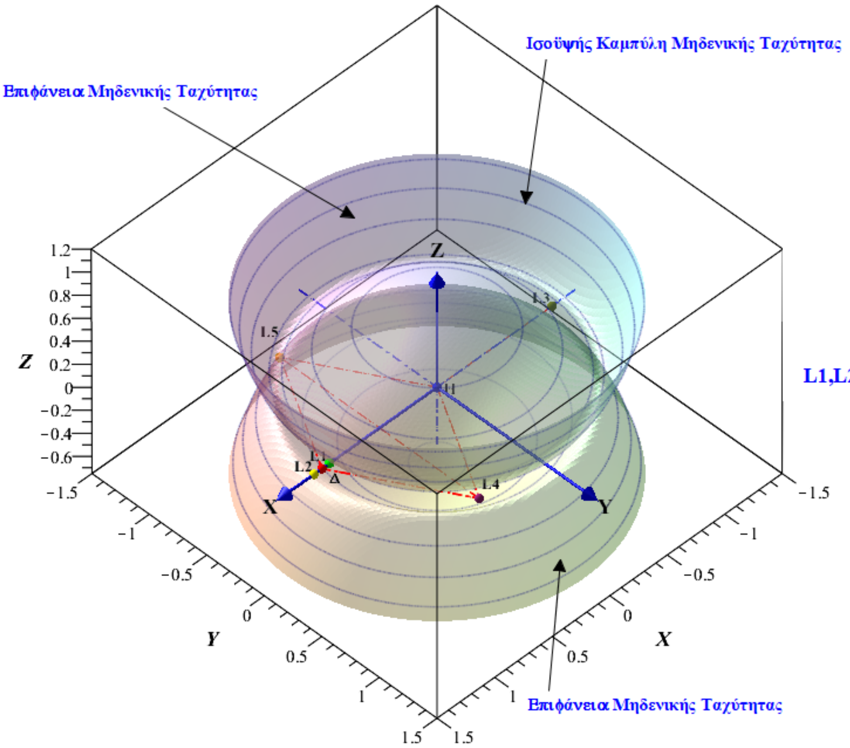
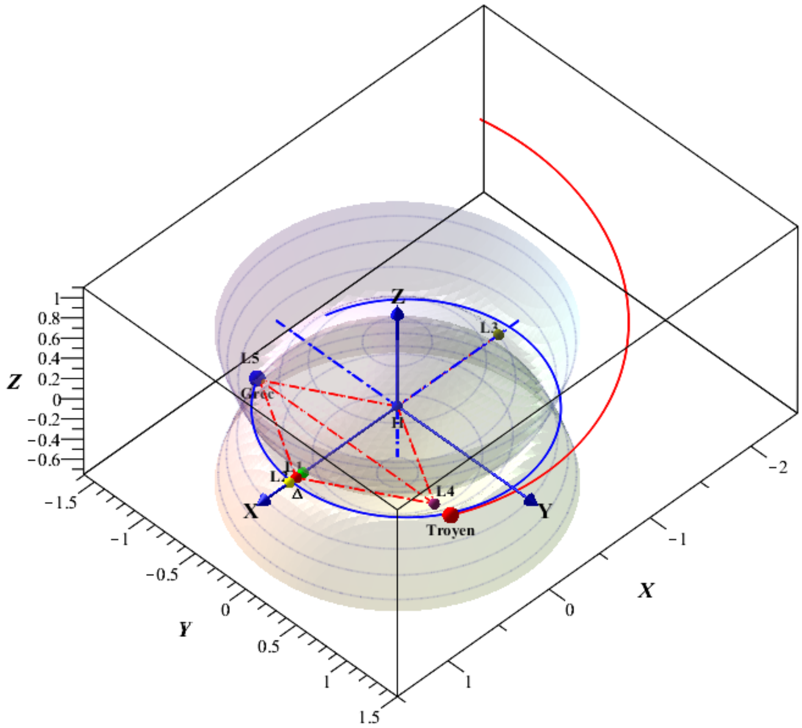
ΑΔΡΑΝΕΙΑΚΟ ΣΥΣΤΗΜΑ ΤΡΟΧΙΑ ΣΥΣΤΗΜΑΤΟΣ
ΗΛΙΟΣ-ΔΙΑΣ , C=2.987997088
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ
ΔΙΑΡΚΕΙΑ ΚΙΝΗΣΗΣ : t=4328.703704 ΗΜΕΡΕΣ



ΥΠΟΜΝΗΜΑ
XYZ : Αδρανειακό Σύστημα Αναφοράς
Δ : ΔΙΑΣ
H : ΗΛΙΟΣ
L1,L2,L3,L4,L5 : Σημεία Ισορροπίας Lagrange

ΑΔΡΑΝΕΙΑΚΟ ΣΥΣΤΗΜΑ ΤΡΟΧΙΑ ΣΥΣΤΗΜΑΤΟΣ
ΗΛΙΟΣ-ΔΙΑΣ , C=2.987997088
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ
ΔΙΑΡΚΕΙΑ ΚΙΝΗΣΗΣ : t=4328.703704 ΗΜΕΡΕΣ



ΥΠΟΜΝΗΜΑ
Η:Ηλιος
Δ:Δίας
L1,L2,L3,L4,L5 :Τα σημεία ισορροπίας Lagrange
Ο Troyen είναι κοντά στο σημείο ισορροπίας L4
με μηδενική ταχύτητα κατά την έναρξη της κίνησης .
Ο Grec είναι κοντά στο σημείο ισορροπίας L5
με μηδενική ταχύτητα κατά την έναρξη της κίνησης .

Το σύστημα ΗΛΙΟΣ-ΔΙΑΣ θεωρείται απομονωμένο .

Αρχικές Συνθήκες

$a\Xi TR := 0:$

$bHTR := +0.159812598$

$cZTR := 0:$

$a\Xi GR := a\Xi TR:$

$bHGR := -0.02239605819$

$cZGR := cZTR:$

$icsTR := \Xi(0) = L4[1] + a\Xi TR, D(\Xi)(0) = 0, H(0) = L4[2] + bHTR, D(H)(0) = 0, Z(0) = cZTR, D(Z)(0) = 0$

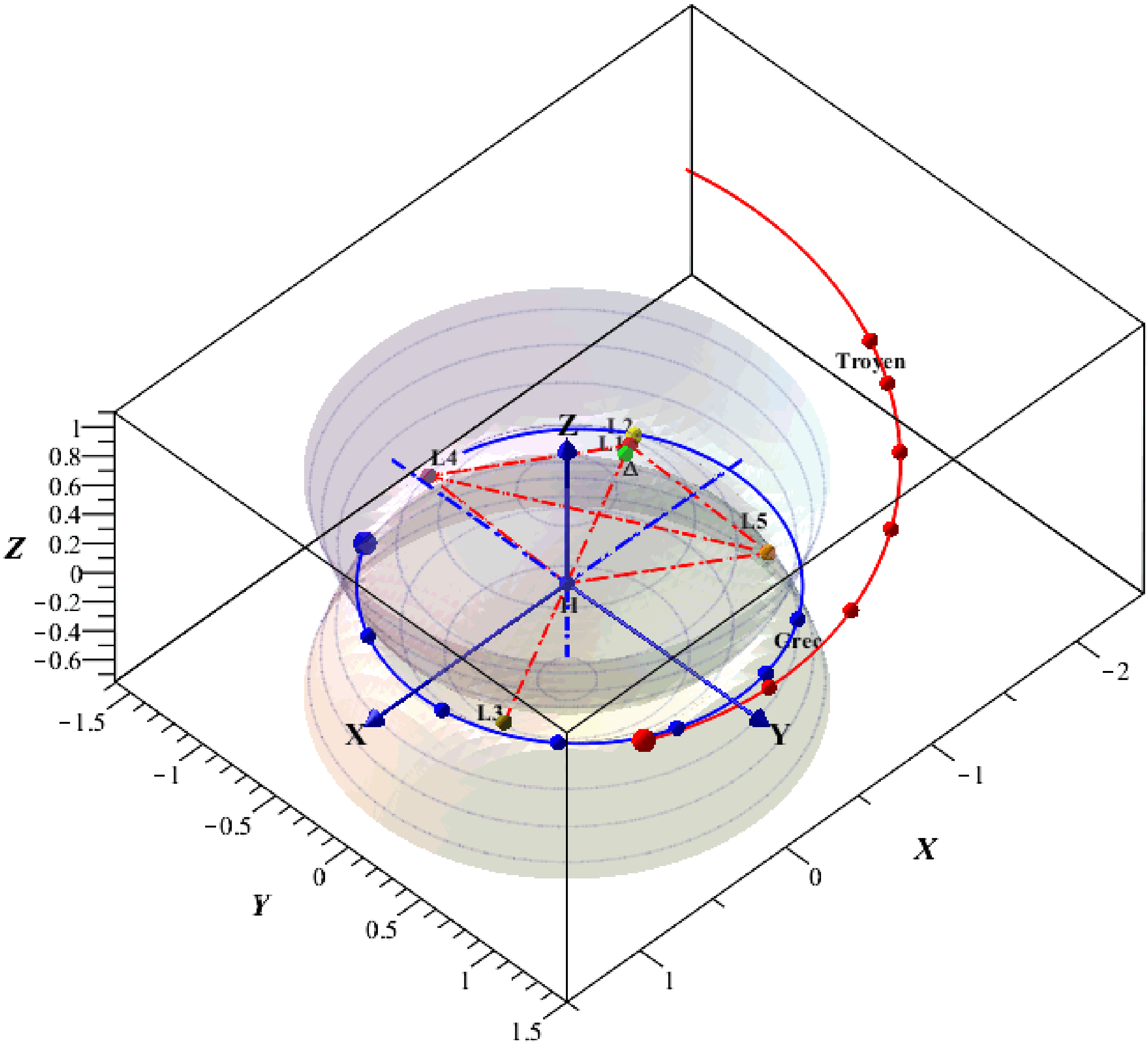
$icsTR := \Xi(0) = 0.4990466155, D(\Xi)(0) = 0, H(0) = 1.025838002, D(H)(0) = 0, Z(0) = 0, D(Z)(0) = 0$

$icsGR := \Xi(0) = L5[1] + a\Xi GR, D(\Xi)(0) = 0, H(0) = L5[2] + bHGR, D(H)(0) = 0, Z(0) = cZGR, D(Z)(0) = 0$

$icsGR := \Xi(0) = 0.4990466155, D(\Xi)(0) = 0, H(0) = -0.8884214622, D(H)(0) = 0, Z(0) = 0, D(Z)(0) = 0$

ΑΔΡΑΝΕΙΑΚΟ ΣΥΣΤΗΜΑ ΤΡΟΧΙΑ ΣΥΣΤΗΜΑΤΟΣ
ΗΛΙΟΣ-ΔΙΑΣ , C=2.987997088
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ
ΔΙΑΡΚΕΙΑ ΚΙΝΗΣΗΣ : t=4328.703704 ΗΜΕΡΕΣ

Ο Troyen απομακρύνεται από τον Ήλιο .
Ο Greec κρατάει σχεδόν σταθερή την αρχική απόστασή του από τον Ήλιο



>

ΔΕΔΟΜΕΝΑ ΓΙΑ ΤΟ ΣΥΣΤΗΜΑ ΗΛΙΟΣ-ΔΙΑΣ

> $G := 6.67259 \cdot 10^{-19} \mathrm{N} \cdot \mathrm{m}^2 \cdot \mathrm{kg}^{-2}$

$$G := \frac{6.672590000\ 10^{-19}}{\mathrm{kg}^2}\ \mathrm{N\ m}^2 \quad \textbf{(1)}$$

> $m_H := 1.991 \cdot 10^{30} \mathrm{kg}$

$$m_H := 1.991000000\ 10^{30}\ \mathrm{kg} \quad \textbf{(2)}$$

> $m_\Delta := 1.90 \cdot 10^{27} \mathrm{kg}$

$$m_{\Delta} := 1.900000000\ 10^{27}\ \mathrm{kg} \qquad (3)$$

> $M := m_{H} + m_{\Delta}$

$$M := 1.992900000\ 10^{30}\ \mathrm{kg} \qquad (4)$$

> $DistH\Delta := 7.78 \cdot 10^{11}\,\mathrm{m}$

$$DistH\Delta := 7.780000000\ 10^{11}\ \mathrm{m} \qquad (5)$$

> $AU := 1.496 \cdot 10^{11}\ \mathrm{m}$

$$AU := 1.496000000\ 10^{11}\ \mathrm{m} \qquad (6)$$

> $\frac{DistH\Delta}{AU}$

$$5.200534759 \qquad (7)$$

> $n := \frac{T}{2 \cdot \mathrm{Pi}}$:

> $T := 3.74 \cdot 10^{8}\mathrm{s}$

$$T := 3.740000000\ 10^{8}\ \mathrm{s} \qquad (8)$$

> $convert(T, units, day)$

$$4328.703704\ \mathrm{d} \qquad (9)$$

> $convert(\mathbf{(9)}, units, year)$

$$11.85159826\ \mathrm{yr} \qquad (10)$$

> $6 \cdot \mathbf{(10)}$

$$71.10958956\ \mathrm{yr} \qquad (11)$$

> $\omega := evalf\left(\frac{2 \cdot \mathrm{Pi}}{T}\right)$

$$\omega := \frac{1.679996071\ 10^{-8}}{\mathrm{s}} \qquad (12)$$

> $A := simplify(G \cdot (m_{H} + m_{\Delta}))$

$$A := 1.329780461\ 10^{12}\ \frac{\mathrm{m}^3}{\mathrm{s}^2} \qquad (13)$$

>

Αδιαστατοποιημένα μεγέθη στο Περιστρεφόμενο Σύστημα .

Τα μεγέθη μαζών είναι διαιρεμένα με : $M := 1.992900000\ 10^{30}$ kg :

Τα μεγέθη χρόνου με : $n := \frac{T}{2 \cdot \mathrm{Pi}} = 5.952394870\ 10^{7}$ s :

Τα μεγέθη μηκών με :$DistH\Delta := 7.780000000\ 10^{11}$ m :

> $\mu_{H} := \frac{m_{H}}{m_{H} + m_{\Delta}}$

$$\mu_{H} := 0.9990466155 \qquad (14)$$

> $\mu_{\Delta} := \frac{m_{\Delta}}{m_{H} + m_{\Delta}}$

$$\mu_{\Delta} := 0.0009533845150 \qquad (15)$$

> $\mu_H + \mu_\Delta$

$$1.000000000 \tag{16}$$

>

> $L4 := evalf\left(\left[\frac{1}{2}\left(\mu_H - \mu_\Delta\right), \frac{1}{2}\sqrt{3}, 0\right]\right)$

$$L4 := [0.4990466155, 0.8660254040, 0.] \tag{17}$$

> $\mathrm{sqrt}\left(\left(L4[1] + \mu_\Delta\right)^2 + (L4[2])^2\right) \cdot \mathbf{(7)}$

$$5.200534759 \tag{18}$$

>

> $L5 := evalf\left(\left[\frac{1}{2}\left(\mu_H - \mu_\Delta\right), -\frac{1}{2}\sqrt{3}, 0\right]\right)$

$$L5 := [0.4990466155, -0.8660254040, 0.] \tag{19}$$

> $-\frac{\mu_H\left(\Xi(\tau) + \mu_\Delta\right)}{\left(\left(\Xi(\tau) + \mu_\Delta\right)^2 + \mathrm{H}(\tau)^2 + \mathrm{Z}(\tau)^2\right)^{3/2}} - \frac{\mu_\Delta\left(\Xi(\tau) - \mu_H\right)}{\left(\left(\Xi(\tau) - \mu_H\right)^2 + \mathrm{H}(\tau)^2 + \mathrm{Z}(\tau)^2\right)^{3/2}} + \Xi(\tau) = 0$

$$-\frac{0.9990466155\,(\Xi(\tau) + 0.0009533845150)}{\left((\Xi(\tau) + 0.0009533845150)^2 + \mathrm{H}(\tau)^2 + \mathrm{Z}(\tau)^2\right)^{3/2}} - \frac{0.0009533845150\,(\Xi(\tau) - 0.9990466155)}{\left((\Xi(\tau) - 0.9990466155)^2 + \mathrm{H}(\tau)^2 + \mathrm{Z}(\tau)^2\right)^{3/2}} + \Xi(\tau) = 0 \tag{20}$$

>

> $subs(\{\mathrm{H}(\tau) = 0, \mathrm{Z}(\tau) = 0\}, \mathbf{(20)})$

$$-\frac{0.9990466155\,(\Xi(\tau) + 0.0009533845150)}{\left((\Xi(\tau) + 0.0009533845150)^2\right)^{3/2}} - \frac{0.0009533845150\,(\Xi(\tau) - 0.9990466155)}{\left((\Xi(\tau) - 0.9990466155)^2\right)^{3/2}} + \Xi(\tau) = 0 \tag{21}$$

> $solve(\mathbf{(21)}, \Xi(\tau))$

$$0.9323772470, 1.068818779, -1.000397244 \tag{22}$$

> $L1 := [\mathbf{(22)}[1], 0, 0]$

$$L1 := [0.9323772470, 0, 0] \tag{23}$$

> $L2 := [\mathbf{(22)}[2], 0, 0]$

$$L2 := [1.068818779, 0, 0] \tag{24}$$

> $L3 := [\mathbf{(22)}[3], 0, 0]$

$$L3 := [-1.000397244, 0, 0] \tag{25}$$

>

> $Soleil := \left[\mu_\Delta, 0, 0\right]$

$$Soleil := [0.0009533845150, 0, 0] \tag{26}$$

> $Jupiter := \left[\mu_H, 0, 0\right]$

$$Jupiter := [0.9990466155, 0, 0] \tag{27}$$

>

> $with(plots):$

> $with(Physics[Vectors])$

$[\&x, `+`, `.`, ChangeBasis, ChangeCoordinates, Component, Curl, DirectionalDiff, Divergence, Gradient, Identify, Laplacian, \nabla, Norm, Setup, diff]$ **(28)**

> $Setup(mathematicalnotation = true)$

$[mathematicalnotation = true]$ **(29)**

>

Έστω TROYEN χωρίς αρχική ταχύτητα (Κοντά) στο τριγωνικό σημείο Ευσταθούς ισορροπίας L4 .
Έστω GREC χωρίς αρχική ταχύτητα (Κοντά) στο τριγωνικό σημείο Ευσταθούς ισορροπίας L5 .
Λύνουμε τις εξισώσεις κίνησης (αριθμητικά !) στο Περιστρεφόμενο Σύστημα Συντεταγμένων (Ξ,Η,Ζ) και μετά κάνουμε μετατροπές για να πάμε στο Αδρανειακό Συστημα Συντεταγμένων (X,Y,Z) .

ΤΑ Ξ,Η,Ζ σε ΕΛΛΗΝΙΚΕΣ ΓΡΑΜΜΑΤΟΣΕΙΡΕΣ . !!!

>

Αλλάζουμε κατά βούληση τις Αρχικές παραμέτρους : $a\Xi TR := 0 :$
$bHTR := -0.159812598 :$
$cZTR := 0 :$
$a\Xi GR := a\Xi TR :$
$bHGR := -bHTR :$
$cZGR := cZTR :$

>

> $a\Xi TR := 0 :$

> $bHTR := +0.159812598$

$bHTR := 0.159812598$ **(30)**

> $cZTR := 0 :$

> $a\Xi GR := a\Xi TR :$

> $bHGR := -0.02239605819$

$bHGR := -0.02239605819$ **(31)**

> $cZGR := cZTR :$

>

$icsTR := \Xi(0) = L4[1] + a\Xi TR, D(\Xi)(0) = 0, H(0) = L4[2] + bHTR, D(H)(0) = 0, Z(0) = cZTR, D(Z)(0) = 0$
$icsGR := \Xi(0) = L5[1] + a\Xi GR, D(\Xi)(0) = 0, H(0) = L5[2] + bHGR, D(H)(0) = 0, Z(0) = cZGR, D(Z)(0) = 0$

Αρχικές Αποστάσεις από τον Ήλιο σε Αστρονομικές Μονάδες (AU):

$distTR := \mathrm{sqrt}\left(\left(rhs(icsTR[1]) + \mu_{\Delta}\right)^2 + (rhs(icsTR[3]))^2 + (rhs(icsTR[5]))^2\right) \cdot (7):$

$distGR := \mathrm{sqrt}\left(\left(rhs(icsGR[1]) + \mu_{\Delta}\right)^2 + (rhs(icsGR[3]))^2 + (rhs(icsGR[5]))^2\right) \cdot (7):$

Αποστάσεις σε κάθε χρόνο από τον Ήλιο σε Αστρονομικές Μονάδες (AU):

$distTR := \mathrm{sqrt}\left(\left(rhs(solTR[2](\tau)) + \mu_{\Delta}\right)^2 + (rhs(solTR[4](\tau)))^2 + (rhs(solTR[6](\tau)))^2\right) \cdot (7):$

$distGR := \mathrm{sqrt}\left(\left(rhs(solGR[2](\tau)) + \mu_{\Delta}\right)^2 + (rhs(solGR[4](\tau)))^2 + (rhs(solGR[6](\tau)))^2\right) \cdot (7):$

>

> $icsTR := \Xi(0) = L4[1] + a\Xi TR, \mathrm{D}(\Xi)(0) = 0, \mathrm{H}(0) = L4[2] + bHTR, \mathrm{D}(\mathrm{H})(0) = 0, \mathrm{Z}(0) = cZTR, \mathrm{D}(\mathrm{Z})(0) = 0$

$icsTR := \Xi(0) = 0.4990466155, \mathrm{D}(\Xi)(0) = 0, \mathrm{H}(0) = 1.025838002, \mathrm{D}(\mathrm{H})(0) = 0, \mathrm{Z}(0) = 0, \mathrm{D}(\mathrm{Z})(0) = 0$ **(32)**

> $icsGR := \Xi(0) = L5[1] + a\Xi GR, \mathrm{D}(\Xi)(0) = 0, \mathrm{H}(0) = L5[2] + bHGR, \mathrm{D}(\mathrm{H})(0) = 0, \mathrm{Z}(0) = cZGR, \mathrm{D}(\mathrm{Z})(0) = 0$

$icsGR := \Xi(0) = 0.4990466155, \mathrm{D}(\Xi)(0) = 0, \mathrm{H}(0) = -0.8884214622, \mathrm{D}(\mathrm{H})(0) = 0, \mathrm{Z}(0) = 0, \mathrm{D}(\mathrm{Z})(0) = 0$ **(33)**

>

Αρχικές Αποστάσεις από τον Ήλιο σε Αστρονομικές Μονάδες (AU):

> $Hector := \mathrm{sqrt}\left(\left(rhs(icsTR[1]) + \mu_{\Delta}\right)^2 + (rhs(icsTR[3]))^2 + (rhs(icsTR[5]))^2\right) \cdot (7)$

$Hector := 5.934864319$ **(34)**

> $Patrocle := \mathrm{sqrt}\left(\left(rhs(icsGR[1]) + \mu_{\Delta}\right)^2 + (rhs(icsGR[3]))^2 + (rhs(icsGR[5]))^2\right) \cdot (7)$

$Patrocle := 5.301721865$ **(35)**

>

Το Σύστημα των Εξισώσεων κίνησης TROYEN & GREC EINAI :

> $U := \frac{\Xi(\tau)^2}{2} + \frac{\mathrm{H}(\tau)^2}{2} + \frac{\mu_{\mathrm{H}}}{\sqrt{\left(\Xi(\tau) + \mu_{\Delta}\right)^2 + \mathrm{H}(\tau)^2 + \mathrm{Z}(\tau)^2}} + \frac{\mu_{\Delta}}{\sqrt{\left(\Xi(\tau) - \mu_{\mathrm{H}}\right)^2 + \mathrm{H}(\tau)^2 + \mathrm{Z}(\tau)^2}}$

$$U := \frac{\Xi(\tau)^2}{2} + \frac{H(\tau)^2}{2} + \frac{0.9990466155}{\sqrt{(\Xi(\tau) + 0.0009533845150)^2 + H(\tau)^2 + Z(\tau)^2}} + \frac{0.0009533845150}{\sqrt{(\Xi(\tau) - 0.9990466155)^2 + H(\tau)^2 + Z(\tau)^2}} \quad \textbf{(36)}$$

\>

\> *diff*(U, Ξ(τ))

$$\Xi(\tau) - \frac{0.4995233078\,(2\,\Xi(\tau) + 0.001906769030)}{\left((\Xi(\tau) + 0.0009533845150)^2 + H(\tau)^2 + Z(\tau)^2\right)^{3/2}} - \frac{0.0004766922575\,(2\,\Xi(\tau) - 1.998093231)}{\left((\Xi(\tau) - 0.9990466155)^2 + H(\tau)^2 + Z(\tau)^2\right)^{3/2}} \quad \textbf{(37)}$$

\> *diff*(U, H(τ))

$$H(\tau) - \frac{0.9990466155\,H(\tau)}{\left((\Xi(\tau) + 0.0009533845150)^2 + H(\tau)^2 + Z(\tau)^2\right)^{3/2}} - \frac{0.0009533845150\,H(\tau)}{\left((\Xi(\tau) - 0.9990466155)^2 + H(\tau)^2 + Z(\tau)^2\right)^{3/2}} \quad \textbf{(38)}$$

\> *diff*(U, Z(τ))

$$-\frac{0.9990466155\,Z(\tau)}{\left((\Xi(\tau) + 0.0009533845150)^2 + H(\tau)^2 + Z(\tau)^2\right)^{3/2}} - \frac{0.0009533845150\,Z(\tau)}{\left((\Xi(\tau) - 0.9990466155)^2 + H(\tau)^2 + Z(\tau)^2\right)^{3/2}} \quad \textbf{(39)}$$

\> *diff*(Ξ(τ), τ$2) − 2·*diff*(H(τ), τ) = *diff*(U, Ξ(τ))

$$\frac{d^2}{d\tau^2}\,\Xi(\tau) - 2\,\frac{d}{d\tau}\,H(\tau) = \Xi(\tau) - \frac{0.4995233078\,(2\,\Xi(\tau) + 0.001906769030)}{\left((\Xi(\tau) + 0.0009533845150)^2 + H(\tau)^2 + Z(\tau)^2\right)^{3/2}} - \frac{0.0004766922575\,(2\,\Xi(\tau) - 1.998093231)}{\left((\Xi(\tau) - 0.9990466155)^2 + H(\tau)^2 + Z(\tau)^2\right)^{3/2}} \quad \textbf{(40)}$$

\> *diff*(H(τ), τ$2) + 2·*diff*(Ξ(τ), τ) = *diff*(U, H(τ))

$$\frac{d^2}{d\tau^2}\,H(\tau) + 2\,\frac{d}{d\tau}\,\Xi(\tau) = H(\tau) - \frac{0.9990466155\,H(\tau)}{\left((\Xi(\tau) + 0.0009533845150)^2 + H(\tau)^2 + Z(\tau)^2\right)^{3/2}} - \frac{0.0009533845150\,H(\tau)}{\left((\Xi(\tau) - 0.9990466155)^2 + H(\tau)^2 + Z(\tau)^2\right)^{3/2}} \quad \textbf{(41)}$$

\> *diff*(Z(τ), τ$2) = *diff*(U, Z(τ))

$$\frac{d^2}{d\tau^2} Z(\tau) = -\frac{0.9990466155\, Z(\tau)}{\left(\left(\Xi(\tau) + 0.0009533845150\right)^2 + H(\tau)^2 + Z(\tau)^2\right)^{3/2}} - \frac{0.0009533845150\, Z(\tau)}{\left(\left(\Xi(\tau) - 0.9990466155\right)^2 + H(\tau)^2 + Z(\tau)^2\right)^{3/2}} \quad \textbf{(42)}$$

\>

\> *sys* := **(40), (41), (42)** :

\> *solTR* := *dsolve*([*sys*, *icsTR*], [Ξ(τ), H(τ), Z(τ)], *numeric*, *output* = *listprocedure*)

$$solTR := \left[\tau = \mathbf{proc}(\tau)\ \ldots\ \mathbf{end\ proc}, \Xi(\tau) = \mathbf{proc}(\tau)\ \ldots\ \mathbf{end\ proc}, \frac{d}{d\tau}\Xi(\tau) = \mathbf{proc}(\tau)\ \ldots\ \mathbf{end\ proc}, H(\tau) = \mathbf{proc}(\tau)\ \ldots\ \mathbf{end\ proc}, \frac{d}{d\tau}H(\tau) = \mathbf{proc}(\tau)\ \ldots\ \mathbf{end\ proc}, Z(\tau) = \mathbf{proc}(\tau)\ \ldots\ \mathbf{end\ proc}, \frac{d}{d\tau}Z(\tau) = \mathbf{proc}(\tau)\ \ldots\ \mathbf{end\ proc}\right] \quad \textbf{(43)}$$

\> *rhs*(*solTR*[2](0))

$$0.499046615500000 \quad \textbf{(44)}$$

\> *rhs*(*solTR*[4](0))

$$1.02583800200000 \quad \textbf{(45)}$$

\> *rhs*(*solTR*[6](0))

$$0. \quad \textbf{(46)}$$

\>

\> *solGR* := *dsolve*([*sys*, *icsGR*], [Ξ(τ), H(τ), Z(τ)], *numeric*, *output* = *listprocedure*)

$$solGR := \left[\tau = \mathbf{proc}(\tau)\ \ldots\ \mathbf{end\ proc}, \Xi(\tau) = \mathbf{proc}(\tau)\ \ldots\ \mathbf{end\ proc}, \frac{d}{d\tau}\Xi(\tau) = \mathbf{proc}(\tau)\ \ldots\ \mathbf{end\ proc}, H(\tau) = \mathbf{proc}(\tau)\ \ldots\ \mathbf{end\ proc}, \frac{d}{d\tau}H(\tau) = \mathbf{proc}(\tau)\ \ldots\ \mathbf{end\ proc}, Z(\tau) = \mathbf{proc}(\tau)\ \ldots\ \mathbf{end\ proc}, \frac{d}{d\tau}Z(\tau) = \mathbf{proc}(\tau)\ \ldots\ \mathbf{end\ proc}\right] \quad \textbf{(47)}$$

\> *rhs*(*solGR*[2](0))

$$0.499046615500000 \quad \textbf{(48)}$$

\> *rhs*(*solGR*[4](0))

$$-0.888421462200000 \quad \textbf{(49)}$$

> $rhs(solGR[6](0))$

$$0. \tag{50}$$

>

Αρχικές Αποστάσεις από τον Ήλιο (ΑΔΙΑΣΤΑΤΕΣ):

> $distTR := \mathrm{sqrt}\left(\left(rhs(icsTR[1]) + \mu_{\Delta}\right)^2 + (rhs(icsTR[3]))^2 + (rhs(icsTR[5]))^2\right)$

$$distTR := 1.141202702 \tag{51}$$

> $distGR := \mathrm{sqrt}\left(\left(rhs(icsGR[1]) + \mu_{\Delta}\right)^2 + (rhs(icsGR[3]))^2 + (rhs(icsGR[5]))^2\right)$

$$distGR := 1.019457058 \tag{52}$$

Αποστάσεις σε κάθε χρόνο από τον Ήλιο (ΑΔΙΑΣΤΑΤΕΣ):

> $distTR := \mathrm{sqrt}\left(\left(rhs(solTR[2](\tau)) + \mu_{\Delta}\right)^2 + \left(rhs(solTR[4](\tau))\right)^2 + \left(rhs(solTR[6](\tau))\right)^2\right):$

> $distGR := \mathrm{sqrt}\left(\left(rhs(solGR[2](\tau)) + \mu_{\Delta}\right)^2 + \left(rhs(solGR[4](\tau))\right)^2 + \left(rhs(solGR[6](\tau))\right)^2\right):$

>

> $\mathrm{sqrt}\left(\left(rhs(solTR[2](25)) + \mu_{\Delta}\right)^2 + (rhs(solTR[4](25)))^2 + (rhs(solTR[6](25)))^2\right)$

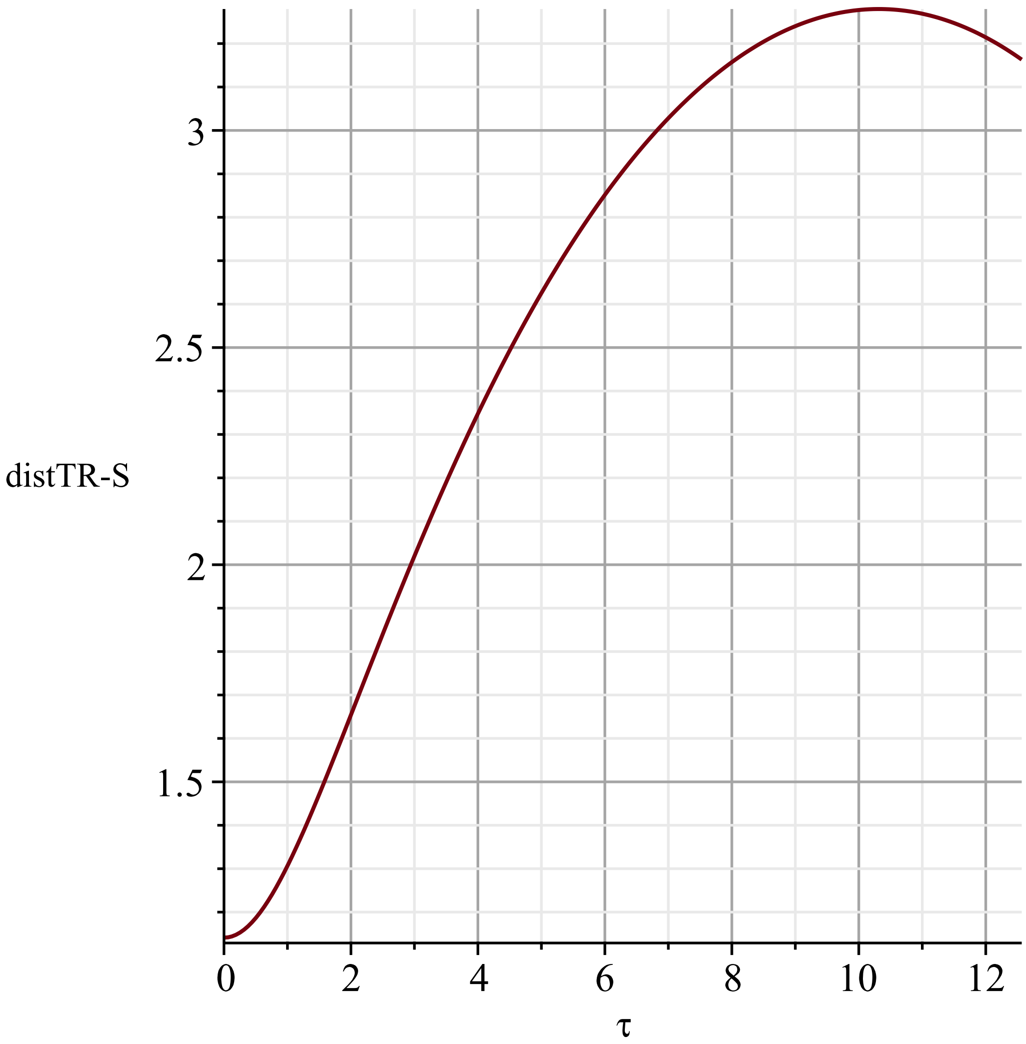
$$2.48159426310303 \tag{53}$$

> $\mathrm{sqrt}\left(\left(rhs(solGR[2](25)) + \mu_{\Delta}\right)^2 + (rhs(solGR[4](25)))^2 + (rhs(solGR[6](25)))^2\right)$

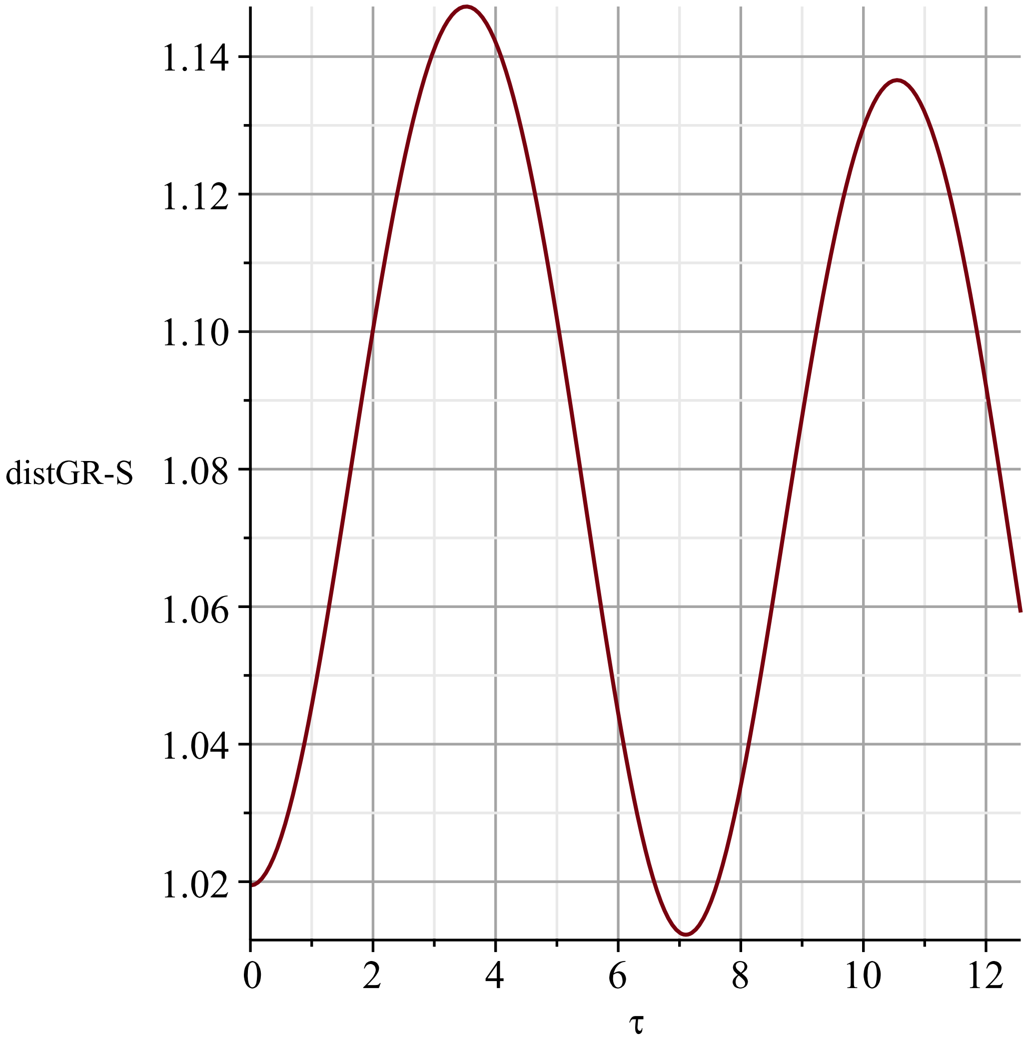
$$1.12261032673163 \tag{54}$$

>

> $plot([\tau, distTR, \tau = 0..4 \cdot \mathrm{Pi}], labels = [\tau, \text{"distTR−S"}], gridlines)$



> *plot*([τ, *distGR*, τ = 0 ..4 · Pi], *labels* = [τ, "distGR−S"], *gridlines*)



>

ΣΥΝΤΕΤΑΓΜΕΝΕΣ ΣΤΟ ΠΕΡΙΣΤΡΕΦΟΜΕΝΟ ΣΥΣΤΗΜΑ :

> $L1 := [0.9323772470, 0, 0]$:

> $L2 := [1.068818779, 0, 0]$:

> $L3 := [-1.000397244, 0, 0]$:

> $L4 := [0.4990466155, 0.8660254040, 0.]$:

> $L5 := [0.4990466155, -0.8660254040, 0.]$:

> $Soleil := [\mu_{\Delta}, 0, 0]$:

> $Jupiter := [\mu_{H}, 0, 0]$:

> **ARSHMEIOTR** := [**rhs**(**icsTR**[**1**]), **rhs**(**icsTR**[**3**]), **rhs**(**icsTR**[**5**])]

$$ARSHMEIOTR := [0.4990466155, 1.025838002, 0] \quad (55)$$

> **ARSHMEIOGR** := [**rhs**(**icsGR**[**1**]), **rhs**(**icsGR**[**3**]), **rhs**(**icsGR**[**5**])]

$$ARSHMEIOGR := [0.4990466155, -0.8884214622, 0] \quad (56)$$

> *animARSHMEIOTR* := [*rhs*(*solTR*[2](τ)), *rhs*(*solTR*[4](τ)), *rhs*(*solTR*[6](τ))] :

> *animARSHMEIOGR* := [*rhs*(*solGR*[2](τ)), *rhs*(*solGR*[4](τ)), *rhs*(*solGR*[6](τ))] :

>

ΜΕΤΑΤΡΟΠΗ ΣΥΝΤΕΤΑΓΜΕΝΩΝ

ΑΠΟ ΤΟ ΠΕΡΙΣΤΡΕΦΟΜΕΝΟ ΣΥΣΤΗΜΑ ΣΤΟ ΑΔΡΑΝΕΙΑΚΟ ΣΥΣΤΗΜΑ ΣΥΝΤΕΤΑΓΜΕΝΩΝ .

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \cos(\tau) & -\sin(\tau) \\ \sin(\tau) & \cos(\tau) \end{bmatrix} \cdot \begin{bmatrix} \Xi \\ \mathrm{H} \end{bmatrix}$$

> $XL1 := \cos(\tau) \cdot L1[1] - \sin(\tau) \cdot L1[2]$

$$XL1 := 0.9323772470 \cos(\tau) \quad (57)$$

> $YL1 := \sin(\tau) \cdot L1[1] + \cos(\tau) \cdot L1[2]$

$$YL1 := 0.9323772470 \sin(\tau) \quad (58)$$

>

> $XL2 := \cos(\tau) \cdot L2[1] - \sin(\tau) \cdot L2[2]$

$$XL2 := 1.068818779 \cos(\tau) \quad (59)$$

> $YL2 := \sin(\tau) \cdot L2[1] + \cos(\tau) \cdot L2[2]$

$$YL2 := 1.068818779 \sin(\tau) \quad (60)$$

>

> $XL3 := \cos(\tau) \cdot L3[1] - \sin(\tau) \cdot L3[2]$

$$XL3 := -1.000397244 \cos(\tau) \quad (61)$$

> $YL3 := \sin(\tau) \cdot L3[1] + \cos(\tau) \cdot L3[2]$

$$YL3 := -1.000397244 \sin(\tau) \quad (62)$$

>

> $XL4 := \cos(\tau) \cdot L4[1] - \sin(\tau) \cdot L4[2]$

$$XL4 := 0.4990466155 \cos(\tau) - 0.8660254040 \sin(\tau) \quad (63)$$

> $YL4 := \sin(\tau) \cdot L4[1] + \cos(\tau) \cdot L4[2]$

$$YL4 := 0.4990466155 \sin(\tau) + 0.8660254040 \cos(\tau) \quad (64)$$

>

> $XL5 := \cos(\tau) \cdot L5[1] - \sin(\tau) \cdot L5[2]$

$$XL5 := 0.4990466155 \cos(\tau) + 0.8660254040 \sin(\tau) \quad (65)$$

> $YL5 := \sin(\tau) \cdot L5[1] + \cos(\tau) \cdot L5[2]$

$$YL5 := 0.4990466155 \sin(\tau) - 0.8660254040 \cos(\tau) \quad (66)$$

>

> $XSoleil := \cos(\tau) \cdot Soleil[1] - \sin(\tau) \cdot Soleil[2]$

$$XSoleil := 0.0009533845150 \cos(\tau) \quad (67)$$

> $YSoleil := \sin(\tau) \cdot Soleil[1] + \cos(\tau) \cdot Soleil[2]$

$$YSoleil := 0.0009533845150 \sin(\tau) \quad (68)$$

>

> $XJupiter := \cos(\tau) \cdot Jupiter[1] - \sin(\tau) \cdot Jupiter[2]$

$$XJupiter := 0.9990466155 \cos(\tau) \quad (69)$$

> $YJupiter := \sin(\tau) \cdot Jupiter[1] + \cos(\tau) \cdot Jupiter[2]$

$$YJupiter := 0.9990466155 \sin(\tau) \quad (70)$$

> $XARSHMEIOTR := \cos(\tau) \cdot ARSHMEIOTR[1] - \sin(\tau) \cdot ARSHMEIOTR[2]$:

> $YARSHMEIOTR := \sin(\tau) \cdot ARSHMEIOTR[1] + \cos(\tau) \cdot ARSHMEIOTR[2]$:

> $ZARSHMEIOTR := ARSHMEIOTR[3]$:

>

> $XARSHMEIOGR := \cos(\tau) \cdot ARSHMEIOGR[1] - \sin(\tau) \cdot ARSHMEIOGR[2]$:

> $YARSHMEIOGR := \sin(\tau) \cdot ARSHMEIOGR[1] + \cos(\tau) \cdot ARSHMEIOGR[2]$:

> $ZARSHMEIOGR := ARSHMEIOGR[3]$:

>

> $XanimARSHMEIOTR := \cos(\tau) \cdot animARSHMEIOTR[1] - \sin(\tau) \cdot animARSHMEIOTR[2]$:

> $YanimARSHMEIOTR := \sin(\tau) \cdot animARSHMEIOTR[1] + \cos(\tau) \cdot animARSHMEIOTR[2]$:

> $ZanimARSHMEIOTR := animARSHMEIOTR[3]$:

>

> $XanimARSHMEIOGR := \cos(\tau) \cdot animARSHMEIOGR[1] - \sin(\tau) \cdot animARSHMEIOGR[2]$:

> $YanimARSHMEIOGR := \sin(\tau) \cdot animARSHMEIOGR[1] + \cos(\tau) \cdot animARSHMEIOGR[2]$:

> $ZanimARSHMEIOGR := animARSHMEIOGR[3]$:

>

ΑΠΕΙΚΟΝΙΣΕΙΣ ΣΤΟ ΑΔΡΑΝΕΙΑΚΟ ΣΥΣΤΗΜΑ ΣΥΝΤΕΤΑΓΜΕΝΩΝ .

>

> $P1 := [XL1, YL1, 0]$:

> $P2 := [XL2, YL2, 0]$:

> $P3 := [XL3, YL3, 0]$:

> $P4 := [XL4, YL4, 0]$:

> $P5 := [XL5, YL5, 0]$:

> $P6 := [XSoleil, YSoleil, 0]$:

> $P7 := [XJupiter, YJupiter, 0]$:

> $points := [P1, P2, P3, P4, P5, P6, P7]$:

> $animP := animate(pointplot3d, [points, color = [green, yellow, olive, maroon, coral, blue, red], symbol = solidcircle, symbolsize = 10], \tau = 0..2 \cdot \mathrm{Pi}, frames = 10, trace = 0)$:

> $T1 := [XL1, YL1 - 0.1, 0, "L1"]$:

> $T2 := [XL2, YL2 - 0.1, 0, "L2"]$:

> $T3 := [XL3, YL3 - 0.1, 0, "L3"]$:

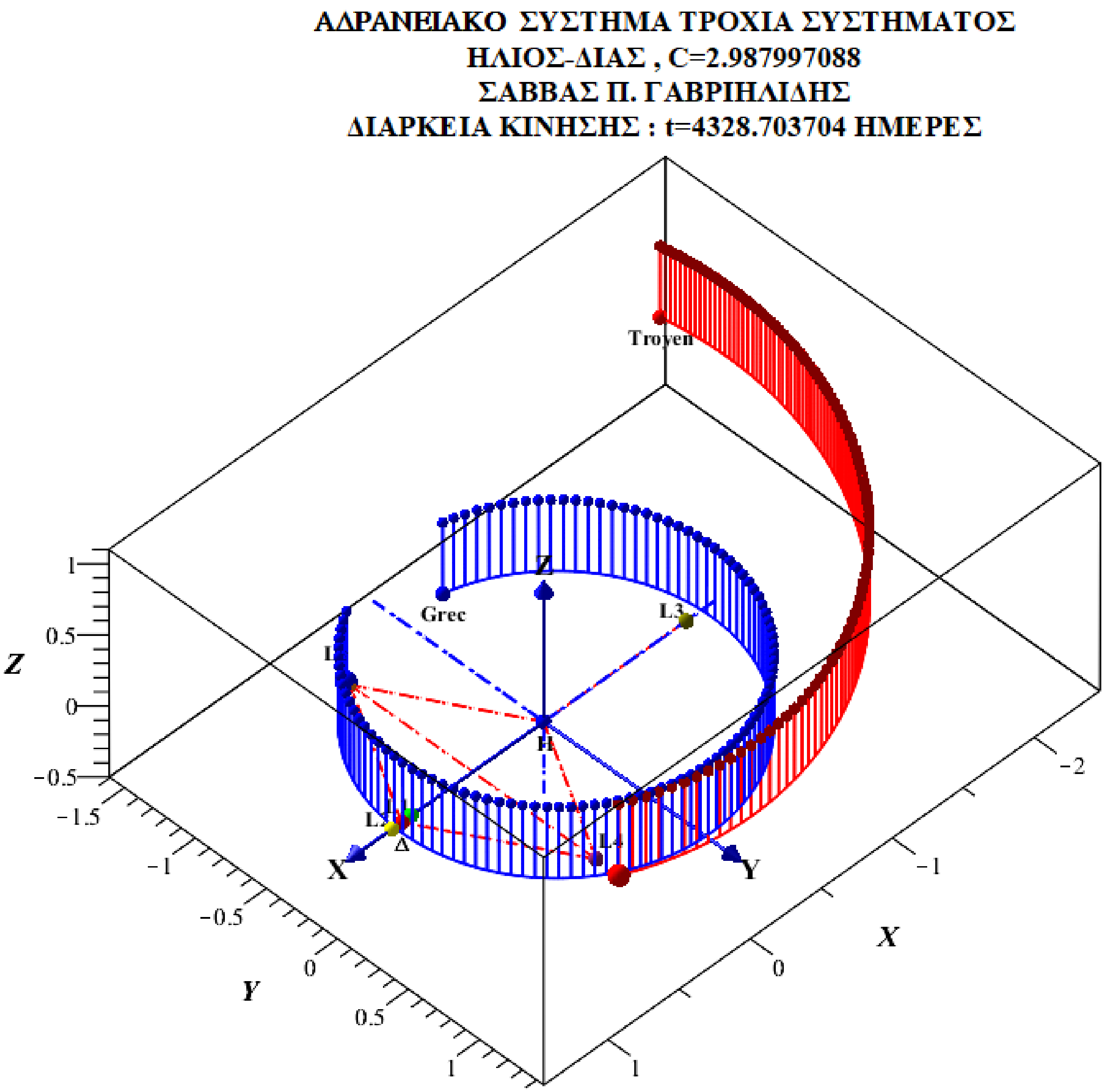
> $T4 := [XL4, YL4 + 0.1, 0.2, "L4"]$:

> $T5 := [XL5, YL5 - 0.1, 0.15, "L5"]$:

```
> T6 := [XSoleil + 0.10, YSoleil + 0.10, 0, "H"] :
> T7 := [XJupiter + 0.10, YJupiter + 0.10, 0, "Δ"] :
> T := [T1, T2, T3, T4, T5, T6, T7] :
> animT := animate(textplot3d, [T, font = [arial, bold, 10]], τ = 0 ..2·Pi, frames = 10, trace
      = 0) :
> ARXH := pointplot3d([0, 0, 0], color = yellow, symbol = solidcircle, symbolsize = 5) :
>
> axonX := spacecurve([x, 0, 0], x =-1.2 ..0.0, color = blue, thickness = 2, linestyle = 4) :
> axonY := spacecurve([0, y, 0], y =-1.2 ..0.0, color = blue, thickness = 2, linestyle = 4) :
> axonZ := spacecurve([0, 0, z], z =-0.5 ..0.0, color = blue, thickness = 2, linestyle = 4) :
>
> axX := arrow(⟨1.4, 0, 0⟩, width = 0.02, head_length = 0.1, head_width = 0.1, color = blue) :

> axY := arrow(⟨0, 1.4, 0⟩, width = 0.02, head_length = 0.1, head_width = 0.1, color = blue) :
> axZ := arrow(⟨0, 0, 1⟩, width = 0.02, head_length = 0.1, head_width = 0.1, color = blue) :
> TaxX := textplot3d([1.45, 0, 0, "X"], font = [arial, bold, 14]) :
> TaxY := textplot3d([0, 1.45, 0, "Y"], font = [arial, bold, 14]) :
> TaxZ := textplot3d([0, 0, 1.10, "Z"], font = [arial, bold, 14]) :
> line1 := animate(spacecurve, [[XL3 + λ·(XL2 − XL3), YL3 + λ·(YL2 − YL3), 0], λ = 0 ..1,
      color = red, linestyle = 4], τ = 0 ..2·Pi, frames = 10, trace = 0) :
> line2 := animate(spacecurve, [[XL4 + λ·(XL5 − XL4), YL4 + λ·(YL5 − YL4), 0], λ = 0 ..1,
      color = red, linestyle = 4], τ = 0 ..2·Pi, frames = 10, trace = 0) :
> line3 := animate(spacecurve, [[XSoleil + λ·(XL4 − XSoleil), YSoleil + λ·(YL4 − YSoleil),
      0], λ = 0 ..1, color = red, linestyle = 4], τ = 0 ..2·Pi, frames = 10, trace = 0) :
> line4 := animate(spacecurve, [[XSoleil + λ·(XL5 − XSoleil), YSoleil + λ·(YL5 − YSoleil),
      0], λ = 0 ..1, color = red, linestyle = 4], τ = 0 ..2·Pi, frames = 10, trace = 0) :
> line5 := animate(spacecurve, [[XJupiter + λ·(XL4 − XJupiter), YJupiter + λ·(YL4
      − YJupiter), 0], λ = 0 ..1, color = red, linestyle = 4], τ = 0 ..2·Pi, frames = 10, trace = 0) :
> line6 := animate(spacecurve, [[XJupiter + λ·(XL5 − XJupiter), YJupiter + λ·(YL5
      − YJupiter), 0], λ = 0 ..1, color = red, linestyle = 4], τ = 0 ..2·Pi, frames = 10, trace = 0) :
>
> display(axX, axY, axZ, TaxX, TaxY, TaxZ, ARXH, axonX, axonY, axonZ, animP, animT,
      line1, line2, line3, line4, line5, line6, title
      = "ΑΔΡΑΝΕΙΑΚΟ  ΣΥΣΤΗΜΑ ΤΡΟΧΙΑ ΣΥΣΤΗΜΑΤΟΣ\nΗΛΙΟΣ−ΔΙΑΣ , C=
      2.987997088\nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ\nΔΙΑΡΚΕΙΑ ΚΙΝΗΣΗΣ : t=4328.703704
      ΗΜΕΡΕΣ", titlefont = [arial, 14, bold], labels = [X, Y, Z], labelfont = [arial, 14, bold],
      orientation = [45, 45, 0], axes = boxed, scaling = constrained) :
> SYNOLO := display(axX, axY, axZ, TaxX, TaxY, TaxZ, ARXH, axonX, axonY, axonZ,
      animP, animT, line1, line2, line3, line4, line5, line6, title
      = "ΑΔΡΑΝΕΙΑΚΟ  ΣΥΣΤΗΜΑ ΤΡΟΧΙΑ ΣΥΣΤΗΜΑΤΟΣ\nΗΛΙΟΣ−ΔΙΑΣ , C=
      2.987997088\nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ\nΔΙΑΡΚΕΙΑ ΚΙΝΗΣΗΣ : t=4328.703704
      ΗΜΕΡΕΣ", titlefont = [arial, 14, bold], labels = [X, Y, Z], labelfont = [arial, 14, bold],
      orientation = [45, 45, 0], axes = boxed, scaling = constrained) :
> TROYEN := pointplot3d(ARSHMEIOTR, color = red, symbol = solidcircle, symbolsize
      = 15) :
> GREC := pointplot3d(ARSHMEIOGR, color = blue, symbol = solidcircle, symbolsize
      = 15) :
> TROXIATR := spacecurve([XanimARSHMEIOTR, YanimARSHMEIOTR,
```

```
ZanimARSHMEIOTR], τ=0..2·Pi, color=red, numpoints=1000) :
> TROXIAGR := spacecurve([XanimARSHMEIOGR, YanimARSHMEIOGR,
    ZanimARSHMEIOGR], τ=0..2·Pi, color=blue, numpoints=1000) :
> ANIMTR := animate(pointplot3d, [[XanimARSHMEIOTR, YanimARSHMEIOTR,
    ZanimARSHMEIOTR], color=red, symbol=solidcircle, symbolsize=10], τ=0..2·Pi,
    frames=10 , trace=0) :
> ANIMGR := animate(pointplot3d, [[XanimARSHMEIOGR, YanimARSHMEIOGR,
    ZanimARSHMEIOGR], color=blue, symbol=solidcircle, symbolsize=10], τ=0..2·Pi,
    frames=10, trace=0) :
> TR := [XanimARSHMEIOTR + 0.1, YanimARSHMEIOTR + 0.1, ZanimARSHMEIOTR,
    "Troyen"], font=[arial, bold, 10] :
> GR := [XanimARSHMEIOGR + 0.1, YanimARSHMEIOGR + 0.1, ZanimARSHMEIOGR,
    "Grec"], font=[arial, bold, 10] :
> textTR := animate(textplot3d, [TR], τ=0..2·Pi, frames=10, trace=0) :
> textGR := animate(textplot3d, [GR], τ=0..2·Pi, frames=10, trace=0) :
>
> line7 := animate(spacecurve, [[XanimARSHMEIOTR, YanimARSHMEIOTR,
    ZanimARSHMEIOTR + λ·0.5], λ=0..1, color=red, linestyle=1], τ=0..2·Pi, frames
    =10, trace=1) :
> line8 := animate(spacecurve, [[XanimARSHMEIOGR, YanimARSHMEIOGR,
    ZanimARSHMEIOGR + λ·0.5], λ=0..1, color=blue, linestyle=1], τ=0..2·Pi, frames
    =10, trace=1) :
> ANIMTR1 := animate(pointplot3d, [[XanimARSHMEIOTR, YanimARSHMEIOTR,
    ZanimARSHMEIOTR + 0.5], color=red, symbol=solidbox, symbolsize=5], τ=0..2
    ·Pi, frames=10 , trace=1) :
> ANIMGR1 := animate(pointplot3d, [[XanimARSHMEIOGR, YanimARSHMEIOGR,
    ZanimARSHMEIOGR + 0.5], color=blue, symbol=solidcircle, symbolsize=7], τ=0..2
    ·Pi, frames=10, trace=1) :
>
> display(SYNOLO, TROYEN, GREC, TROXIATR, TROXIAGR, ANIMTR, ANIMGR, textTR,
    textGR, line7, line8, ANIMTR1, ANIMGR1) :
> SYNOLO1 := display(SYNOLO, TROYEN, GREC, TROXIATR, TROXIAGR, ANIMTR,
    ANIMGR, textTR, textGR) :
```



>

> $a := [1.4, 1.4925, 1.49399854, 1.4999985438, 1.501483, 1.5060735567, 1.526, 1.586080083, 1.589, 1.5941703866, 1.63343]$

$a := [1.4, 1.4925, 1.49399854, 1.4999985438, 1.501483, 1.5060735567, 1.526, 1.586080083, 1.589, 1.5941703866, 1.63343]$ **(71)**

> $C := 2 \cdot a$

$C := [2.8, 2.9850, 2.98799708, 2.999997088, 3.002966, 3.012147114, 3.052, 3.172160166, 3.178, 3.188340774, 3.26686]$ **(72)**

>

> $$UA := X^2 + Y^2 + \frac{2 \cdot 0.9990467000}{\sqrt{(X + 0.0009533000000 \cos(\tau))^2 + (Y + 0.0009533000000 \sin(\tau))^2 + Z^2}} + \frac{2 \cdot 0.0009533000000}{\sqrt{(X - 0.9990467000 \cos(\tau))^2 + (Y - 0.9990467000 \sin(\tau))^2 + Z^2}} - C[3] = 0 :$$

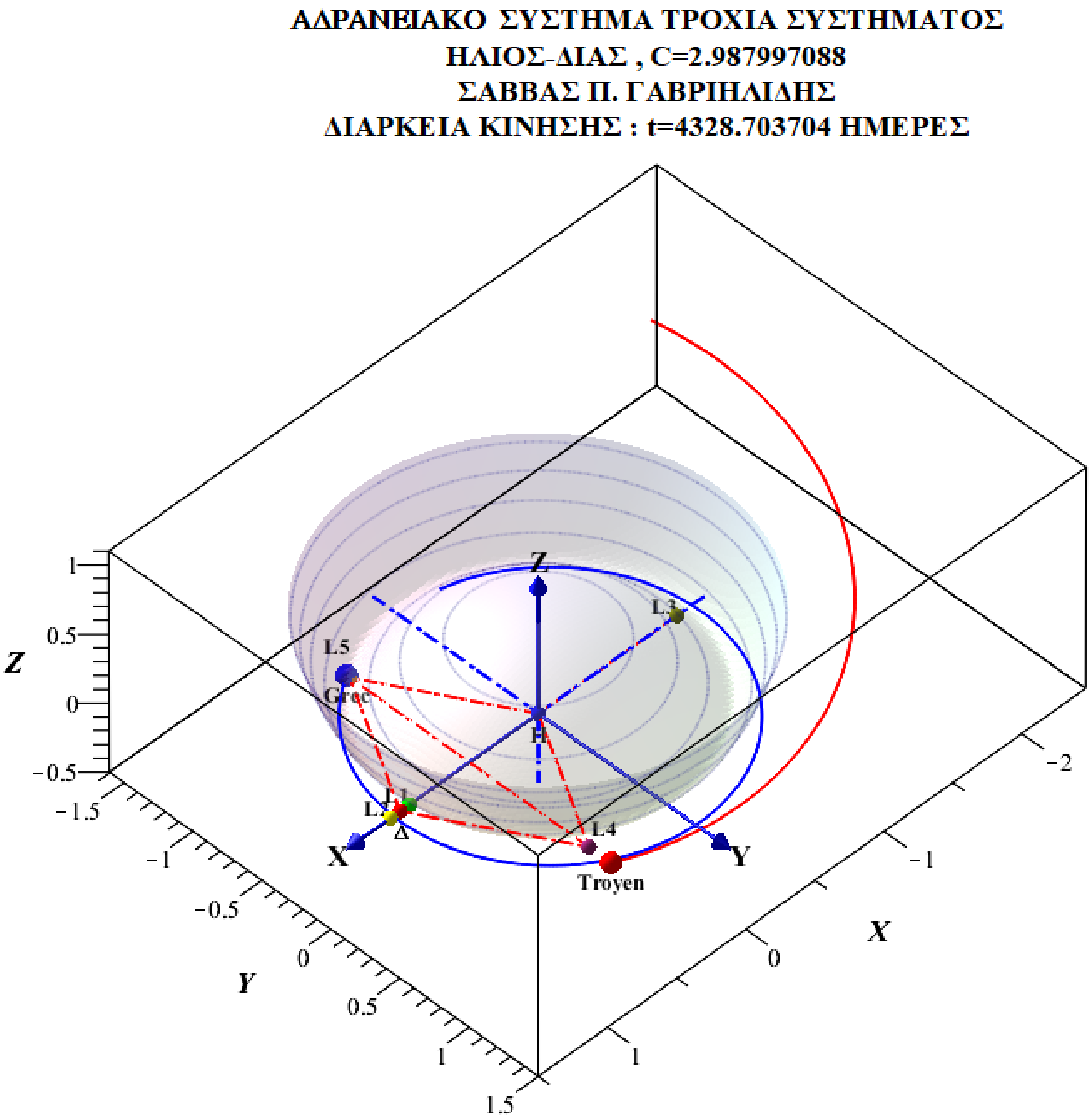
>

> $animUA := animate(implicitplot3d, [UA, X=-1.5..1.5, Y=-1.5..1.5, Z=0..0.75, style = surfacecontour, numpoints = 100, transparency = 0.750], \tau = 0..2\cdot \mathrm{Pi}, frames = 2)$:

>

> $display(SYNOLO1, animUA)$:

>



>

ΤΡΟΧΙΕΣ ΣΤΟ ΠΕΡΙΣΤΡΕΦΟΜΕΝΟ ΣΥΣΤΗΜΑ (Ξ,Η,Ζ) .

> $ARXIKOSHMEIOTR := pointplot3d([rhs(icsTR[1]), rhs(icsTR[3]), rhs(icsTR[5])], color = red, symbol = solidcircle, symbolsize = 20)$:

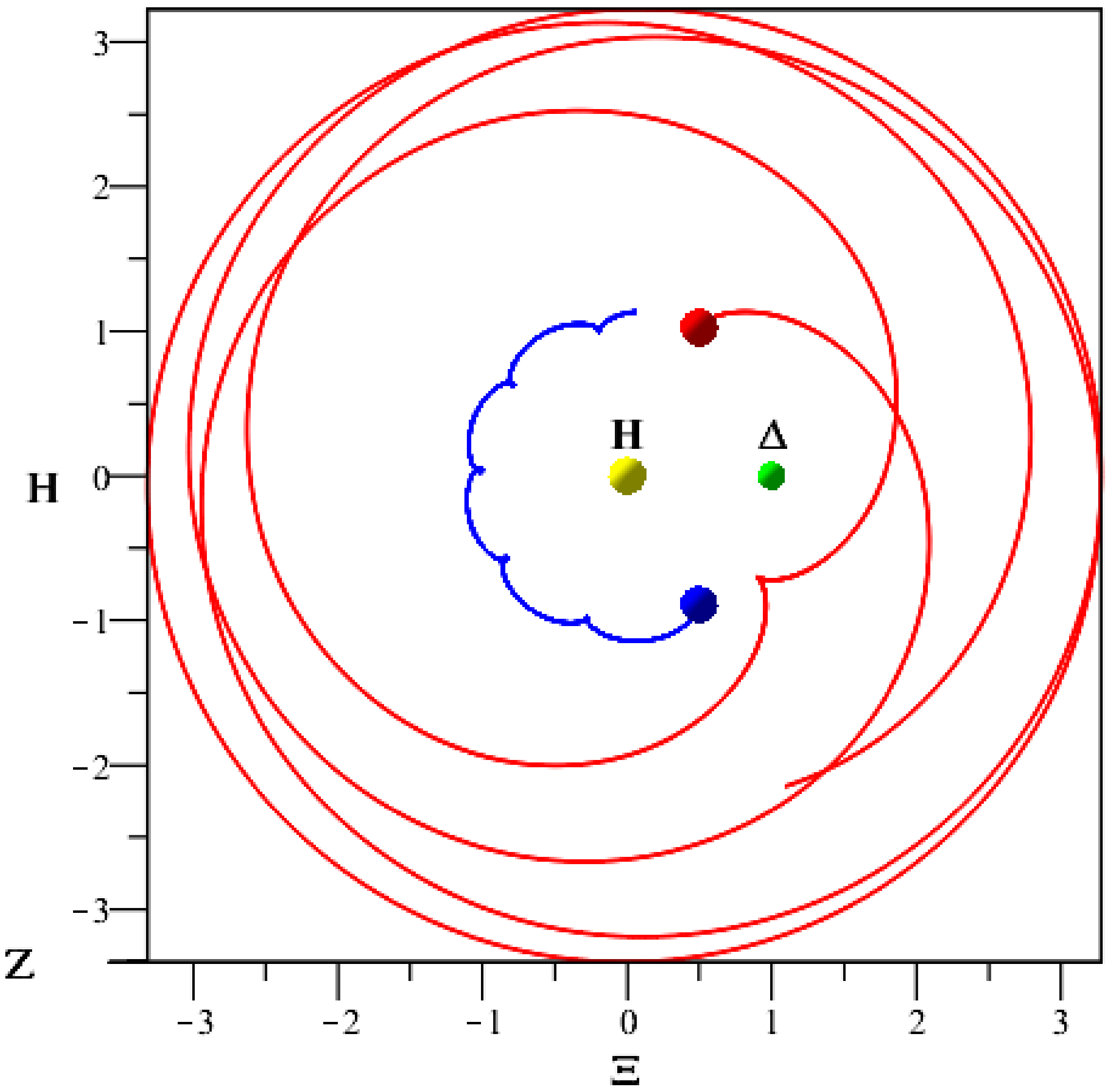
> $ARXIKOSHMEIOGR := pointplot3d([rhs(icsGR[1]), rhs(icsGR[3]), rhs(icsGR[5])], color = blue, symbol = solidcircle, symbolsize = 20)$:

> $TROXTR := spacecurve([rhs(solTR[2](\tau)), rhs(solTR[4](\tau)), rhs(solTR[6](\tau))], \tau = 0..12\cdot \mathrm{Pi}, color = red, labels = [\Xi, \mathrm{H}, \mathrm{Z}], labelfont = [arial, bold, 14], numpoints = 100)$:

> $animTR := animate(pointplot3d, [[rhs(solTR[2](\tau)), rhs(solTR[4](\tau)), rhs(solTR[6](\tau))], \mathrm{color} = \mathrm{red}, \mathrm{symbol} = \mathrm{solidcircle}, \mathrm{symbolsize} = 10], \tau = 0..12\cdot \mathrm{Pi},$

```
    frames = 2, trace = 2) :
> TROXGR := spacecurve([rhs(solGR[2](τ)), rhs(solGR[4](τ)), rhs(solGR[6](τ))], τ
     = 0 ..12·Pi, color = blue, labels = [Ξ, H, Z], labelfont = [arial, bold, 14], numpoints
     = 100) :
> animGR := animate(pointplot3d, [[rhs(solGR[2](τ)), rhs(solGR[4](τ)),
    rhs(solGR[6](τ))], color = blue, symbol = solidcircle, symbolsize = 10], τ = 0 ..12·Pi,
    frames = 2, trace = 2) :
> display(ARXIKOSHMEIOTR, ARXIKOSHMEIOGR, TROXTR, animTR, TROXGR, animGR,
    title
     = "ΤΡΟΧΙΕΣ ΣΤΟ ΠΕΡΙΣΤΡΕΦΟΜΕΝΟ ΣΥΣΤΗΜΑ\nΣΑΒΒΑΣ Π.
    ΓΑΒΡΙΗΛΙΔΗΣ\nΔΙΑΡΚΕΙΑ ΚΙΝΗΣΗΣ :71.10958956 ΧΡΟΝΙΑ", titlefont = [arial, bold,
    14], scaling = constrained) :
> HLIOS := pointplot3d(Soleil, color = yellow, symbol = solidcircle, symbolsize = 20) :
> DIAS := pointplot3d(Jupiter, color = green, symbol = solidcircle, symbolsize = 15) :
> THLIOS := textplot3d([μ_Δ, 0 + 0.3, 0, "Η"], font = [arial, bold, 14]) :
> TDIAS := textplot3d([μ_Η, 0 + 0.3, 0, "Δ"], font = [arial, bold, 14]) :
> display(ARXIKOSHMEIOTR, ARXIKOSHMEIOGR, TROXTR, HLIOS, DIAS, THLIOS,
    TDIAS, animTR, TROXGR, animGR, title
     = "ΠΡΟΒΟΛΗ ΣΤΟ ΕΠΙΠΕΔΟ (Ξ0Η)\nΤΡΟΧΙΕΣ ΣΤΟ ΠΕΡΙΣΤΡΕΦΟΜΕΝΟ
    ΣΥΣΤΗΜΑ\nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ\nΔΙΑΡΚΕΙΑ ΚΙΝΗΣΗΣ :71.10958956
    ΧΡΟΝΙΑ", titlefont = [arial, bold, 14], scaling = constrained, orientation = [-90, 0, 0]) :
```

ΠΡΟΒΟΛΗ ΣΤΟ ΕΠΙΠΕΔΟ (Ξ0Η)
ΤΡΟΧΙΕΣ ΣΤΟ ΠΕΡΙΣΤΡΕΦΟΜΕΝΟ ΣΥΣΤΗΜΑ
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ
ΔΙΑΡΚΕΙΑ ΚΙΝΗΣΗΣ :71.10958956 ΧΡΟΝΙΑ



>