

$$\begin{bmatrix} \vec{e}_1 \\ \vec{e}_2 \\ \vec{e}_3 \end{bmatrix} = \begin{bmatrix} \cos(a) \hat{i} + \cos(b) \hat{j} + \cos(c) \hat{k} \\ \cos(d) \hat{i} + \cos(e) \hat{j} + \cos(f) \hat{k} \\ \cos(g) \hat{i} + \cos(h) \hat{j} + \cos(i) \hat{k} \end{bmatrix}$$

$$\vec{e}_1 = \cos(a) \hat{i} + \cos(b) \hat{j} + \cos(c) \hat{k}$$

$$\vec{e}_2 = \cos(d) \hat{i} + \cos(e) \hat{j} + \cos(f) \hat{k}$$

$$\vec{e}_3 = \cos(g) \hat{i} + \cos(h) \hat{j} + \cos(i) \hat{k}$$

$$\begin{bmatrix} \hat{i} \\ \hat{j} \\ \hat{k} \end{bmatrix} = \begin{bmatrix} \cos(a) \vec{e}_1 + \cos(d) \vec{e}_2 + \cos(g) \vec{e}_3 \\ \cos(b) \vec{e}_1 + \cos(e) \vec{e}_2 + \cos(h) \vec{e}_3 \\ \cos(c) \vec{e}_1 + \cos(f) \vec{e}_2 + \cos(i) \vec{e}_3 \end{bmatrix}$$

$$\hat{i} = \cos(a) \vec{e}_1 + \cos(d) \vec{e}_2 + \cos(g) \vec{e}_3$$

$$\hat{j} = \cos(b) \vec{e}_1 + \cos(e) \vec{e}_2 + \cos(h) \vec{e}_3$$

$$\hat{k} = \cos(c) \vec{e}_1 + \cos(f) \vec{e}_2 + \cos(i) \vec{e}_3$$

ΜΕΤΑΣΧΗΜΑΤΙΣΜΟΣ ΣΥΣΤΗΜΑΤΩΝ

> with(plots) :

> with(plottools) :

> with(Student[VectorCalculus]) :

> with(Physics[Vectors]) :

> Setup(mathematicalnotation = true) :

> PL1 := plot([[2 + λ, 1 - λ², λ = -1 .. 1], [-2 + λ, -1 + λ², λ = -1 .. 1], [λ, 0, λ = -1 .. 1],

color = [red, blue, green], thickness = [3, 3, 3], title

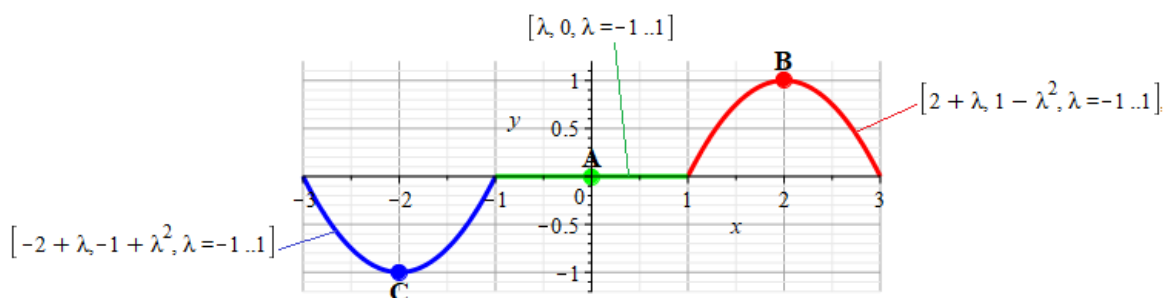
= "ΣΥΜΠΛΕΓΜΑ : ΠΑΡΑΒΟΛΕΣ ΜΕ ΕΥΘΕΙΑ ΕΝΩΝΟΥΣΑ", titlefont = [arial, bold, 14], gridlines, scaling = constrained) :

> PL2 := pointplot([[2, 1], [0, 0], [-2, -1]], color = [red, green, blue], symbol = solidcircle, symbolsize = 20, gridlines, scaling = constrained) :

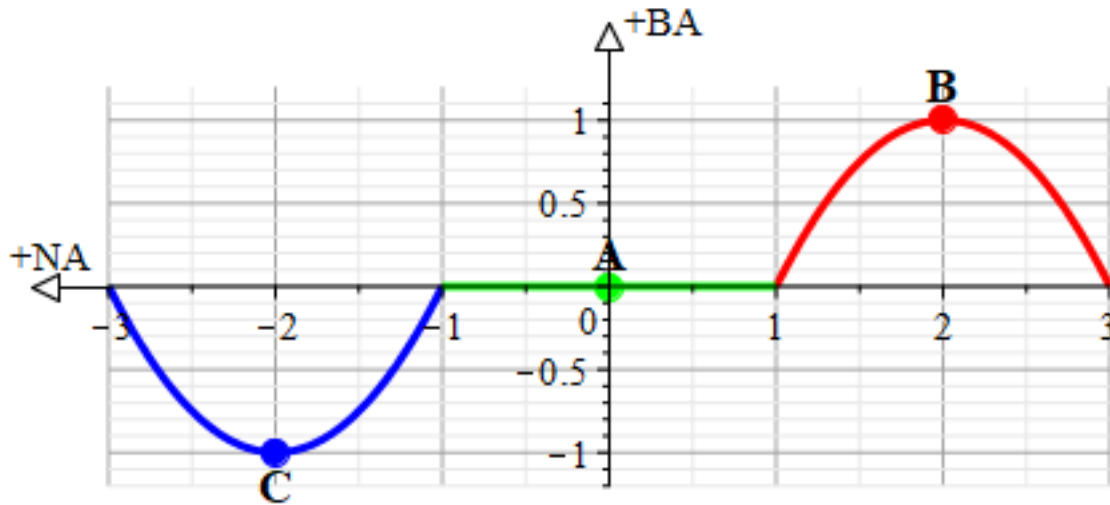
> PL3 := textplot([[2, 1.2, "B"], [0, 0.2, "A"], [-2, -1.2, "C"]], font = [arial, bold, 14]) :

> display(PL1, PL2, PL3, labels = [x, y]) :

ΣΥΜΠΛΕΓΜΑ : ΠΑΡΑΒΟΛΕΣ ΜΕ ΕΥΘΕΙΑ ΕΝΩΝΟΥΣΑ



ΣΥΜΠΛΕΓΜΑ : ΠΑΡΑΒΟΛΕΣ ΜΕ ΕΥΘΕΙΑ ΕΝΩΝΟΥΣΑ



>

Θέμα :

1. Έστω Ελλειπτικός Κώνος με παραμετρικές εξισώσεις :

$$[z \cdot a \cdot \cos(\phi), z \cdot b \cdot \sin(\phi), z], z = 0 \dots h, \phi = 0 \dots 2 \cdot \text{Pi}$$

Μία τυχαία καμπύλη πάνω στην παράπλευρη επιφάνεια του πύο πάνω κώνου έχει παραμετρικές εξισώσεις :

$$[z(t) \cdot a \cdot \cos(\phi(t)), z(t) \cdot b \cdot \sin(\phi(t)), z(t)] :$$

Οι παραμετρικές εξισώσεις Ελλειπτικής Κωνοειδούς Έλικας, πάνω στην παράπλευρη επιφάνεια του πύο πάνω κώνου, με βήμα έλικας c , αριθμό στροφών n και ύψος έλικας h είναι :

Παραμετρικές εξισώσεις Ελλειπτικής κωνοειδούς Έλικας :

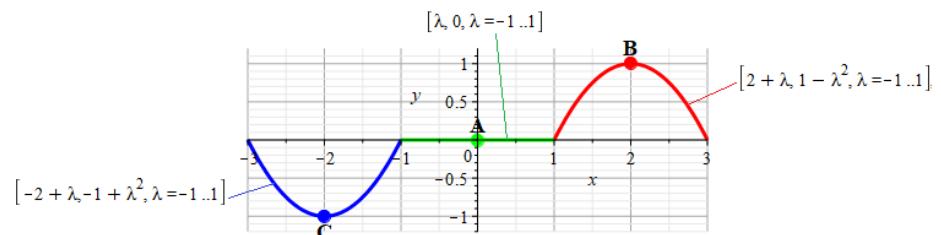
$$\left[t \cdot a \cdot \cos\left(\frac{2 \cdot \text{Pi}}{c} \cdot t\right), t \cdot b \cdot \sin\left(\frac{2 \cdot \text{Pi}}{c} \cdot t\right), t \right], t = c \dots h$$

Συμβολισμοί :

- a = Μεγάλος ημιάξονας έλλειψης
- b = Μικρός ημιάξονας έλλειψης
- c = Βήμα Έλικας
- n = Αριθμός στροφών έλικας
- h = $c \cdot n$ Ύψος Έλικας

ΣΥΜΠΛΕΓΜΑ : ΠΑΡΑΒΟΛΕΣ ΜΕ ΕΥΘΕΙΑ ΕΝΩΝΟΥΣΑ

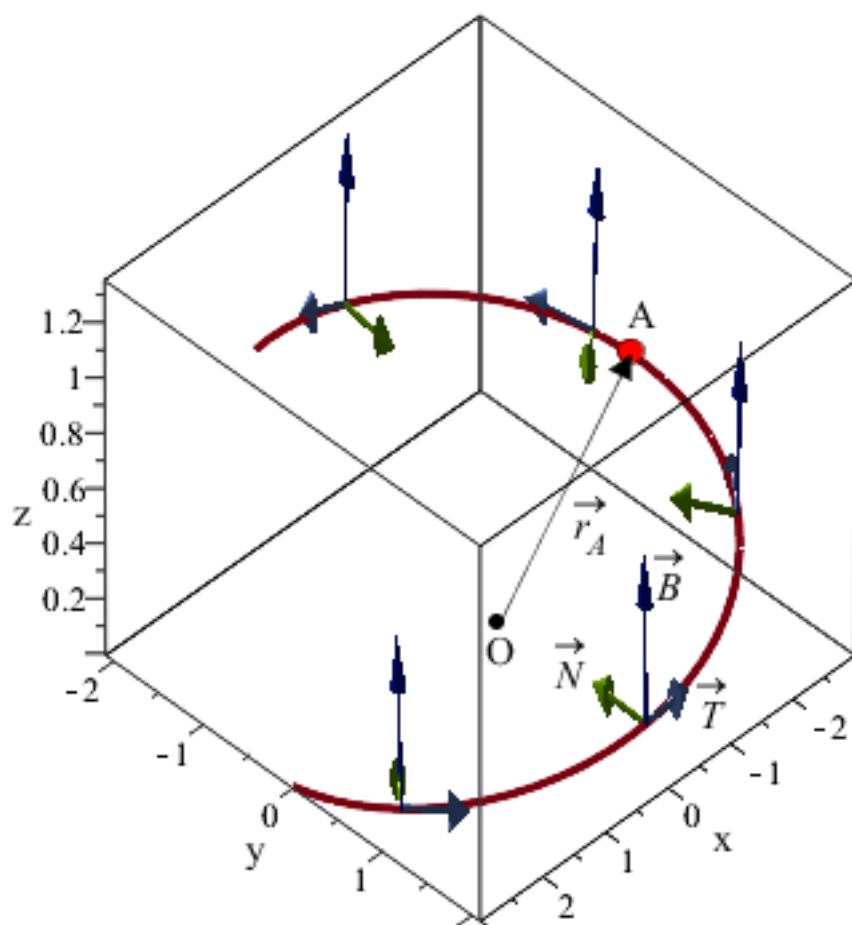
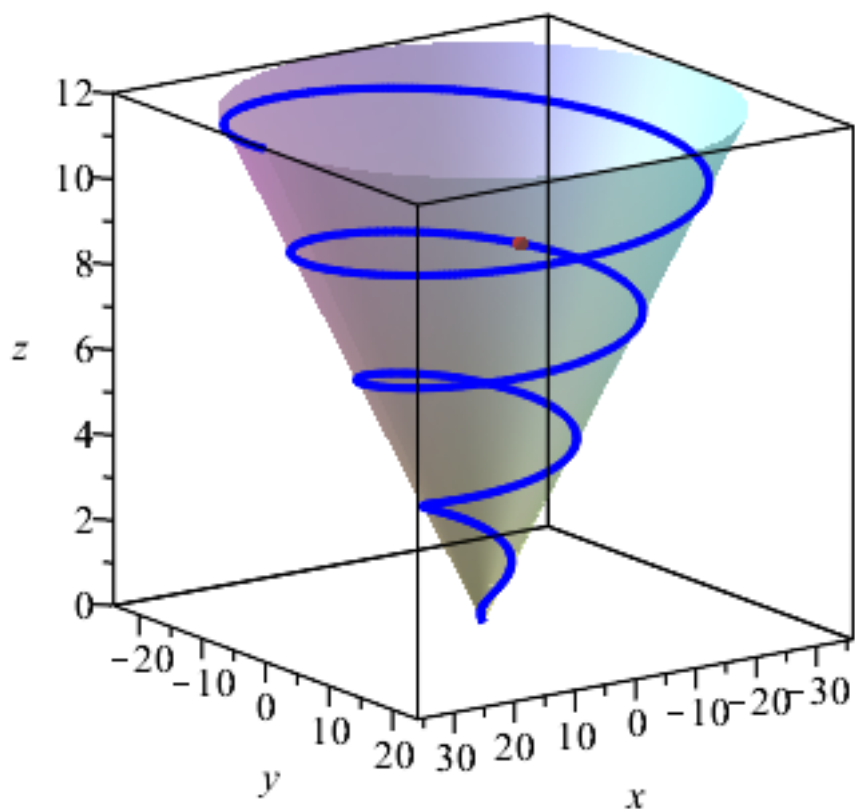
2. Έστω το σύμπλεγμα του σχήματος :



**ΝΑ ΒΡΟΥΜΕ ΤΗΝ ΕΙΣΩΣΗ ΤΗΣ ΕΠΙΦΑΝΕΙΑΣ
ΠΟΥ ΔΙΑΓΡΑΦΕΙ ΤΟ ΣΥΜΠΛΕΓΜΑ
ΟΤΑΝ ΚΙΝΕΙΤΑΙ ΚΑΘΕΤΑ στην εφαπτομένη της
ΕΛΛΕΙΠΤΙΚΗΣ ΚΩΝΟΕΙΔΟΥΣ ΕΛΙΚΑΣ .**

ΕΛΛΕΙΠΤΙΚΗ ΚΩΝΟΕΙΔΗΣ ΕΛΙΚΑ

ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ



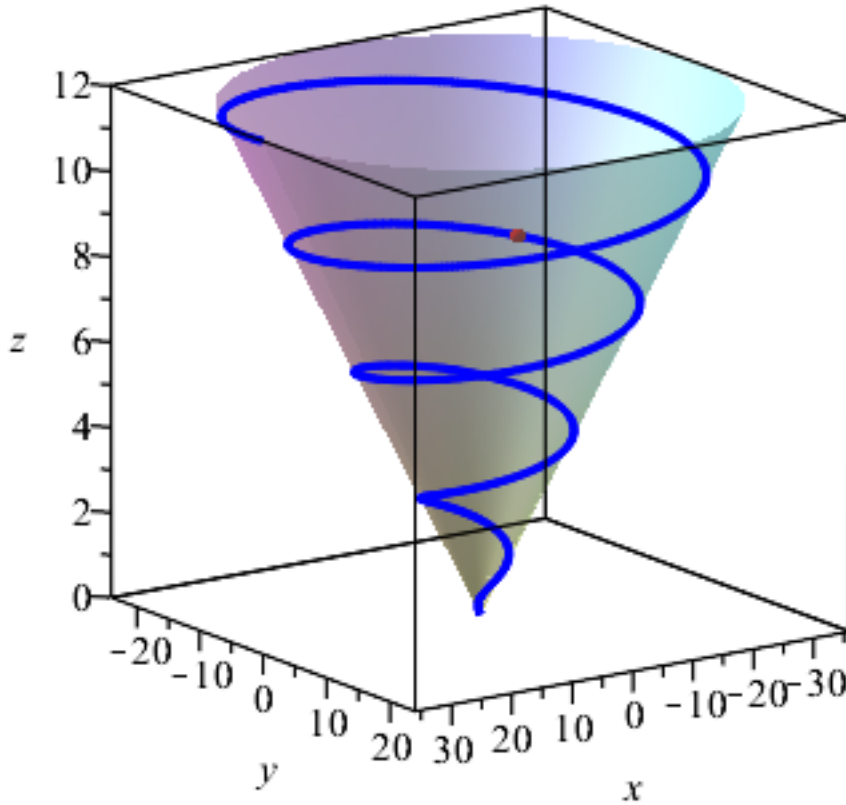
```

>
>
> a := 3
a := 3 (1)
> b := 2
b := 2 (2)
> c := 3
c := 3 (3)
> n := 4
n := 4 (4)
> h := c·n
h := 12 (5)
> Helix := <t·a·cos(2·Pi/c·t), t·b·sin(2·Pi/c·t), t> :
> Helix1 := [t·a·cos(2·Pi/c·t), t·b·sin(2·Pi/c·t), t] :
> s1 := spacecurve([t·a·cos(2·Pi/c·t), t·b·sin(2·Pi/c·t), t], t=0..h, color=blue,
linestyle=1, thickness=3, numpoints=1000, labels=[x, y, z]) :
> s2 := point([5·Pi/2·a·cos(2·Pi/c·5·Pi/2), 5·Pi/2·b·sin(2·Pi/c·5·Pi/2), 5·Pi/2], symbol
=solidcircle, symbolsize=15, color=red) :
> konos := plot3d([z·a·cos(φ), z·b·sin(φ), z], z=0..h, φ=0..2·Pi, style=surface,
transparency=0.5) :
> display(s1, s2, konos, scaling=unconstrained, title
="ΕΛΛΕΙΠΤΙΚΗ ΚΩΝΟΕΙΔΗΣ ΕΛΙΚΑ\nΣΑΒΒΑΣ ΠΙ. ΓΑΒΡΙΗΛΙΔΗΣ", titlefont=[arial,
bold, 14])

```

ΕΛΛΕΠΤΙΚΗ ΚΩΝΟΕΙΔΗΣ ΕΛΙΚΑ

ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ



> *TNBFrame(Helix)*

$$\left[\left[- \left(3 \left(2 t \pi \sin \left(\frac{2 \pi t}{3} \right) - 3 \cos \left(\frac{2 \pi t}{3} \right) \right) \right) / \right. \right. \\ \left. \left(-20 t^2 \pi^2 \cos \left(\frac{2 \pi t}{3} \right)^2 + 36 \pi^2 t^2 - 60 \sin \left(\frac{2 \pi t}{3} \right) t \pi \cos \left(\frac{2 \pi t}{3} \right) \right. \right. \\ \left. \left. + 45 \cos \left(\frac{2 \pi t}{3} \right)^2 + 45 \right)^{1/2} \right], \\ \left[\left(2 \left(2 t \pi \cos \left(\frac{2 \pi t}{3} \right) + 3 \sin \left(\frac{2 \pi t}{3} \right) \right) \right) / \right. \\ \left. \left(-20 t^2 \pi^2 \cos \left(\frac{2 \pi t}{3} \right)^2 + 36 \pi^2 t^2 - 60 \sin \left(\frac{2 \pi t}{3} \right) t \pi \cos \left(\frac{2 \pi t}{3} \right) \right. \right. \\ \left. \left. + 45 \cos \left(\frac{2 \pi t}{3} \right)^2 + 45 \right)^{1/2} \right], \\ \left[3 / \right.$$

(6)

$$\left(-20 t^2 \pi^2 \cos\left(\frac{2 \pi t}{3}\right)^2 + 36 \pi^2 t^2 - 60 \sin\left(\frac{2 \pi t}{3}\right) t \pi \cos\left(\frac{2 \pi t}{3}\right) \right. \\ \left. + 45 \cos\left(\frac{2 \pi t}{3}\right)^2 + 45 \right)^{1/2} \left[- \left(16 \pi^3 \cos\left(\frac{2 \pi t}{3}\right) t^3 \right. \right.$$

$$\left. + 24 \pi^2 \sin\left(\frac{2 \pi t}{3}\right) t^2 + 81 t \pi \cos\left(\frac{2 \pi t}{3}\right) + 135 \sin\left(\frac{2 \pi t}{3}\right) \right) \right]$$

$$\left(\left(\left(320 \pi^6 \cos\left(\frac{2 \pi t}{3}\right)^2 t^6 - 576 \pi^6 t^6 \right. \right. \right.$$

$$\left. + 960 \pi^5 \cos\left(\frac{2 \pi t}{3}\right) \sin\left(\frac{2 \pi t}{3}\right) t^5 + 100 \pi^4 \cos\left(\frac{2 \pi t}{3}\right)^4 t^4 \right.$$

$$\left. + 2060 \pi^4 \cos\left(\frac{2 \pi t}{3}\right)^2 t^4 + 900 \pi^3 \cos\left(\frac{2 \pi t}{3}\right)^3 \sin\left(\frac{2 \pi t}{3}\right) t^3 - 6048 t^4 \pi^4 \right.$$

$$\left. + 7800 \pi^3 \cos\left(\frac{2 \pi t}{3}\right) \sin\left(\frac{2 \pi t}{3}\right) t^3 - 2925 \pi^2 \cos\left(\frac{2 \pi t}{3}\right)^4 t^2 \right.$$

$$\left. + 4635 t^2 \pi^2 \cos\left(\frac{2 \pi t}{3}\right)^2 - 4050 \pi \cos\left(\frac{2 \pi t}{3}\right)^3 \sin\left(\frac{2 \pi t}{3}\right) t - 21240 \pi^2 t^2 \right.$$

$$\left. + 22950 \sin\left(\frac{2 \pi t}{3}\right) t \pi \cos\left(\frac{2 \pi t}{3}\right) + 2025 \cos\left(\frac{2 \pi t}{3}\right)^4 - 16200 \cos\left(\frac{2 \pi t}{3}\right)^2 \right.$$

$$\left. - 18225 \right) \left(-91125 - 270000 \pi^4 \cos\left(\frac{2 \pi t}{3}\right)^6 t^4 + 745200 \pi^4 \cos\left(\frac{2 \pi t}{3}\right)^4 t^4 \right.$$

$$\left. - 369360 \pi^4 \cos\left(\frac{2 \pi t}{3}\right)^2 t^4 + 607500 \pi^2 \cos\left(\frac{2 \pi t}{3}\right)^6 t^2 + 24300 \pi^2 \cos\left(\frac{2 \pi t}{3}\right)^4 t^2 \right.$$

$$\left. + 8000 \pi^6 \cos\left(\frac{2 \pi t}{3}\right)^6 t^6 - 43200 \pi^6 \cos\left(\frac{2 \pi t}{3}\right)^4 t^6 + 77760 \pi^6 \cos\left(\frac{2 \pi t}{3}\right)^2 t^6 \right)$$

$$\begin{aligned}
& + 364500 \sin\left(\frac{2\pi t}{3}\right) t \pi \cos\left(\frac{2\pi t}{3}\right) + 364500 \pi \cos\left(\frac{2\pi t}{3}\right)^5 \sin\left(\frac{2\pi t}{3}\right) t \\
& + 729000 \pi \cos\left(\frac{2\pi t}{3}\right)^3 \sin\left(\frac{2\pi t}{3}\right) t + 233280 \pi^5 \cos\left(\frac{2\pi t}{3}\right) \sin\left(\frac{2\pi t}{3}\right) t^5 \\
& - 540000 \pi^3 \cos\left(\frac{2\pi t}{3}\right)^5 \sin\left(\frac{2\pi t}{3}\right) t^3 + 475200 \pi^3 \cos\left(\frac{2\pi t}{3}\right)^3 \sin\left(\frac{2\pi t}{3}\right) t^3 \\
& + 583200 \pi^3 \cos\left(\frac{2\pi t}{3}\right) \sin\left(\frac{2\pi t}{3}\right) t^3 + 72000 \pi^5 \cos\left(\frac{2\pi t}{3}\right)^5 \sin\left(\frac{2\pi t}{3}\right) t^5 \\
& - 259200 \pi^5 \cos\left(\frac{2\pi t}{3}\right)^3 \sin\left(\frac{2\pi t}{3}\right) t^5 - 218700 \pi^2 t^2 - 46656 \pi^6 t^6 - 174960 t^4 \pi^4 \\
& - 91125 \cos\left(\frac{2\pi t}{3}\right)^6 - 273375 \cos\left(\frac{2\pi t}{3}\right)^4 - 801900 t^2 \pi^2 \cos\left(\frac{2\pi t}{3}\right)^2 \\
& \quad \left. - 273375 \cos\left(\frac{2\pi t}{3}\right)^2\right) \left(-20 t^2 \pi^2 \cos\left(\frac{2\pi t}{3}\right)^2 + 36 \pi^2 t^2 \right. \\
& \quad \left. - 60 \sin\left(\frac{2\pi t}{3}\right) t \pi \cos\left(\frac{2\pi t}{3}\right) + 45 \cos\left(\frac{2\pi t}{3}\right)^2 + 45\right)^{3/2} \Bigg], \\
& \left[- \left(6 \left(4 \pi^3 \sin\left(\frac{2\pi t}{3}\right) t^3 - 6 \pi^2 \cos\left(\frac{2\pi t}{3}\right) t^2 + 19 t \pi \sin\left(\frac{2\pi t}{3}\right) \right. \right. \right. \\
& \quad \left. \left. \left. - 30 \cos\left(\frac{2\pi t}{3}\right) \right) \right) \right. \\
& \left. \left(\left(\left(320 \pi^6 \cos\left(\frac{2\pi t}{3}\right)^2 t^6 - 576 \pi^6 t^6 \right. \right. \right. \right. \\
& \quad \left. \left. \left. + 960 \pi^5 \cos\left(\frac{2\pi t}{3}\right) \sin\left(\frac{2\pi t}{3}\right) t^5 + 100 \pi^4 \cos\left(\frac{2\pi t}{3}\right)^4 t^4 \right. \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& + 2060 \pi^4 \cos\left(\frac{2 \pi t}{3}\right)^2 t^4 + 900 \pi^3 \cos\left(\frac{2 \pi t}{3}\right)^3 \sin\left(\frac{2 \pi t}{3}\right) t^3 - 6048 t^4 \pi^4 \\
& + 7800 \pi^3 \cos\left(\frac{2 \pi t}{3}\right) \sin\left(\frac{2 \pi t}{3}\right) t^3 - 2925 \pi^2 \cos\left(\frac{2 \pi t}{3}\right)^4 t^2 \\
& + 4635 t^2 \pi^2 \cos\left(\frac{2 \pi t}{3}\right)^2 - 4050 \pi \cos\left(\frac{2 \pi t}{3}\right)^3 \sin\left(\frac{2 \pi t}{3}\right) t - 21240 \pi^2 t^2 \\
& + 22950 \sin\left(\frac{2 \pi t}{3}\right) t \pi \cos\left(\frac{2 \pi t}{3}\right) + 2025 \cos\left(\frac{2 \pi t}{3}\right)^4 - 16200 \cos\left(\frac{2 \pi t}{3}\right)^2 \\
& - 18225 \left. \right) / \left(-91125 - 270000 \pi^4 \cos\left(\frac{2 \pi t}{3}\right)^6 t^4 + 745200 \pi^4 \cos\left(\frac{2 \pi t}{3}\right)^4 t^4 \right. \\
& - 369360 \pi^4 \cos\left(\frac{2 \pi t}{3}\right)^2 t^4 + 607500 \pi^2 \cos\left(\frac{2 \pi t}{3}\right)^6 t^2 + 24300 \pi^2 \cos\left(\frac{2 \pi t}{3}\right)^4 t^2 \\
& + 8000 \pi^6 \cos\left(\frac{2 \pi t}{3}\right)^6 t^6 - 43200 \pi^6 \cos\left(\frac{2 \pi t}{3}\right)^4 t^6 + 77760 \pi^6 \cos\left(\frac{2 \pi t}{3}\right)^2 t^6 \\
& + 364500 \sin\left(\frac{2 \pi t}{3}\right) t \pi \cos\left(\frac{2 \pi t}{3}\right) + 364500 \pi \cos\left(\frac{2 \pi t}{3}\right)^5 \sin\left(\frac{2 \pi t}{3}\right) t \\
& + 729000 \pi \cos\left(\frac{2 \pi t}{3}\right)^3 \sin\left(\frac{2 \pi t}{3}\right) t + 233280 \pi^5 \cos\left(\frac{2 \pi t}{3}\right) \sin\left(\frac{2 \pi t}{3}\right) t^5 \\
& - 540000 \pi^3 \cos\left(\frac{2 \pi t}{3}\right)^5 \sin\left(\frac{2 \pi t}{3}\right) t^3 + 475200 \pi^3 \cos\left(\frac{2 \pi t}{3}\right)^3 \sin\left(\frac{2 \pi t}{3}\right) t^3 \\
& + 583200 \pi^3 \cos\left(\frac{2 \pi t}{3}\right) \sin\left(\frac{2 \pi t}{3}\right) t^3 + 72000 \pi^5 \cos\left(\frac{2 \pi t}{3}\right)^5 \sin\left(\frac{2 \pi t}{3}\right) t^5 \\
& - 259200 \pi^5 \cos\left(\frac{2 \pi t}{3}\right)^3 \sin\left(\frac{2 \pi t}{3}\right) t^5 - 218700 \pi^2 t^2 - 46656 \pi^6 t^6 - 174960 t^4 \pi^4
\end{aligned}$$

$$\begin{aligned}
& -91125 \cos\left(\frac{2\pi t}{3}\right)^6 - 273375 \cos\left(\frac{2\pi t}{3}\right)^4 - 801900 t^2 \pi^2 \cos\left(\frac{2\pi t}{3}\right)^2 \\
& \quad \left. - 273375 \cos\left(\frac{2\pi t}{3}\right)^2\right)^{1/2} \left(-20 t^2 \pi^2 \cos\left(\frac{2\pi t}{3}\right)^2 + 36 \pi^2 t^2 \right. \\
& \quad \left. - 60 \sin\left(\frac{2\pi t}{3}\right) t \pi \cos\left(\frac{2\pi t}{3}\right) + 45 \cos\left(\frac{2\pi t}{3}\right)^2 + 45 \right)^{3/2} \Bigg], \\
& \left[- \left(10 t^2 \pi^2 \cos\left(\frac{2\pi t}{3}\right) \sin\left(\frac{2\pi t}{3}\right) - 45 t \pi \cos\left(\frac{2\pi t}{3}\right)^2 + 42 \pi t \right. \right. \\
& \quad \left. \left. - 45 \sin\left(\frac{2\pi t}{3}\right) \cos\left(\frac{2\pi t}{3}\right) \right) \right] / \\
& \left(\left(\left(320 \pi^6 \cos\left(\frac{2\pi t}{3}\right)^2 t^6 - 576 \pi^6 t^6 \right. \right. \right. \\
& \quad \left. \left. + 960 \pi^5 \cos\left(\frac{2\pi t}{3}\right) \sin\left(\frac{2\pi t}{3}\right) t^5 + 100 \pi^4 \cos\left(\frac{2\pi t}{3}\right)^4 t^4 \right. \right. \\
& \quad \left. \left. + 2060 \pi^4 \cos\left(\frac{2\pi t}{3}\right)^2 t^4 + 900 \pi^3 \cos\left(\frac{2\pi t}{3}\right)^3 \sin\left(\frac{2\pi t}{3}\right) t^3 - 6048 t^4 \pi^4 \right. \right. \\
& \quad \left. \left. + 7800 \pi^3 \cos\left(\frac{2\pi t}{3}\right) \sin\left(\frac{2\pi t}{3}\right) t^3 - 2925 \pi^2 \cos\left(\frac{2\pi t}{3}\right)^4 t^2 \right. \right. \\
& \quad \left. \left. + 4635 t^2 \pi^2 \cos\left(\frac{2\pi t}{3}\right)^2 - 4050 \pi \cos\left(\frac{2\pi t}{3}\right)^3 \sin\left(\frac{2\pi t}{3}\right) t - 21240 \pi^2 t^2 \right. \right. \\
& \quad \left. \left. + 22950 \sin\left(\frac{2\pi t}{3}\right) t \pi \cos\left(\frac{2\pi t}{3}\right) + 2025 \cos\left(\frac{2\pi t}{3}\right)^4 - 16200 \cos\left(\frac{2\pi t}{3}\right)^2 \right. \right. \\
& \quad \left. \left. - 18225 \right) \right) / \left(-91125 - 270000 \pi^4 \cos\left(\frac{2\pi t}{3}\right)^6 t^4 + 745200 \pi^4 \cos\left(\frac{2\pi t}{3}\right)^4 t^4 \right)
\end{aligned}$$

$$\begin{aligned}
& - 369360 \pi^4 \cos\left(\frac{2 \pi t}{3}\right)^2 t^4 + 607500 \pi^2 \cos\left(\frac{2 \pi t}{3}\right)^6 t^2 + 24300 \pi^2 \cos\left(\frac{2 \pi t}{3}\right)^4 t^2 \\
& + 8000 \pi^6 \cos\left(\frac{2 \pi t}{3}\right)^6 t^6 - 43200 \pi^6 \cos\left(\frac{2 \pi t}{3}\right)^4 t^6 + 77760 \pi^6 \cos\left(\frac{2 \pi t}{3}\right)^2 t^6 \\
& + 364500 \sin\left(\frac{2 \pi t}{3}\right) t \pi \cos\left(\frac{2 \pi t}{3}\right) + 364500 \pi \cos\left(\frac{2 \pi t}{3}\right)^5 \sin\left(\frac{2 \pi t}{3}\right) t \\
& + 729000 \pi \cos\left(\frac{2 \pi t}{3}\right)^3 \sin\left(\frac{2 \pi t}{3}\right) t + 233280 \pi^5 \cos\left(\frac{2 \pi t}{3}\right) \sin\left(\frac{2 \pi t}{3}\right) t^5 \\
& - 540000 \pi^3 \cos\left(\frac{2 \pi t}{3}\right)^5 \sin\left(\frac{2 \pi t}{3}\right) t^3 + 475200 \pi^3 \cos\left(\frac{2 \pi t}{3}\right)^3 \sin\left(\frac{2 \pi t}{3}\right) t^3 \\
& + 583200 \pi^3 \cos\left(\frac{2 \pi t}{3}\right) \sin\left(\frac{2 \pi t}{3}\right) t^3 + 72000 \pi^5 \cos\left(\frac{2 \pi t}{3}\right)^5 \sin\left(\frac{2 \pi t}{3}\right) t^5 \\
& - 259200 \pi^5 \cos\left(\frac{2 \pi t}{3}\right)^3 \sin\left(\frac{2 \pi t}{3}\right) t^5 - 218700 \pi^2 t^2 - 46656 \pi^6 t^6 - 174960 t^4 \pi^4 \\
& - 91125 \cos\left(\frac{2 \pi t}{3}\right)^6 - 273375 \cos\left(\frac{2 \pi t}{3}\right)^4 - 801900 t^2 \pi^2 \cos\left(\frac{2 \pi t}{3}\right)^2 \\
& \quad \left. - 273375 \cos\left(\frac{2 \pi t}{3}\right)^2 \right)^{1/2} \left(-20 t^2 \pi^2 \cos\left(\frac{2 \pi t}{3}\right)^2 + 36 \pi^2 t^2 \right. \\
& \quad \left. - 60 \sin\left(\frac{2 \pi t}{3}\right) t \pi \cos\left(\frac{2 \pi t}{3}\right) + 45 \cos\left(\frac{2 \pi t}{3}\right)^2 + 45 \right)^{3/2} \Bigg] \Bigg| \\
& - \left(2 \left(20 \pi^3 \cos\left(\frac{2 \pi t}{3}\right)^2 \sin\left(\frac{2 \pi t}{3}\right) t^3 - 36 \pi^3 \sin\left(\frac{2 \pi t}{3}\right) t^3 - 120 \pi^2 \cos\left(\frac{2 \pi t}{3}\right)^3 t^2 \right. \right.
\end{aligned}$$

$$+ 168 \pi^2 \cos\left(\frac{2 \pi t}{3}\right) t^2 - 225 \pi \cos\left(\frac{2 \pi t}{3}\right)^2 \sin\left(\frac{2 \pi t}{3}\right) t - 45 t \pi \sin\left(\frac{2 \pi t}{3}\right)$$

$$+ 135 \cos\left(\frac{2 \pi t}{3}\right)^3 + 135 \cos\left(\frac{2 \pi t}{3}\right)\right)$$

$$\left(\left(\left(320 \pi^6 \cos\left(\frac{2 \pi t}{3}\right)^2 t^6 - 576 \pi^6 t^6 \right. \right. \right.$$

$$+ 960 \pi^5 \cos\left(\frac{2 \pi t}{3}\right) \sin\left(\frac{2 \pi t}{3}\right) t^5 + 100 \pi^4 \cos\left(\frac{2 \pi t}{3}\right)^4 t^4$$

$$+ 2060 \pi^4 \cos\left(\frac{2 \pi t}{3}\right)^2 t^4 + 900 \pi^3 \cos\left(\frac{2 \pi t}{3}\right)^3 \sin\left(\frac{2 \pi t}{3}\right) t^3 - 6048 t^4 \pi^4$$

$$+ 7800 \pi^3 \cos\left(\frac{2 \pi t}{3}\right) \sin\left(\frac{2 \pi t}{3}\right) t^3 - 2925 \pi^2 \cos\left(\frac{2 \pi t}{3}\right)^4 t^2$$

$$+ 4635 t^2 \pi^2 \cos\left(\frac{2 \pi t}{3}\right)^2 - 4050 \pi \cos\left(\frac{2 \pi t}{3}\right)^3 \sin\left(\frac{2 \pi t}{3}\right) t - 21240 \pi^2 t^2$$

$$+ 22950 \sin\left(\frac{2 \pi t}{3}\right) t \pi \cos\left(\frac{2 \pi t}{3}\right) + 2025 \cos\left(\frac{2 \pi t}{3}\right)^4 - 16200 \cos\left(\frac{2 \pi t}{3}\right)^2$$

$$- 18225 \left) / \left(-91125 - 270000 \pi^4 \cos\left(\frac{2 \pi t}{3}\right)^6 t^4 + 745200 \pi^4 \cos\left(\frac{2 \pi t}{3}\right)^4 t^4 \right.$$

$$- 369360 \pi^4 \cos\left(\frac{2 \pi t}{3}\right)^2 t^4 + 607500 \pi^2 \cos\left(\frac{2 \pi t}{3}\right)^6 t^2 + 24300 \pi^2 \cos\left(\frac{2 \pi t}{3}\right)^4 t^2$$

$$+ 8000 \pi^6 \cos\left(\frac{2 \pi t}{3}\right)^6 t^6 - 43200 \pi^6 \cos\left(\frac{2 \pi t}{3}\right)^4 t^6 + 77760 \pi^6 \cos\left(\frac{2 \pi t}{3}\right)^2 t^6$$

$$+ 364500 \sin\left(\frac{2 \pi t}{3}\right) t \pi \cos\left(\frac{2 \pi t}{3}\right) + 364500 \pi \cos\left(\frac{2 \pi t}{3}\right)^5 \sin\left(\frac{2 \pi t}{3}\right) t$$

$$\begin{aligned}
& + 729000 \pi \cos\left(\frac{2 \pi t}{3}\right)^3 \sin\left(\frac{2 \pi t}{3}\right) t + 233280 \pi^5 \cos\left(\frac{2 \pi t}{3}\right) \sin\left(\frac{2 \pi t}{3}\right) t^5 \\
& - 540000 \pi^3 \cos\left(\frac{2 \pi t}{3}\right)^5 \sin\left(\frac{2 \pi t}{3}\right) t^3 + 475200 \pi^3 \cos\left(\frac{2 \pi t}{3}\right)^3 \sin\left(\frac{2 \pi t}{3}\right) t^3 \\
& + 583200 \pi^3 \cos\left(\frac{2 \pi t}{3}\right) \sin\left(\frac{2 \pi t}{3}\right) t^3 + 72000 \pi^5 \cos\left(\frac{2 \pi t}{3}\right)^5 \sin\left(\frac{2 \pi t}{3}\right) t^5 \\
& - 259200 \pi^5 \cos\left(\frac{2 \pi t}{3}\right)^3 \sin\left(\frac{2 \pi t}{3}\right) t^5 - 218700 \pi^2 t^2 - 46656 \pi^6 t^6 - 174960 t^4 \pi^4 \\
& - 91125 \cos\left(\frac{2 \pi t}{3}\right)^6 - 273375 \cos\left(\frac{2 \pi t}{3}\right)^4 - 801900 t^2 \pi^2 \cos\left(\frac{2 \pi t}{3}\right)^2 \\
& \quad \left. - 273375 \cos\left(\frac{2 \pi t}{3}\right)^2\right)^{1/2} \left(400 \pi^4 \cos\left(\frac{2 \pi t}{3}\right)^4 t^4 - 1440 \pi^4 \cos\left(\frac{2 \pi t}{3}\right)^2 t^4 \right. \\
& + 2400 \pi^3 \cos\left(\frac{2 \pi t}{3}\right)^3 \sin\left(\frac{2 \pi t}{3}\right) t^3 + 1296 t^4 \pi^4 \\
& - 4320 \pi^3 \cos\left(\frac{2 \pi t}{3}\right) \sin\left(\frac{2 \pi t}{3}\right) t^3 - 5400 \pi^2 \cos\left(\frac{2 \pi t}{3}\right)^4 t^2 \\
& + 5040 t^2 \pi^2 \cos\left(\frac{2 \pi t}{3}\right)^2 - 5400 \pi \cos\left(\frac{2 \pi t}{3}\right)^3 \sin\left(\frac{2 \pi t}{3}\right) t + 3240 \pi^2 t^2 \\
& - 5400 \sin\left(\frac{2 \pi t}{3}\right) t \pi \cos\left(\frac{2 \pi t}{3}\right) + 2025 \cos\left(\frac{2 \pi t}{3}\right)^4 + 4050 \cos\left(\frac{2 \pi t}{3}\right)^2 \\
& \left. + 2025\right)^{1/2} \left. \right], \\
& \left[\left(3 \left(20 \pi^3 \cos\left(\frac{2 \pi t}{3}\right)^3 t^3 - 36 \pi^3 \cos\left(\frac{2 \pi t}{3}\right) t^3 \right. \right. \right. \\
& \left. \left. \left. + 120 \pi^2 \cos\left(\frac{2 \pi t}{3}\right)^2 \sin\left(\frac{2 \pi t}{3}\right) t^2 - 108 \pi^2 \sin\left(\frac{2 \pi t}{3}\right) t^2 - 225 \pi \cos\left(\frac{2 \pi t}{3}\right)^3 t \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& + 135 t \pi \cos\left(\frac{2 \pi t}{3}\right) - 135 \cos\left(\frac{2 \pi t}{3}\right)^2 \sin\left(\frac{2 \pi t}{3}\right) - 135 \sin\left(\frac{2 \pi t}{3}\right) \Big) \\
& \left(\left(\left(320 \pi^6 \cos\left(\frac{2 \pi t}{3}\right)^2 t^6 - 576 \pi^6 t^6 \right. \right. \right. \\
& + 960 \pi^5 \cos\left(\frac{2 \pi t}{3}\right) \sin\left(\frac{2 \pi t}{3}\right) t^5 + 100 \pi^4 \cos\left(\frac{2 \pi t}{3}\right)^4 t^4 \\
& + 2060 \pi^4 \cos\left(\frac{2 \pi t}{3}\right)^2 t^4 + 900 \pi^3 \cos\left(\frac{2 \pi t}{3}\right)^3 \sin\left(\frac{2 \pi t}{3}\right) t^3 - 6048 t^4 \pi^4 \\
& + 7800 \pi^3 \cos\left(\frac{2 \pi t}{3}\right) \sin\left(\frac{2 \pi t}{3}\right) t^3 - 2925 \pi^2 \cos\left(\frac{2 \pi t}{3}\right)^4 t^2 \\
& + 4635 t^2 \pi^2 \cos\left(\frac{2 \pi t}{3}\right)^2 - 4050 \pi \cos\left(\frac{2 \pi t}{3}\right)^3 \sin\left(\frac{2 \pi t}{3}\right) t - 21240 \pi^2 t^2 \\
& + 22950 \sin\left(\frac{2 \pi t}{3}\right) t \pi \cos\left(\frac{2 \pi t}{3}\right) + 2025 \cos\left(\frac{2 \pi t}{3}\right)^4 - 16200 \cos\left(\frac{2 \pi t}{3}\right)^2 \\
& - 18225 \Big) / \left(-91125 - 270000 \pi^4 \cos\left(\frac{2 \pi t}{3}\right)^6 t^4 + 745200 \pi^4 \cos\left(\frac{2 \pi t}{3}\right)^4 t^4 \right. \\
& - 369360 \pi^4 \cos\left(\frac{2 \pi t}{3}\right)^2 t^4 + 607500 \pi^2 \cos\left(\frac{2 \pi t}{3}\right)^6 t^2 + 24300 \pi^2 \cos\left(\frac{2 \pi t}{3}\right)^4 t^2 \\
& + 8000 \pi^6 \cos\left(\frac{2 \pi t}{3}\right)^6 t^6 - 43200 \pi^6 \cos\left(\frac{2 \pi t}{3}\right)^4 t^6 + 77760 \pi^6 \cos\left(\frac{2 \pi t}{3}\right)^2 t^6 \\
& + 364500 \sin\left(\frac{2 \pi t}{3}\right) t \pi \cos\left(\frac{2 \pi t}{3}\right) + 364500 \pi \cos\left(\frac{2 \pi t}{3}\right)^5 \sin\left(\frac{2 \pi t}{3}\right) t \\
& \left. + 729000 \pi \cos\left(\frac{2 \pi t}{3}\right)^3 \sin\left(\frac{2 \pi t}{3}\right) t + 233280 \pi^5 \cos\left(\frac{2 \pi t}{3}\right) \sin\left(\frac{2 \pi t}{3}\right) t^5 \right)
\end{aligned}$$

$$\begin{aligned}
& + 960 \pi^5 \cos\left(\frac{2 \pi t}{3}\right) \sin\left(\frac{2 \pi t}{3}\right) t^5 + 100 \pi^4 \cos\left(\frac{2 \pi t}{3}\right)^4 t^4 \\
& + 2060 \pi^4 \cos\left(\frac{2 \pi t}{3}\right)^2 t^4 + 900 \pi^3 \cos\left(\frac{2 \pi t}{3}\right)^3 \sin\left(\frac{2 \pi t}{3}\right) t^3 - 6048 t^4 \pi^4 \\
& + 7800 \pi^3 \cos\left(\frac{2 \pi t}{3}\right) \sin\left(\frac{2 \pi t}{3}\right) t^3 - 2925 \pi^2 \cos\left(\frac{2 \pi t}{3}\right)^4 t^2 \\
& + 4635 t^2 \pi^2 \cos\left(\frac{2 \pi t}{3}\right)^2 - 4050 \pi \cos\left(\frac{2 \pi t}{3}\right)^3 \sin\left(\frac{2 \pi t}{3}\right) t - 21240 \pi^2 t^2 \\
& + 22950 \sin\left(\frac{2 \pi t}{3}\right) t \pi \cos\left(\frac{2 \pi t}{3}\right) + 2025 \cos\left(\frac{2 \pi t}{3}\right)^4 - 16200 \cos\left(\frac{2 \pi t}{3}\right)^2 \\
& - 18225 \Big) / \Big(-91125 - 270000 \pi^4 \cos\left(\frac{2 \pi t}{3}\right)^6 t^4 + 745200 \pi^4 \cos\left(\frac{2 \pi t}{3}\right)^4 t^4 \\
& - 369360 \pi^4 \cos\left(\frac{2 \pi t}{3}\right)^2 t^4 + 607500 \pi^2 \cos\left(\frac{2 \pi t}{3}\right)^6 t^2 + 24300 \pi^2 \cos\left(\frac{2 \pi t}{3}\right)^4 t^2 \\
& + 8000 \pi^6 \cos\left(\frac{2 \pi t}{3}\right)^6 t^6 - 43200 \pi^6 \cos\left(\frac{2 \pi t}{3}\right)^4 t^6 + 77760 \pi^6 \cos\left(\frac{2 \pi t}{3}\right)^2 t^6 \\
& + 364500 \sin\left(\frac{2 \pi t}{3}\right) t \pi \cos\left(\frac{2 \pi t}{3}\right) + 364500 \pi \cos\left(\frac{2 \pi t}{3}\right)^5 \sin\left(\frac{2 \pi t}{3}\right) t \\
& + 729000 \pi \cos\left(\frac{2 \pi t}{3}\right)^3 \sin\left(\frac{2 \pi t}{3}\right) t + 233280 \pi^5 \cos\left(\frac{2 \pi t}{3}\right) \sin\left(\frac{2 \pi t}{3}\right) t^5 \\
& - 540000 \pi^3 \cos\left(\frac{2 \pi t}{3}\right)^5 \sin\left(\frac{2 \pi t}{3}\right) t^3 + 475200 \pi^3 \cos\left(\frac{2 \pi t}{3}\right)^3 \sin\left(\frac{2 \pi t}{3}\right) t^3 \\
& + 583200 \pi^3 \cos\left(\frac{2 \pi t}{3}\right) \sin\left(\frac{2 \pi t}{3}\right) t^3 + 72000 \pi^5 \cos\left(\frac{2 \pi t}{3}\right)^5 \sin\left(\frac{2 \pi t}{3}\right) t^5
\end{aligned}$$


```
> X3 := Component(rp2_, 1) :
> Y3 := Component(rp2_, 2) :
> Z3 := Component(rp2_, 3) :
```

4. ΝΑ ΒΡΟΥΜΕ ΤΙΣ ΣΥΝΤΕΤΑΓΜΕΝΕΣ ΤΟΥ ΣΗΜΕΙΟΥ Β .

```
> rB_ := rA_ + TA_·0 + NA_·(-2) + BA_·(+1) :
> XB := Component(rB_, 1) :
> YB := Component(rB_, 2) :
> ZB := Component(rB_, 3) :
```

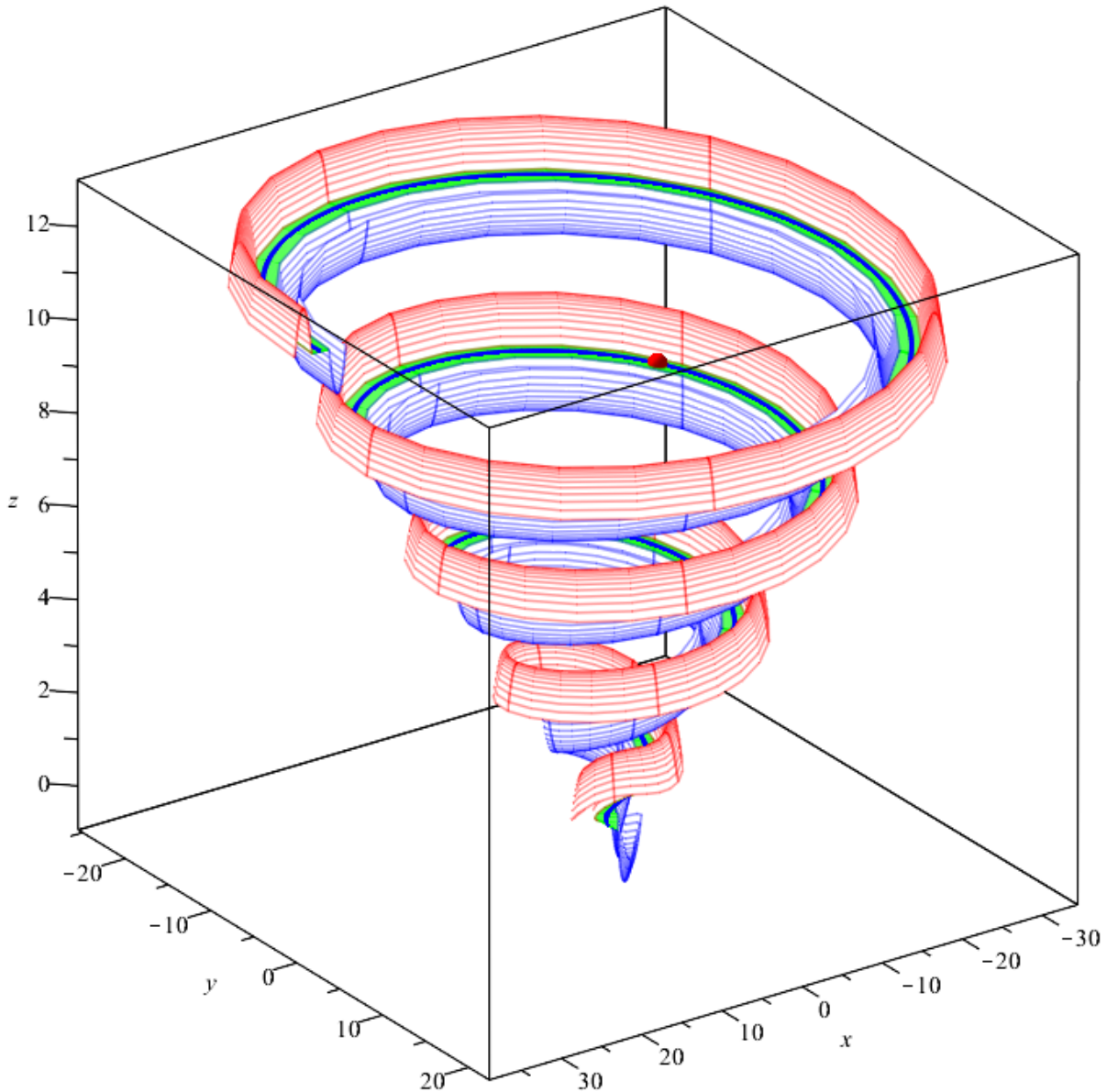
5. ΝΑ ΒΡΟΥΜΕ ΤΙΣ ΣΥΝΤΕΤΑΓΜΕΝΕΣ ΤΟΥ ΣΗΜΕΙΟΥ C .

```
> rC_ := rA_ + TA_·0 + NA_·(+2) + BA_·(-1) :
> XC := Component(rC_, 1) :
> YC := Component(rC_, 2) :
> ZC := Component(rC_, 3) :
```

ΠΑΡΟΥΣΙΑΣΗ

```
> p1 := plot3d([X1, Y1, Z1], λ=-1..1, t=0..h, color=green, numpoints=10000, style
=hidden) :
> p2 := plot3d([X2, Y2, Z2], λ=-1..1, t=0..h, color=blue, numpoints=10000, style
=hidden) :
> p3 := plot3d([X3, Y3, Z3], λ=-1..1, t=0..h, color=red, numpoints=10000, style
=hidden) :
> display(s1, s2, p1, p2, p3, scaling=unconstrained, orientation=[55, 65, 0], title
="ΕΠΙΦΑΝΕΙΑ ΔΙΑΓΡΑΦΟΜΕΝΗ \nΑΠΟ ΤΟ ΣΥΜΠΛΕΓΜΑ\nΣΑΒΒΑΣ Π.
ΓΑΒΡΙΗΛΙΔΗΣ", titlefont=[arial, 14, bold]) :
```

**ΕΠΙΦΑΝΕΙΑ ΔΙΑΓΡΑΦΟΜΕΝΗ
ΑΠΟ ΤΟ ΣΥΜΠΛΕΓΜΑ
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ**



- ```

> p1A := animate(plot3d, [[X1, Y1, Z1], λ=-1..1, t=0..C, color=green, numpoints=10000,
 style=hidden], C=0..h, frames=20) :
> p2A := animate(plot3d, [[X2, Y2, Z2], λ=-1..1, t=0..C, color=blue, numpoints=10000,
 style=hidden], C=0..h, frames=20) :
> p3A := animate(plot3d, [[X3, Y3, Z3], λ=-1..1, t=0..C, color=red, numpoints=10000,
 style=hidden], C=0..h, frames=20) :
> pointA := animate(pointplot3d, [Helix1, symbol=solidcircle, symbolsize=10, color
 =green], t=0..h, frames=20, trace=10) :
> pointB := animate(pointplot3d, [[XB, YB, ZB], symbol=solidcircle, symbolsize=10, color
 =red], t=0..h, frames=20, trace=10) :
> pointC := animate(pointplot3d, [[XC, YC, ZC], symbol=solidcircle, symbolsize=10, color
 =blue], t=0..h, frames=20, trace=10) :
>
> display(s1, s2, p1A, p2A, p3A, pointA, pointB, pointC, konos, scaling=unconstrained,
 orientation=[55, 65, 0], title

```

= "ANIMATION\nto σύστημα Frenet-Serret κινείται\nΣάββας Π.  
Γαβριηλίδης", titlefont = [arial, bold, 12]) :

**ANIMATION  
TO ΣΥΣΤΗΜΑ FRENET-SERRET ΚΙΝΕΙΤΑΙ  
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ**

