cdot \cos(\omega) + 7000 \\ &\quad \cdot \cos\left(\frac{(90 - 45)}{180} \cdot \text{Pi}\right) \cdot \sin(\omega) \\ &\quad xBERtxt := \frac{49866087 \cos(\psi)}{500} + 3211.188289 \sqrt{2} \cos(0.3875538510 + \phi) \\ &\quad + 1392.217574 \sqrt{2} \end{aligned} \quad (62)$$

$$\begin{aligned} &> yBERtxt := yEL + 7000 \cdot \sin\left(\frac{(90 - 45)}{180} \cdot \text{Pi}\right) \cdot \sin\left(\frac{22.2052}{180} \cdot \text{Pi} + \phi\right) \\ &\quad yBERtxt := \frac{149577371 \sin(\psi)}{1500} + 3500 \sqrt{2} \sin(0.3875538510 + \phi) \end{aligned} \quad (63)$$

$$\begin{aligned} &> zBERtxt := zEL - 7000 \cdot \sin\left(\frac{(90 - 45)}{180} \cdot \text{Pi}\right) \cdot \cos\left(\frac{22.2052}{180} \cdot \text{Pi} + \phi\right) \cdot \sin(\omega) + 7000 \\ &\quad \cdot \cos\left(\frac{(90 - 45)}{180} \cdot \text{Pi}\right) \cdot \cos(\omega) \\ &\quad zBERtxt := -1392.217574 \sqrt{2} \cos(0.3875538510 + \phi) + 3211.188289 \sqrt{2} \end{aligned} \quad (64)$$

$$\begin{aligned} &> evalf(subs(\psi = \psi_{af}, xBERtxt)) \\ &\quad -97763.28103 + 4541.306028 \cos(0.3875538510 + \phi) \end{aligned} \quad (65)$$

$$\begin{aligned} &> evalf(subs(\psi = \psi_{af}, yBERtxt)) \\ &\quad -0.00004090509931 + 4949.747467 \sin(0.3875538510 + \phi) \end{aligned} \quad (66)$$

$$\begin{aligned} &> evalf(subs(\psi = \psi_{af}, zBERtxt)) \\ &\quad -1968.892974 \cos(0.3875538510 + \phi) + 4541.306028 \end{aligned} \quad (67)$$

> BEROIAaf := animate(pointplot3d, [[(59), (60), (61)], symbol=solidcircle, symbolsize=5, color=green], $\phi = 0 \dots 2 \cdot \text{Pi}$, frames=49) :

> Btextaf := animate(textplot3d, [[(65), (66), (67)], "BEPOIA"], font=[arial, bold, 8], color=blue, $\phi = 0 \dots 2 \cdot \text{Pi}$, frames=49) :

>

Συντεταγμένες του ΠΑΡΑΛΛΗΛΟΥ ΚΥΚΛΟΥ της ΓΗΣ του διερχόμενου από την ΒΕΡΟΙΑ , με κεκλιμένο κατά γωνία ω άξονα :

$$> XBER := xEL + R[e] \cdot \sin\left(\frac{(90 - 40.5194)}{180} \cdot \text{Pi}\right) \cdot \cos(\phi) \cdot \cos(\omega) + R[e]$$

$$\cdot \cos\left(\frac{(90 - 40.5194)}{180} \cdot \text{Pi}\right) \cdot \sin(\omega)$$

$$XBER := \frac{49866087 \cos(\psi)}{500} + 4443.500284 \cos(\phi) + 1646.505561 \quad (68)$$

$$> YBER := yEL + R[e] \cdot \sin\left(\frac{(90 - 40.5194)}{180} \cdot \text{Pi}\right) \cdot \sin(\phi)$$

$$YBER := \frac{149577371 \sin(\psi)}{1500} + 4843.145152 \sin(\phi) \quad (69)$$

$$> ZBER := zEL - R[e] \cdot \sin\left(\frac{(90 - 40.5194)}{180} \cdot \text{Pi}\right) \cdot \cos(\phi) \cdot \sin(\omega) + R[e]$$

$$\cdot \cos\left(\frac{(90 - 40.5194)}{180} \cdot \text{Pi}\right) \cdot \cos(\omega)$$

$$ZBER := 3797.710557 - 1926.489083 \cos(\phi) \quad (70)$$

$$> evalf(subs(\psi = \psi_{af}, XBER))$$

$$-98085.66844 + 4443.500284 \cos(\phi) \quad (71)$$

$$> evalf(subs(\psi = \psi_{af}, YBER))$$

$$-0.00004090509931 + 4843.145152 \sin(\phi) \quad (72)$$

$$> evalf(subs(\psi = \psi_{af}, ZBER))$$

$$3797.710557 - 1926.489083 \cos(\phi) \quad (73)$$

$$> CYCLEBEROIA_{af} := spacecurve([(71), (72), (73)], \phi = 0 .. 2 \cdot \text{Pi}, color = green, linestyle = 1, thickness = 1) :$$

>

Συντεταγμένες του ΚΕΝΤΡΟΥ του ΠΑΡΑΛΛΗΛΟΥ ΚΥΚΛΟΥ της ΓΗΣ του διερχόμενου από την ΒΕΡΟΙΑ , με κεκλιμένο κατά γωνία ω άξονα :

$$> Xcenter := evalf\left(subs\left(\psi = \psi_{af}, xEL + R[e] \cdot \cos\left(\frac{(90 - 40.5194)}{180} \cdot \text{Pi}\right) \cdot \sin(\omega)\right)\right)$$

$$Xcenter := -98085.66844 \quad (74)$$

$$> Ycenter := evalf(subs(\psi = \psi_{af}, yEL))$$

$$Ycenter := -0.00004090509931 \quad (75)$$

$$> Zcenter := subs\left(\psi = \psi_{af}, zEL + R[e] \cdot \cos\left(\frac{(90 - 40.5194)}{180} \cdot \text{Pi}\right) \cdot \cos(\omega)\right)$$

$$Zcenter := 3797.710557 \quad (76)$$

$$> CENTERBEROIA := pointplot3d([(74), (75), (76)], symbol = solidcircle, symbolsize = 5, color = black) :$$

>

ΕΠΙΠΕΔΟ ΕΚΛΕΙΠΤΙΚΗΣ .

$$> ePEKLEIPTIKHS_{af} := cylinder([subs(\psi = \psi_{af}, xEL), subs(\psi = \psi_{af}, yEL), subs(\psi = \psi_{af}, zEL)], 1.5 \cdot R[e], -1, strips = 100, transparency = 0.75) :$$

$$> XEKL := xEL + R[e] \cdot \sin\left(\frac{\text{Pi}}{2}\right) \cdot \cos(\phi) \cdot \cos(0) + R[e] \cdot \cos\left(\frac{\text{Pi}}{2}\right) \cdot \sin(0)$$

$$XEKL := \frac{49866087 \cos(\psi)}{500} + 6371 \cos(\phi) \quad (77)$$

$$\begin{aligned}
 &> YEKL := yEL + R[e] \cdot \sin\left(\frac{\text{Pi}}{2}\right) \cdot \sin(\phi) \\
 &\quad YEKL := \frac{149577371 \sin(\psi)}{1500} + 6371 \sin(\phi)
 \end{aligned} \tag{78}$$

$$\begin{aligned}
 &> ZEKL := zEL - R[e] \cdot \sin\left(\frac{\text{Pi}}{2}\right) \cdot \cos(\phi) \cdot \sin(0) + R[e] \cdot \cos\left(\frac{\text{Pi}}{2}\right) \cdot \cos(0) \\
 &\quad ZEKL := 0
 \end{aligned} \tag{79}$$

$$\begin{aligned}
 &> evalf(\text{subs}(\psi = \psi af, ZEKL)) \\
 &\quad -99732.17400 + 6371 \cdot \cos(\phi)
 \end{aligned} \tag{80}$$

$$\begin{aligned}
 &> evalf(\text{subs}(\psi = \psi af, YEKL)) \\
 &\quad -0.00004090509931 + 6371 \cdot \sin(\phi)
 \end{aligned} \tag{81}$$

$$\begin{aligned}
 &> evalf(\text{subs}(\psi = \psi af, ZEKL)) \\
 &\quad 0.
 \end{aligned} \tag{82}$$

$$\begin{aligned}
 &> TOMHaf := \text{spacecurve}([(80), (81), (82)], \phi = 0 .. 2 \cdot \text{Pi}, \text{color} = \text{black}, \text{linestyle} = 1, \text{thickness} \\
 &\quad = 3) :
 \end{aligned}$$

>

ΑΞΟΝΑΣ ΤΩΝ ΚΟΜΒΩΝ (Nodes) , παράλληλος στον y-y της ΕΚΛΕΙΠΤΙΚΗΣ που περνάει από το κέντρο της ΓΗΣ .

>

$$\begin{aligned}
 &> AXnodes1af := \text{spacecurve}([\text{subs}(\psi = \psi af, xEL), \text{subs}(\psi = \psi af, -\lambda \cdot 1), 0], \lambda = -15000 \\
 &\quad .. 15000, \text{color} = \text{black}, \text{thickness} = 1, \text{linestyle} = 4) :
 \end{aligned}$$

>

ΑΞΟΝΑΣ παράλληλος στον x-x της ΕΚΛΕΙΠΤΙΚΗΣ που περνάει από το κέντρο της ΓΗΣ .

$$\begin{aligned}
 &> AKTINESaf := \text{spacecurve}([\text{subs}(\psi = \psi af, xEL + R[e] + \lambda \cdot (a \cdot \varepsilon - xEL - R[e])), \text{subs}(\psi \\
 &\quad = \psi af, yEL), 0], \lambda = 0 .. 0.1, \text{color} = \text{red}, \text{thickness} = 1, \text{linestyle} = 1) :
 \end{aligned}$$

$$\begin{aligned}
 &> AXONASxaf := \text{spacecurve}([\text{subs}(\psi = \psi af, xEL + \lambda \cdot (xEL + R[e] - xEL)), \text{subs}(\psi = \psi af, \\
 &\quad yEL), 0], \lambda = -2 .. 2, \text{color} = \text{black}, \text{thickness} = 1, \text{linestyle} = 4) :
 \end{aligned}$$

>

$$\begin{aligned}
 &> evalf(\text{subs}(\psi = \psi af, xEL + R[e])) \\
 &\quad -93361.17400
 \end{aligned} \tag{83}$$

$$\begin{aligned}
 &> evalf(\text{subs}(\psi = \psi af, xEL + 2 \cdot R[e])) \\
 &\quad -86990.17400
 \end{aligned} \tag{84}$$

$$\begin{aligned}
 &> BELOS := \text{arrow}([(83) + 1000, 0, 0], \text{Vector}([-1000, 0, 0]), 200, 400, 0.5, \\
 &\quad \text{cylindrical_arrow}, \text{color} = \text{red}) :
 \end{aligned}$$

>

ΑΞΟΝΑΣ παράλληλος στον z-z της ΕΚΛΕΙΠΤΙΚΗΣ που περνάει από το κέντρο της ΓΗΣ .

$$\begin{aligned}
 &> AXONASzaf := \text{spacecurve}([\text{subs}(\psi = \psi af, xEL), \text{subs}(\psi = \psi af, yEL), \lambda \cdot 1], \lambda = -10000 \\
 &\quad .. 10000, \text{color} = \text{black}, \text{thickness} = 1, \text{linestyle} = 4) :
 \end{aligned}$$

$$\begin{aligned}
 &> KATHETOS := \text{spacecurve}([\text{subs}(\psi = \psi af, xEL + R[e]), \text{subs}(\psi = \psi af, yEL), \lambda \cdot 1], \lambda = \\
 &\quad -10000 .. 10000, \text{color} = \text{green}, \text{thickness} = 1, \text{linestyle} = 1) :
 \end{aligned}$$

```
> SHMPROSPTAKTINAS := pointplot3d( [subs(ψ = ψaf, xEL + R[e]), 0, 0], symbol
= solidcircle, symbolsize = 5, color = blue) :
```

Συντεταγμένες του ΦΩΤΙΖΟΜΕΝΟΥ ΜΕΓΙΣΤΟΥ ΚΥΚΛΟΥ της ΓΗΣ .

```
> μ[φ] := 0
```

$$\mu_{\phi} := 0 \quad (85)$$

```
> XΦ := xEL + R[e]·sin(θ)·cos( (Pi/2 + μ[φ]) )
```

$$X\Phi := \frac{49866087 \cos(\psi)}{500} \quad (86)$$

```
> YΦ := yEL + R[e]·sin(θ)·sin( (Pi/2 + μ[φ]) )
```

$$Y\Phi := \frac{149577371 \sin(\psi)}{1500} + 6371 \sin(\theta) \quad (87)$$

```
> ZΦ := zEL + R[e]·cos(θ)
```

$$Z\Phi := 6371 \cos(\theta) \quad (88)$$

```
> evalf( subs(ψ = ψaf, XΦ) )
```

$$-99732.17400 \quad (89)$$

```
> evalf( subs(ψ = ψaf, YΦ) )
```

$$-0.00004090509931 + 6371. \sin(\theta) \quad (90)$$

```
> evalf( subs(ψ = ψaf, ZΦ) )
```

$$6371. \cos(\theta) \quad (91)$$

```
> FMEGISTOSKYKLOSaf := spacecurve( [(89), (90), (91)], θ = 0 .. 2·Pi, color = blue, thickness
= 1) :
```

ΕΥΡΕΣΗ ΤΩΝ ΣΗΜΕΙΩΝ ΤΟΜΗΣ Α , Β : ΠΑΡΑΛΛΗΛΟΥ ΚΥΚΛΟΥ της ΓΗΣ του διερχόμενου από την ΒΕΡΟΙΑ & ΦΩΤΙΖΟΜΕΝΟΥ ΜΕΓΙΣΤΟΥ ΚΥΚΛΟΥ .

ΕΠΙΛΥΣΗ ΤΟΥ ΣΥΣΤΗΜΑΤΟΣ

```
> (71) = (89)
```

$$-98085.66844 + 4443.500284 \cos(\phi) = -99732.17400 \quad (92)$$

```
> (72) = (90)
```

$$-0.00004090509931 + 4843.145152 \sin(\phi) = -0.00004090509931 + 6371. \sin(\theta) \quad (93)$$

```
> (73) = (91)
```

$$3797.710557 - 1926.489083 \cos(\phi) = 6371. \cos(\theta) \quad (94)$$

```
> allvalues(solve( {(92), (93)}, {φ, θ} ))
{φ = 1.950389322, θ = 0.7839366496} (95)
```

```
> evalf(subs((95)[1], (71)))
-99732.17400 (96)
```

```
> evalf(subs((95)[1], (72)))
4498.388366 (97)
```

```
> evalf(subs((95)[1], (73)))
4511.556576 (98)
```

```
>
>
> A := [(96), (97), (98)]
A := [-99732.17400, 4498.388366, 4511.556576] (99)
```

```
> B := [(96), -(97), (98)]
B := [-99732.17400, -4498.388366, 4511.556576] (100)
```

```
>
> pointA := pointplot3d(A, symbol=solidcircle, symbolsize=5, color=red) :
> pointB := pointplot3d(B, symbol=solidcircle, symbolsize=5, color=cyan) :
```

L_{AB} ΜΗΚΟΣ ΧΟΡΔΗΣ AB :

```
> L[AB] := sqrt((A[1] - B[1])^2 + (A[2] - B[2])^2 + (A[3] - B[3])^2)
L_AB := 8996.776732 (101)
```

```
>
>
ΑΚΤΙΝΑ  $r_{BER}$  ΤΟΥ ΠΑΡΑΛΛΗΛΟΥ ΚΥΚΛΟΥ της ΓΗΣ του διερχόμενου από την ΒΕΡΟΙΑ :
```

```
> r[BER] := R[e]·sin( (90 - 40.5194) / 180 · Pi )
r_BER := 4843.145152 (102)
```

```
>
>
ΕΠΙΚΕΝΤΡΟΣ ΓΩΝΙΑ  $a_{BER}$  που βλέπει την χορδή AB (NYXTA στην ΒΕΡΟΙΑ) :
```

```
> a[BER] := 2·sin-1( L[AB] / (2·r[BER]) )
a_BER := 2.382406616 (103)
```

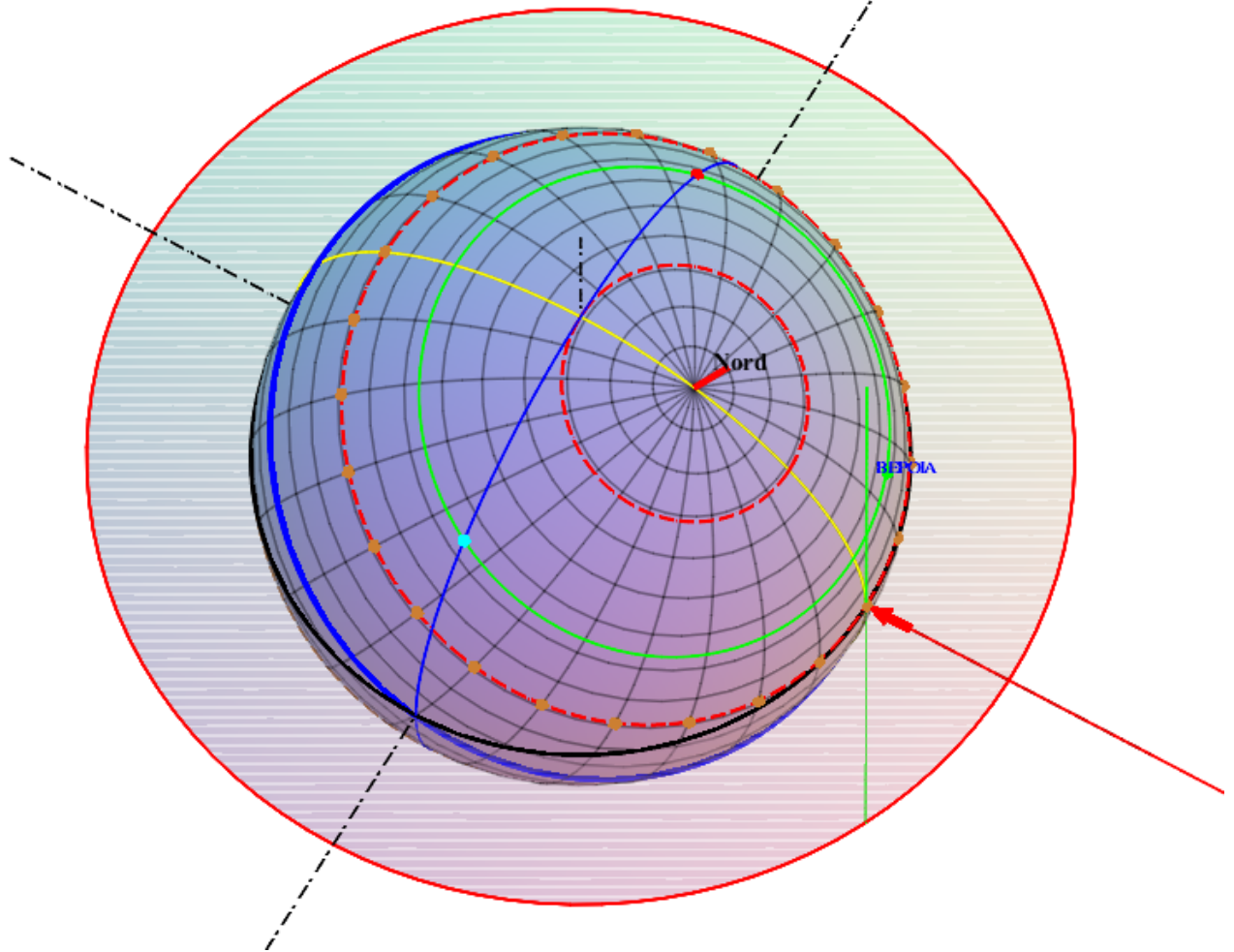
```
> NYXTA := a[BER] / (2·Pi) · 24h
NYXTA := 9.100122944 h (104)
```

```
> HMERΑ := 24h - NYXTA
HMERΑ := 14.89987706 h (105)
```

```
>
> display(anSFaf, axonSaf, ISHMERINOSaf, animMESHMBRINOSaf, BEROIAaf, Btextaf,
ePEKLEIPTIKHSaf, TOMHaf, TEXTaxonaf, CANCEraf, CAPRICORNEaf, AXnodes1af,
AXONASxaf, AKTINESaf, AXONASzaf, KATHETOS, SHMPROSPTAKTINAS,
ARKTIKOSaf, ANTARKTIKOSaf, BELOS, animCANCER, CYCLEBEROIAaf,
FMEGISTOSKYKLOSaf, pointA, pointB, CENTERBEROIA, orientation = [-20, 60, 0],
```

```
> display(anSFaf, axonSaf, ISHMERINOSaf, animMESHMBRINOSaf, BEROIAaf, Btextaf,
> ePEKLEIPTIKHSaf, TOMHaf, TEXTaxonaf, CANCERaf, CAPRICORNEaf, AXnodes1af,
> AXONASxaf, AKTINESaf, AXONASzaf, KATHETOS, SHMPROSPTAKTINAS,
> ARKTIKOSaf, ANTARKTIKOSaf, BELOS, animCANCER, CYCLEBEROIAaf,
> FMEGISTOSKYKLOSaf, pointA, pointB, CENTERBEROIA, orientation = [-60, 25, 0],
> axes = none, title
> = "ΤΙΡΟΒΟΛΗ [-60,25,0]\nΚΑΛΟΚΑΙΡΙ ΣΤΟ ΒΟΡΕΙΟ ΗΜΙΣΦΑΙΡΙΟ\nΗ ΓΗ ΣΤΟ
> ΑΦΗΛΙΟ\nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ", titlefont = [arial, bold, 12], scaling
> = constrained, lightmodel = none) :
```


ΠΡΟΒΟΛΗ [-60,25,0]
ΚΑΛΟΚΑΙΡΙ ΣΤΟ ΒΟΡΕΙΟ ΗΜΙΣΦΑΙΡΙΟ
Η ΓΗ ΣΤΟ ΑΦΗΛΙΟ
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ



> **ΕΥΡΕΣΗ ΔΙΑΡΚΕΙΑΣ ΗΜΕΡΑΣ-ΝΥΧΤΑΣ ΣΕ
ΟΛΗΝ ΤΗΝ ΔΙΑΡΚΕΙΑ ΤΟΥ ΕΤΟΥΣ .**

> **Συντεταγμένες του ΠΑΡΑΛΛΗΛΟΥ ΚΥΚΛΟΥ της ΓΗΣ του
διερχόμενου από την ΒΕΡΟΙΑ , με κεκλιμένο κατά γωνία ω άξονα :**

>
$$XBERI := + R[e] \cdot \sin\left(\frac{(90 - 40.5194)}{180} \cdot \text{Pi}\right) \cdot \cos(\phi) \cdot \cos(\omega) + R[e]$$

$$\cdot \cos\left(\frac{(90 - 40.5194)}{180} \cdot \text{Pi}\right) \cdot \sin(\omega)$$

$$XBERI := 4443.500284 \cos(\phi) + 1646.505561$$

(106)

>
$$YBERI := + R[e] \cdot \sin\left(\frac{(90 - 40.5194)}{180} \cdot \text{Pi}\right) \cdot \sin(\phi)$$

$$YBERI := 4843.145152 \sin(\phi)$$

(107)

>
$$ZBERI := -R[e] \cdot \sin\left(\frac{(90 - 40.5194)}{180} \cdot \text{Pi}\right) \cdot \cos(\phi) \cdot \sin(\omega) + R[e] \cdot \cos\left(\frac{(90 - 40.5194)}{180}\right)$$

$$\cdot \text{Pi}) \cdot \cos(\omega)$$

$$ZBER1 := 3797.710557 - 1926.489083 \cos(\phi) \quad (108)$$

Συντεταγμένες του ΦΩΤΙΖΟΜΕΝΟΥ ΜΕΓΙΣΤΟΥ ΚΥΚΛΟΥ της ΓΗΣ .

$$\mu l := 0..2 \cdot \text{Pi}$$

$$\begin{aligned} > X\Phi l := + R[e] \cdot \sin(\theta) \cdot \cos\left(\frac{\text{Pi}}{2} + \mu l\right) \\ & X\Phi l := -6371 \sin(\theta) \sin(\mu l) \end{aligned} \quad (109)$$

$$\begin{aligned} > Y\Phi l := + R[e] \cdot \sin(\theta) \cdot \sin\left(\frac{\text{Pi}}{2} + \mu l\right) \\ & Y\Phi l := 6371 \sin(\theta) \cos(\mu l) \end{aligned} \quad (110)$$

$$\begin{aligned} > Z\Phi l := + R[e] \cdot \cos(\theta) \\ & Z\Phi l := 6371 \cos(\theta) \end{aligned} \quad (111)$$

ΕΠΙΛΥΣΗ ΤΟΥ ΣΥΣΤΗΜΑΤΟΣ

$$\begin{aligned} > XBER1 = X\Phi l \\ & 4443.500284 \cos(\phi) + 1646.505561 = -6371 \sin(\theta) \sin(\mu l) \end{aligned} \quad (112)$$

$$\begin{aligned} > YBER1 = Y\Phi l \\ & 4843.145152 \sin(\phi) = 6371 \sin(\theta) \cos(\mu l) \end{aligned} \quad (113)$$

$$\begin{aligned} > ZBER1 = Z\Phi l \\ & 3797.710557 - 1926.489083 \cos(\phi) = 6371 \cos(\theta) \end{aligned} \quad (114)$$

$$\begin{aligned} > sol := \text{allvalues}(\text{solve}(\{(112), (113)\}, \{\phi, \theta\})) : \\ & \text{evalf}(\text{subs}(\mu l = 0, sol[1])) \\ & \quad \{\phi = 1.950389322, \theta = 0.7839366497\} \end{aligned} \quad (115)$$

$$\begin{aligned} > \text{evalf}(\text{subs}(\mu l = 0, sol[2])) \\ & \quad \{\phi = -1.950389322, \theta = -0.7839366497\} \end{aligned} \quad (116)$$

$$> A1 := [\text{subs}(sol[1][1], XBER1), \text{subs}(sol[1][1], YBER1), \text{subs}(sol[1][1], ZBER1)] :$$

$$> B1 := [\text{subs}(sol[2][1], XBER1), \text{subs}(sol[2][1], YBER1), \text{subs}(sol[2][1], ZBER1)] :$$

$$\begin{aligned} > \text{evalf}(\text{subs}(\mu l = 0, A1)) \\ & [0., 4498.388408, 4511.556576] \end{aligned} \quad (117)$$

$$\begin{aligned} > \text{evalf}(\text{subs}(\mu l = 0, B1)) \\ & [0., -4498.388408, 4511.556576] \end{aligned} \quad (118)$$

$$\begin{aligned} > \text{evalf}\left(\text{subs}\left(\mu l = \frac{\text{Pi}}{2}, A1\right)\right) \\ & [-2796.994723, -5.736730733 \cdot 10^{-7}, 5724.199640] \end{aligned} \quad (119)$$

$$\begin{aligned} > \text{evalf}\left(\text{subs}\left(\mu l = \frac{\text{Pi}}{2}, B1\right)\right) \\ & [6090.005845, 1.249080787 \cdot 10^{-6}, 1871.221474] \end{aligned} \quad (120)$$

L_{AIB1} ΜΗΚΟΣ ΧΟΡΔΗΣ ΑΙΒ1 .

> $L[AIB1] := \text{sqrt}((AI[1] - BI[1])^2 + (AI[2] - BI[2])^2 + (AI[3] - BI[3])^2) :$
>

ΑΚΤΙΝΑ r_{BER} ΤΟΥ ΠΑΡΑΛΛΗΛΟΥ ΚΥΚΛΟΥ της ΓΗΣ του διερχόμενου από την ΒΕΡΟΙΑ :

> $r[BER] := R[e] \cdot \sin\left(\frac{(90 - 40.5194)}{180} \cdot \text{Pi}\right)$
 $r_{BER} := 4843.145152$ (121)

> $a[BER1] := 2 \cdot \sin^{-1}\left(\frac{L[AIB1]}{2 \cdot r[BER]}\right) :$
>

$$\text{a) } \mu 1 := \frac{3 \cdot \text{Pi}}{2} \dots \frac{\text{Pi}}{2}$$

> $NYXTA1a := \frac{a[BER1]}{2 \cdot \text{Pi}} \cdot 24\text{h} :$

> **$HMER1a := 24\text{h} - NYXTA1a :$**

> $TEST := \text{evalf}(\text{subs}(\mu 1 = 0, NYXTA1a))$
 $TEST := 9.100123121 \text{ h}$ (122)

> $\text{evalf}\left(\text{subs}\left(\mu 1 = \frac{\text{Pi}}{2}, NYXTA1a\right)\right)$
 11.99973536 h (123)

> $\text{evalf}\left(\text{subs}\left(\mu 1 = \frac{3 \cdot \text{Pi}}{2}, NYXTA1a\right)\right)$
 11.99973536 h (124)

$$\text{b) } \mu 1 := \frac{\text{Pi}}{2} \dots \frac{3 \cdot \text{Pi}}{2}$$

> **$HMER1b := \frac{a[BER1]}{2 \cdot \text{Pi}} \cdot 24\text{h} :$**

> **$NYXTA1b := 24\text{h} - HMER1b :$**

> $\text{evalf}(\text{subs}(\mu 1 = \text{Pi}, HMER1b))$
 9.100123111 h (125)

$$\text{evalf}\left(\text{subs}\left(\mu l = \frac{\text{Pi}}{2}, \text{HMERAI}b\right)\right) = 11.99973536 \text{ h} \quad (126)$$

$$\text{evalf}\left(\text{subs}\left(\mu l = \frac{3 \cdot \text{Pi}}{2}, \text{HMERAI}b\right)\right) = 11.99973536 \text{ h} \quad (127)$$

>

ΕΠΑΛΗΘΕΥΣΗ ΜΕ ΕΦΑΡΜΟΓΗ ΤΟΥ ΤΥΠΟΥ : $\text{DIARKEIANYXTAS} := \frac{\cos^{-1}\left(\tan\left(\frac{40.5194}{180} \cdot \text{Pi}\right) \cdot \tan(\omega) \cdot \cos(\mu)\right) \cdot 24}{\text{Pi}} \text{ h}$

>

ΠΡΟΣΟΧΗ !!! ΤΟ , $\mu=0$, ΟΤΑΝ Η ΓΗ ΒΡΙΣΚΕΤΑΙ ΣΤΟ ΘΕΡΙΝΟ ΗΛΙΟΣΤΑΣΙΟ .

>

$$\text{DIARKEIANYXTAS} := \frac{\cos^{-1}\left(\tan\left(\frac{40.5194}{180} \cdot \text{Pi}\right) \cdot \tan(\omega) \cdot \cos(\mu)\right) \cdot 24}{\text{Pi}}$$

$$\text{DIARKEIANYXTAS} := \frac{24 \arccos(0.3705424681 \cos(\mu))}{\pi} \quad (128)$$

$$\text{evalf}\left(\text{subs}(\mu = 0, (128))\right) = 9.100123121 \quad (129)$$

$$\text{evalf}\left(\text{subs}\left(\mu = \frac{\text{Pi}}{2}, (128)\right)\right) = 12.00000000 \quad (130)$$

$$\text{evalf}\left(\text{subs}(\mu = \text{Pi}, (128))\right) = 14.89987687 \quad (131)$$

$$\text{evalf}\left(\text{subs}\left(\mu = \frac{3 \cdot \text{Pi}}{2}, (128)\right)\right) = 12.00000000 \quad (132)$$

$$\text{evalf}\left(\text{subs}(\mu = 2 \cdot \text{Pi}, (128))\right) = 9.100123121 \quad (133)$$

>

$pB1 := \text{plot}([\text{DIARKEIANYXTAS}, 24 - \text{DIARKEIANYXTAS}], \mu = 0 .. 2 \cdot \text{Pi}, \text{color} = [\text{black}, \text{red}], \text{thickness} = 2, \text{view} = [\text{default}, 7 .. 18], \text{labels} = ["\mu", "\text{διάρκεια ώρες}"], \text{gridlines}, \text{legend} = ["\text{Νύχτα}", "\text{Ημέρα}"], \text{title} = "\text{ΒΕΡΟΙΑ : ΔΙΑΡΚΕΙΑ ΗΜΕΡΑΣ-ΝΥΧΤΑΣ \n ΚΑΤΑ ΤΗΝ ΠΕΡΙΦΟΡΑ ΤΗΣ ΓΗΣ ΠΕΡΙ ΤΟΝ ΗΛΙΟ \n ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ}", \text{titlefont} = [\text{arial}, \text{bold}, 14]) :$

>

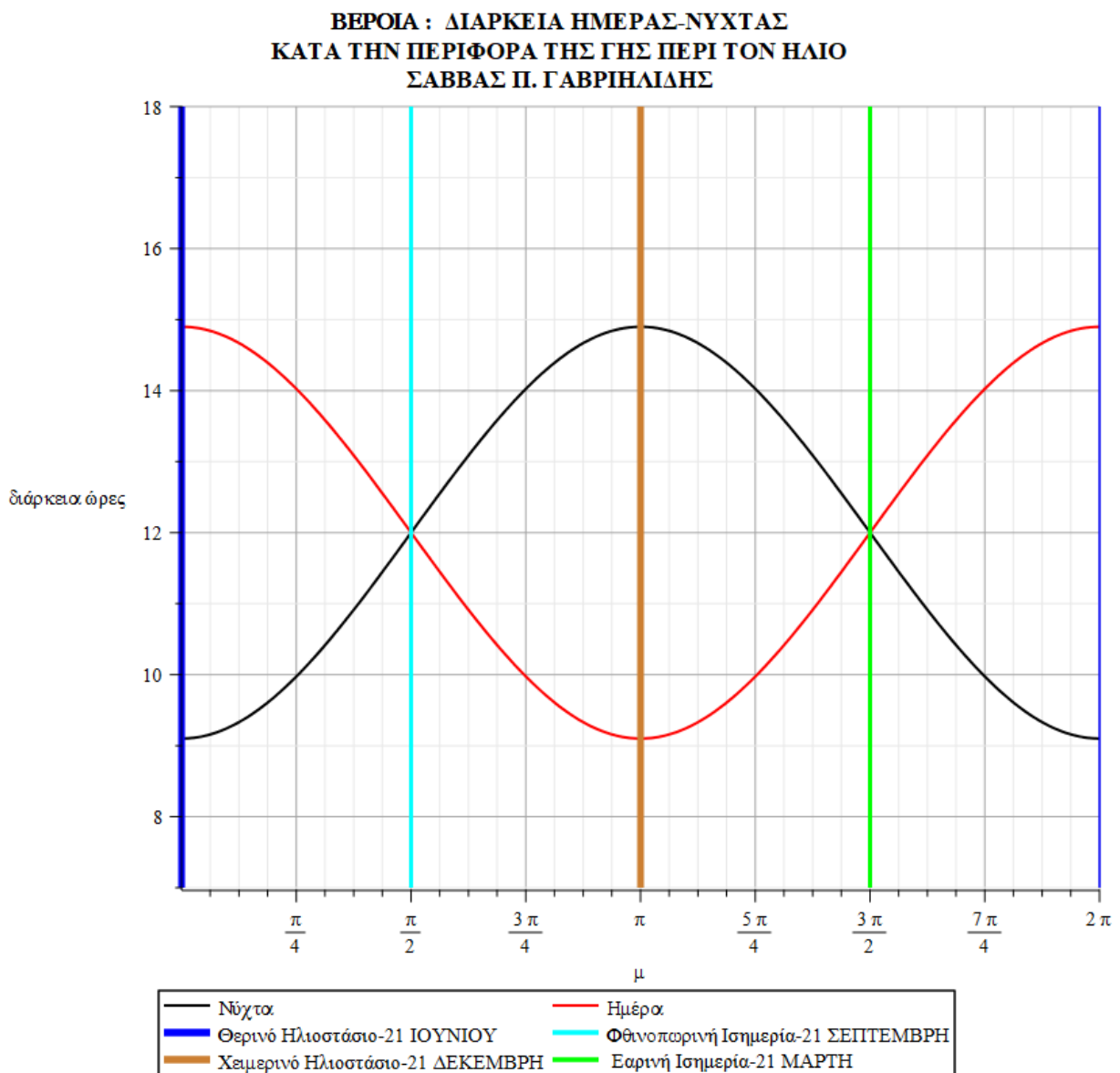
$\text{ETESOLSTICE1B} := \text{plot}([0, \lambda, \lambda = 7 .. 18], \text{linestyle} = 1, \text{thickness} = 5, \text{color} = \text{blue}, \text{legend} = ["\text{Θερινό Ηλιοστάσιο-21 ΙΟΥΝΙΟΥ}"] :$

$\text{ETESOLSTICE2B} := \text{plot}([2 \cdot \text{Pi}, \lambda, \lambda = 7 .. 18], \text{linestyle} = 1, \text{thickness} = 1, \text{color} = \text{blue}) :$

```

> EQUINOX1B := plot( [  $\frac{\text{Pi}}{2}$ , λ, λ = 7 .. 18 ], linestyle = 1, thickness = 3, color = cyan, legend
= [ "Φθινοπωρινή Ισημερία-21 ΣΕΠΤΕΜΒΡΗ" ] ) :
> hiverSOLSTICEB := plot( [ Pi, λ, λ = 7 .. 18 ], linestyle = 1, thickness = 5, color = gold, legend
= [ "Χειμερινό Ηλιοστάσιο-21 ΔΕΚΕΜΒΡΗ" ] ) :
> EQUINOX2B := plot( [  $\frac{3 \cdot \text{Pi}}{2}$ , λ, λ = 7 .. 18 ], linestyle = 1, thickness = 3, color = green,
legend = [ "Εαρινή Ισημερία-21 ΜΑΡΤΗ" ] ) :
>
>
> display(pB1, ETESOLSTICE1B, ETESOLSTICE2B, EQUINOX1B, hiverSOLSTICEB,
EQUINOX2B) :

```



```

>
> pB := plot( [ DIARKEIANYXTAS, 24 - DIARKEIANYXTAS ], μ = 0 .. 2 · Pi, color = [ black,
red ], thickness = 2, view = [ default, 7 .. 18 ], labels = [ "μ", "διάρκεια ώρες" ], gridlines,
legend = [ "Νύχτα", "Ημέρα" ], title

```

= "ΒΕΡΟΙΑ : ANIMATION : ΔΙΑΡΚΕΙΑ ΗΜΕΡΑΣ-NYXΤΑΣ\ nΚΑΤΑ ΤΗΝ ΠΕΡΙΦΟΡΑ ΤΗΣ ΓΗΣ ΠΕΡΙ ΤΟΝ ΗΛΙΟ\ nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ", titlefont = [arial, bold, 14]) :

- > back := display(pB, ETESOLSTICE1B, ETESOLSTICE2B, EQUINOX1B, hiverSOLSTICEB, EQUINOX2B) :
- > HMERΑ := animate(pointplot, [[μ, 24 - DIARKEIANYXTAS], symbol=solidcircle, symbolsize=10, color=red], μ=0 ..2·Pi, frames=17, trace=16) :
- > NYXTA := animate(pointplot, [[μ, DIARKEIANYXTAS], symbol=solidcircle, symbolsize=10, color=black], μ=0 ..2·Pi, frames=17, trace=16) :
- > LIGNE := animate(plot, [[μ, DIARKEIANYXTAS + λ·(24 - DIARKEIANYXTAS - DIARKEIANYXTAS), λ=0 ..1], linestyle=4, thickness=2, color=blue], μ=0 ..2·Pi, frames=17, trace=16) :
- > display(back, HMERΑ, NYXTA, LIGNE) :

**ΒΕΡΟΙΑ : ANIMATION : ΔΙΑΡΚΕΙΑ ΗΜΕΡΑΣ-NYXΤΑΣ
ΚΑΤΑ ΤΗΝ ΠΕΡΙΦΟΡΑ ΤΗΣ ΓΗΣ ΠΕΡΙ ΤΟΝ ΗΛΙΟ
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ**

