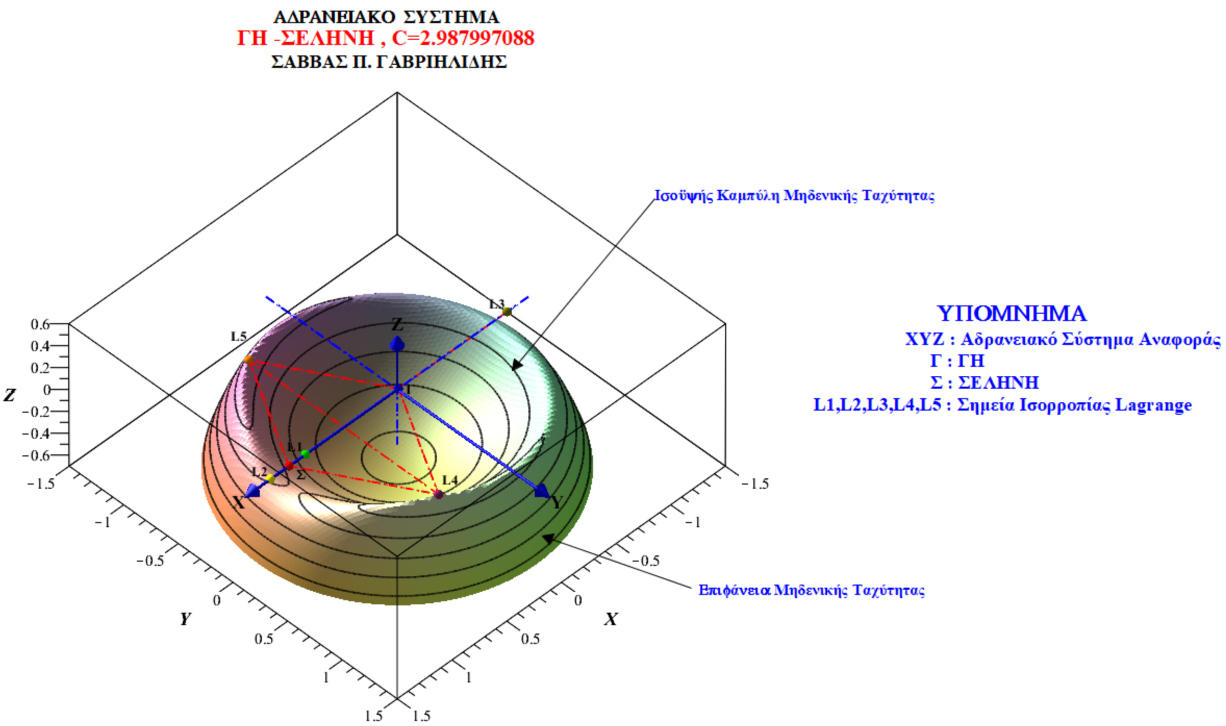
>



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ΣΥΣΤΗΜΑ-ΓΗ-ΣΕΛΗΝΗ . ΣΗΜΕΙΑ ΙΣΟΡΡΟΠΙΑΣ LAGRANGE .

Περιστρεφόμενο Σύστημα Συντεταγμένων (ΞΗΖ) .!!!!!! Αδιαστατοποιημένα μεγέθη .

Διαφορικές Εξισώσεις κίνησης του ΤΡΙΤΟΥ ΣΩΜΑΤΟΣ ως προς το Περιστρεφόμενο Σύστημα (ΩΞΗΖ) γράφονται :

$$\ddot{\Xi}(\tau) - 2\cdot\dot{H}(\tau) = \frac{\partial}{\partial \Xi(t)} U$$

$$\ddot{H}(\tau) + 2\cdot\dot{\Xi}(\tau) = \frac{\partial}{\partial H(\tau)} U$$

$$\ddot{Z}(\tau) = \frac{\partial}{\partial Z(\tau)} U$$

ΔΥΝΑΜΙΚΗ ΣΥΝΑΡΤΗΣΗ U . $$U := \frac{\Xi(\tau)^2}{2} + \frac{H(\tau)^2}{2} + \frac{1-\mu}{\sqrt{(\Xi(\tau)+\mu)^2 + H(\tau)^2 + Z(\tau)^2}} + \frac{\mu}{\sqrt{(\Xi(\tau)-(1-\mu))^2 + H(\tau)^2 + Z(\tau)^2}}$$

Πολλαπλασιάζοντας τις παραπάνω εξισώσεις αντίστοιχα επί $2\cdot\frac{d}{d\tau}\Xi(\tau), 2\cdot\frac{d}{d\tau}H(\tau), 2\cdot\frac{d}{d\tau}Z(\tau)$ και προσθέτοντας κατά μέλη και ολοκληρώνοντας έχουμε το **ολοκλήρωμα Jacobi** : $v^2 = 2\cdot U - C$

όπου **v** το μέτρο της ταχύτητας του ΤΡΙΤΟΥ ΣΩΜΑΤΟΣ ,U η Δυναμική Συνάρτηση και C Σταθερά ολοκλήρωσης (Ενέργειας) .

Άρα η κίνηση του τρίτου σώματος είναι δυνατή ΜΟΝΟ σε σημεία του χώρου γιά τα οποία ικανοποιείται η σχέση : $2\cdot U - C \geq 0$.

ΣΗΜΕΙΑ ΙΣΟΡΡΟΠΙΑΣ

Ορίζονται σαν τα σημεία εκείνα στα οποία ΤΟΠΟΘΕΤΟΥΜΕΝΟ με μηδενική ταχύτητα v το τρίτο σώμα θα μείνει ΑΚΙΝΗΤΟ λόγω "εξισορρόπησης" των έλξεων των ΔΥΟ άλλων σωμάτων .

Δηλαδή θα ισχύει : $\dot{\Xi}(\tau) = 0, \dot{H}(\tau) = 0, \dot{Z}(\tau) = 0, \ddot{\Xi}(\tau) = 0, \ddot{H}(\tau) = 0, \ddot{Z}(\tau) = 0$.

ΑΡΑ γιά τα Σημεία Ισορροπίας θα έχουμε : $\frac{\partial}{\partial \Xi(\tau)} U = 0 , \frac{\partial}{\partial H(\tau)} U = 0 , \frac{\partial}{\partial Z(\tau)} U = 0$.

>

$\Downarrow$
μ

Objets	M1	M2	mu
1 sol-mer	1.99E30	3.30E23	1.66E-7
2 sat-dio	5.68E26	1.05E21	1.84E-6
⇨ 3 sol-ter	1.99E30	5.97E24	3.00E-6
4 sat-tit	5.68E26	1.34E23	2.35E-4
5 sol-sat	1.99E30	5.68E26	2.85E-4
⇨ 6 sol-jup	1.99E30	1.90E27	9.53E-4
⇨ 7 ter-lun	5.97E24	7.35E22	1.21E-2
8 plu-cha	1.27E22	1.90E21	1.30E-1

$\mu = 3.224509257\,10^{-7}$ ΗΛΙΟΣ-ΑΡΗΣ

Το μ είναι η Αδιαστατοποιημένη μάζα του πλανήτη (του μικρότερου εκ των δύο (2) σωμάτων του συστήματος).!!!!!!

>

> with(Physics[Vectors])

[&x, `+`, `.`, ChangeBasis, ChangeCoordinates, Component, Curl, DirectionalDiff, Divergence, Gradient, Identify, Laplacian, ∇, Norm, Setup, diff] **(1)**

> Setup(mathematicalnotation = true)

[mathematicalnotation = true] **(2)**

> $\mu := [1.66\cdot 10^{-7}, 1.84\cdot 10^{-6}, 3.00\cdot 10^{-6}, 2.35\cdot 10^{-4}, 2.85\cdot 10^{-4}, \mathbf{9.533\cdot 10^{-4}}, \mathbf{1.2148\cdot 10^{-2}}, 1.30\cdot 10^{-1}, \mathbf{3.224509257\,10^{-7}}, \mathbf{0.5}]$

$\mu := [1.660000000\,10^{-7}, 1.840000000\,10^{-6}, 3.000000000\,10^{-6}, 0.0002350000000, 0.0002850000000, 0.0009533000000, 0.01214800000, 0.1300000000, 3.224509257\,10^{-7}, 0.5]$ **(3)**

> $$U := \frac{1}{2}\Xi^2 + \frac{1}{2}H^2 + \frac{1-\mu[7]}{\sqrt{(\Xi+\mu[7])^2 + H^2 + Z^2}} + \frac{\mu[7]}{\sqrt{(\Xi-(1-\mu[7]))^2 + H^2 + Z^2}}$$

$$U := \frac{\Xi^2}{2} + \frac{H^2}{2} + \frac{0.9878520000}{\sqrt{(\Xi+0.01214800000)^2 + H^2 + Z^2}} + \frac{0.01214800000}{\sqrt{(\Xi-0.9878520000)^2 + H^2 + Z^2}} \quad \textbf{(4)}$$

> $diff(U, \Xi) = 0$

$$\Xi - \frac{0.4939260000\,(2\,\Xi + 0.02429600000)}{((\Xi+0.01214800000)^2 + H^2 + Z^2)^{3/2}} - \frac{0.006074000000\,(2\,\Xi - 1.975704000)}{((\Xi-0.9878520000)^2 + H^2 + Z^2)^{3/2}} = 0 \quad \textbf{(5)}$$

> *diff*(*U*, H) = 0

$$H - \frac{0.9878520000\,H}{\left((\Xi + 0.01214800000)^2 + H^2 + Z^2\right)^{3/2}} - \frac{0.01214800000\,H}{\left((\Xi - 0.9878520000)^2 + H^2 + Z^2\right)^{3/2}} = 0 \qquad (6)$$

> *diff*(*U*, Z) = 0

$$-\frac{0.9878520000\,Z}{\left((\Xi + 0.01214800000)^2 + H^2 + Z^2\right)^{3/2}} - \frac{0.01214800000\,Z}{\left((\Xi - 0.9878520000)^2 + H^2 + Z^2\right)^{3/2}} = 0 \qquad (7)$$

>

> *sol* := *solve*([**(5)**, **(6)**, **(7)**], [Ξ, H, Z])

sol := [[Ξ = 0.8369278491, H = 0., Z = 0.], [Ξ = 1.155672220, H = 0., Z = 0.], [Ξ = −1.005061569, H = 0., Z = 0.], [Ξ = −0.2948290742 + 0.3506388343 I, H = 0., Z = 0.3761024464 + 1.260733219 I], [Ξ = −0.2948290742 + 0.3506388343 I, H = 0., Z = −0.3761024464 − 1.260733219 I], [Ξ = 0.9978986774 − 0.01709062021 I, H = 0., Z = 0.1147295074 + 0.1998751729 I], [Ξ = 0.9978986774 − 0.01709062021 I, H = 0., Z = −0.1147295074 − 0.1998751729 I], [Ξ = 0.9978986774 + 0.01709062021 I, H = 0., Z = 0.1147295074 − 0.1998751729 I], [Ξ = 0.9978986774 + 0.01709062021 I, H = 0., Z = −0.1147295074 + 0.1998751729 I], [Ξ = −0.2948290742 − 0.3506388343 I, H = 0., Z = 0.3761024464 − 1.260733219 I], [Ξ = −0.2948290742 − 0.3506388343 I, H = 0., Z = −0.3761024464 + 1.260733219 I], [Ξ = 0.4878520000, H = 0.8660254038, Z = 0.], [Ξ = 0.4878520000, H = −0.8660254038, Z = 0.]] **(8)**

> *realsol* := [*sol*[1], *sol*[2], *sol*[3], *sol*[12], *sol*[13]]

realsol := [[Ξ = 0.8369278491, H = 0., Z = 0.], [Ξ = 1.155672220, H = 0., Z = 0.], [Ξ = −1.005061569, H = 0., Z = 0.], [Ξ = 0.4878520000, H = 0.8660254038, Z = 0.], [Ξ = 0.4878520000, H = −0.8660254038, Z = 0.]] **(9)**

>

> *L1* := *sol*[1]

L1 := [Ξ = 0.8369278491, H = 0., Z = 0.] **(10)**

> *L2* := *sol*[2]

L2 := [Ξ = 1.155672220, H = 0., Z = 0.] **(11)**

> *L3* := *sol*[3]

L3 := [Ξ = −1.005061569, H = 0., Z = 0.] **(12)**

> *L4* := *sol*[12]

L4 := [Ξ = 0.4878520000, H = 0.8660254038, Z = 0.] **(13)**

> *L5* := *sol*[13]

L5 := [Ξ = 0.4878520000, H = −0.8660254038, Z = 0.] **(14)**

>

>

Ακολουθώντας τα βήματα της θεωρίας .

>

> $subs(\{Z=0, H=0\}, (5))$

$$\Xi - \frac{0.4939260000\,(2\,\Xi + 0.02429600000)}{\left((\Xi + 0.01214800000)^2\right)^{3/2}} - \frac{0.006074000000\,(2\,\Xi - 1.975704000)}{\left((\Xi - 0.9878520000)^2\right)^{3/2}} = 0 \quad \textbf{(15)}$$

> $solve((15), \Xi)$

$$0.8369278491,\ 1.155672220,\ -1.005061569 \quad \textbf{(16)}$$

> $L1\vartheta := [\Xi = (16)[1], H=0, Z=0]$

$$L1\vartheta := [\Xi = 0.8369278491, H=0, Z=0] \quad \textbf{(17)}$$

> $L2\vartheta := [(16)[2], H=0, Z=0]$

$$L2\vartheta := [1.155672220, H=0, Z=0] \quad \textbf{(18)}$$

> $L3\vartheta := [(16)[3], H=0, Z=0]$

$$L3\vartheta := [-1.005061569, H=0, Z=0] \quad \textbf{(19)}$$

> $subs(Z=0, (5))$

$$\Xi - \frac{0.4939260000\,(2\,\Xi + 0.02429600000)}{\left((\Xi + 0.01214800000)^2 + H^2\right)^{3/2}} - \frac{0.006074000000\,(2\,\Xi - 1.975704000)}{\left((\Xi - 0.9878520000)^2 + H^2\right)^{3/2}} = 0 \quad \textbf{(20)}$$

>

> $\left((\Xi + 0.01214800000)^2 + H^2\right)^{3/2} = 1$

$$\left((\Xi + 0.01214800000)^2 + H^2\right)^{3/2} = 1 \quad \textbf{(21)}$$

> $\left((\Xi - 0.9878520000)^2 + H^2\right)^{3/2} = 1$

$$\left((\Xi - 0.9878520000)^2 + H^2\right)^{3/2} = 1 \quad \textbf{(22)}$$

> $(\Xi + 0.01214800000)^2 + H^2 = 1$

$$(\Xi + 0.01214800000)^2 + H^2 = 1 \quad \textbf{(23)}$$

> $(\Xi - 0.9878520000)^2 + H^2 = 1$

$$(\Xi - 0.9878520000)^2 + H^2 = 1 \quad \textbf{(24)}$$

> $simplify((23)-(24))$

$$2.000000000\,\Xi - 0.9757040000 = 0 \quad \textbf{(25)}$$

> $solve((25), \Xi)$

$$0.4878520000 \quad \textbf{(26)}$$

> $subs(\Xi = (26), (23))$

$$0.2500000000 + H^2 = 1 \quad \textbf{(27)}$$

> $solve((27), H)$

$$0.8660254038,\ -0.8660254038 \quad \textbf{(28)}$$

> $L4\vartheta := [\Xi = (26), H = (28)[1], Z=0]$

$$L4\vartheta := [\Xi = 0.4878520000, H = 0.8660254038, Z=0] \quad \textbf{(29)}$$

> $L5\vartheta := [\Xi = (26), H = (28)[2], Z=0]$

$$L5\vartheta := [\Xi = 0.4878520000, H = -0.8660254038, Z=0] \quad \textbf{(30)}$$

>

>

Έλεγχος Σωστά .!!!!

Αλλάζουμε τα frames !!!!.

>

> $with(plots)$:

> $a := [1.4, 1.4939985438, 1.505, 1.5060735567, 1.526, 1.586080083, 1.589, 1.5941703866, 1.63343]$

$$a := [1.4, 1.4939985438, 1.505, 1.5060735567, 1.526, 1.586080083, 1.589, 1.5941703866, 1.63343] \quad (31)$$

> $C := 2 \cdot a$

$$C := [2.8, 2.987997088, 3.010, 3.012147114, 3.052, 3.172160166, 3.178, 3.188340774, 3.26686] \quad (32)$$

> $b := [0.0, 0.05, 0.1, 0.15, 0.2, 0.21, 0.22, 0.23, 0.24, 0.245, 0.25, 0.3, 0.35, 0.4, 0.45, 0.50, 0.55, 0.60, 0.65, 0.70]$

$$b := [0., 0.05, 0.1, 0.15, 0.2, 0.21, 0.22, 0.23, 0.24, 0.245, 0.25, 0.3, 0.35, 0.4, 0.45, 0.50, 0.55, 0.60, 0.65, 0.70] \quad (33)$$

>

>

> $2 \cdot U - C[3] = 0$

$$\Xi^2 + H^2 + \frac{1.975704000}{\sqrt{(\Xi + 0.01214800000)^2 + H^2 + Z^2}} + \frac{0.02429600000}{\sqrt{(\Xi - 0.9878520000)^2 + H^2 + Z^2}} - 3.010 = 0 \quad (34)$$

> $subs(\{\Xi = 0, H = 0\}, \mathbf{(34)})$

$$-3.010 + \frac{1.975704000}{\sqrt{0.0001475739040 + Z^2}} + \frac{0.02429600000}{\sqrt{0.9758515739 + Z^2}} = 0 \quad (35)$$

> $isolate(\ \mathbf{(35)}, Z\ )$

$$Z = -0.6607568579 \quad (36)$$

> $2 \cdot U - C[3] > 0$

$$0 < \Xi^2 + H^2 + \frac{1.975704000}{\sqrt{(\Xi + 0.01214800000)^2 + H^2 + Z^2}} + \frac{0.02429600000}{\sqrt{(\Xi - 0.9878520000)^2 + H^2 + Z^2}} - 3.010 \quad (37)$$

> $subs(Z = b[1], \mathbf{(34)})$

$$\Xi^2 + H^2 + \frac{1.975704000}{\sqrt{(\Xi + 0.01214800000)^2 + H^2}} + \frac{0.02429600000}{\sqrt{(\Xi - 0.9878520000)^2 + H^2}} - 3.010 = 0 \quad (38)$$

> $subs(Z = b[1], \mathbf{(37)})$

(39)

$$0 < \Xi^2 + H^2 + \frac{1.975704000}{\sqrt{(\Xi + 0.01214800000)^2 + H^2}} + \frac{0.02429600000}{\sqrt{(\Xi - 0.9878520000)^2 + H^2}} - 3.010 \quad (39)$$

>

Προσέχουμε τις φανταστικές τιμές .

>

>

> f := (34)

$$f := \Xi^2 + H^2 + \frac{1.975704000}{\sqrt{(\Xi + 0.01214800000)^2 + H^2 + Z^2}} + \frac{0.02429600000}{\sqrt{(\Xi - 0.9878520000)^2 + H^2 + Z^2}} - 3.010 = 0 \quad (40)$$

> diff(f, Ξ)

$$2\,\Xi - \frac{0.9878520000\,(2\,\Xi + 0.02429600000)}{((\Xi + 0.01214800000)^2 + H^2 + Z^2)^{3/2}} - \frac{0.01214800000\,(2\,\Xi - 1.975704000)}{((\Xi - 0.9878520000)^2 + H^2 + Z^2)^{3/2}} = 0 \quad (41)$$

> diff(f, H)

$$2\,H - \frac{1.975704000\,H}{((\Xi + 0.01214800000)^2 + H^2 + Z^2)^{3/2}} - \frac{0.02429600000\,H}{((\Xi - 0.9878520000)^2 + H^2 + Z^2)^{3/2}} = 0 \quad (42)$$

> diff(f, Z)

$$-\frac{1.975704000\,Z}{((\Xi + 0.01214800000)^2 + H^2 + Z^2)^{3/2}} - \frac{0.02429600000\,Z}{((\Xi - 0.9878520000)^2 + H^2 + Z^2)^{3/2}} = 0 \quad (43)$$

> diff(lhs((41)), Ξ$2)

$$-\frac{3.704445000\,(2\,\Xi + 0.02429600000)^3}{((\Xi + 0.01214800000)^2 + H^2 + Z^2)^{7/2}} + \frac{8.890668000\,(2\,\Xi + 0.02429600000)}{((\Xi + 0.01214800000)^2 + H^2 + Z^2)^{5/2}} - \frac{0.04555500000\,(2\,\Xi - 1.975704000)^3}{((\Xi - 0.9878520000)^2 + H^2 + Z^2)^{7/2}} + \frac{0.1093320000\,(2\,\Xi - 1.975704000)}{((\Xi - 0.9878520000)^2 + H^2 + Z^2)^{5/2}} \quad (44)$$

> diff(lhs((41)), H$2)

(45)

$$-\frac{14.81778000\,(2\,\Xi+0.02429600000)\,H^2}{((\Xi+0.01214800000)^2+H^2+Z^2)^{7/2}}+\frac{2.963556000\,(2\,\Xi+0.02429600000)}{((\Xi+0.01214800000)^2+H^2+Z^2)^{5/2}}-\frac{0.1822200000\,(2\,\Xi-1.975704000)\,H^2}{((\Xi-0.9878520000)^2+H^2+Z^2)^{7/2}}+\frac{0.03644400000\,(2\,\Xi-1.975704000)}{((\Xi-0.9878520000)^2+H^2+Z^2)^{5/2}} \quad \textbf{(45)}$$

> *diff*(*lhs*(**(41)**), Ξ, H)

$$-\frac{7.408890000\,(2\,\Xi+0.02429600000)^2\,H}{((\Xi+0.01214800000)^2+H^2+Z^2)^{7/2}}+\frac{5.927112000\,H}{((\Xi+0.01214800000)^2+H^2+Z^2)^{5/2}}-\frac{0.09111000000\,(2\,\Xi-1.975704000)^2\,H}{((\Xi-0.9878520000)^2+H^2+Z^2)^{7/2}}+\frac{0.07288800000\,H}{((\Xi-0.9878520000)^2+H^2+Z^2)^{5/2}} \quad \textbf{(46)}$$

>

> *solve*({**(40)**, **(41)**, **(42)**}, {Ξ, H, Z})

$\{H=0., \Xi=-0.01425743553, Z=0.6608651504\}, \{H=0., \Xi=-0.01425743553, Z=-0.6608651504\}, \{H=0., \Xi=0.9483374001, Z=0.2233378496\}, \{H=0., \Xi=0.9483374001, Z=-0.2233378496\}, \{H=0., \Xi=-1.003989513, Z=0.04606381432\}, \{H=0., \Xi=-1.003989513, Z=-0.04606381432\}, \{H=0., \Xi=0.9864035850+0.1319076471\,I, Z=0.2083195035+0.01276769394\,I\}, \{H=0., \Xi=0.9864035850+0.1319076471\,I, Z=-0.2083195035-0.01276769394\,I\}, \{H=0., \Xi=0.9864035850-0.1319076471\,I, Z=0.2083195035-0.01276769394\,I\}, \{H=0., \Xi=0.9864035850-0.1319076471\,I, Z=-0.2083195035+0.01276769394\,I\}, \{H=0.8786355479, \Xi=0.4878520000, Z=0.1483254061\,I\}, \{H=-0.8786355479, \Xi=0.4878520000, Z=0.1483254061\,I\}, \{H=0.8786355479, \Xi=0.4878520000, Z=-0.1483254061\,I\}, \{H=-0.8786355479, \Xi=0.4878520000, Z=-0.1483254061\,I\}$ **(47)**

>

> *realsolMAXMIN* := [**(47)**[1], **(47)**[2], **(47)**[3], **(47)**[4], **(47)**[5], **(47)**[6]]

realsolMAXMIN := $[\{H=0., \Xi=-0.01425743553, Z=0.6608651504\}, \{H=0., \Xi=-0.01425743553, Z=-0.6608651504\}, \{H=0., \Xi=0.9483374001, Z=0.2233378496\}, \{H=0., \Xi=0.9483374001, Z=-0.2233378496\}, \{H=0., \Xi=-1.003989513, Z=0.04606381432\}, \{H=0., \Xi=-1.003989513, Z=-0.04606381432\}]$ **(48)**

>

Προσδιορισμός της τιμής $\left.\frac{\partial}{\partial Z} f\right|_{(48)[i]}$

>

> $subs((\mathbf{48})[1], lhs((\mathbf{43})))$

$$-4.532938220 \quad (49)$$

> $subs((\mathbf{48})[2], lhs((\mathbf{43})))$

$$4.532938220 \quad (50)$$

> $subs((\mathbf{48})[3], lhs((\mathbf{43})))$

$$-0.9252429007 \quad (51)$$

> $subs((\mathbf{48})[4], lhs((\mathbf{43})))$

$$0.9252429007 \quad (52)$$

> $subs((\mathbf{48})[5], lhs((\mathbf{43})))$

$$-0.09311332841 \quad (53)$$

> $subs((\mathbf{48})[6], lhs((\mathbf{43})))$

$$0.09311332841 \quad (54)$$

>

>

Προσδιορισμός των τιμών $\left.\frac{\partial^2}{\partial \Xi^2} f\right|_{(48)[i]}$

>

> $subs((\mathbf{48})[1], (\mathbf{44}))$

$$-0.2833422370 \quad (55)$$

> $subs((\mathbf{48})[2], (\mathbf{44}))$

$$-0.2833422370 \quad (56)$$

> $subs((\mathbf{48})[3], (\mathbf{44}))$

$$-24.31298209 \quad (57)$$

> $subs((\mathbf{48})[4], (\mathbf{44}))$

$$-24.31298209 \quad (58)$$

> $subs((\mathbf{48})[5], (\mathbf{44}))$

$$12.12697426 \quad (59)$$

> $subs((\mathbf{48})[6], (\mathbf{44}))$

$$12.12697426 \quad (60)$$

>

Προσδιορισμός των τιμών $\left.\frac{\partial^2}{\partial \mathrm{H}^2} f\right|_{(48)[i]}$

> $subs((\mathbf{48})[1], (\mathbf{45}))$

$$-0.1284869447 \tag{61}$$

> $subs((\mathbf{48})[2], (\mathbf{45}))$

$$-0.1284869447 \tag{62}$$

> $subs((\mathbf{48})[3], (\mathbf{45}))$

$$1.306479563 \tag{63}$$

> $subs((\mathbf{48})[4], (\mathbf{45}))$

$$1.306479563 \tag{64}$$

> $subs((\mathbf{48})[5], (\mathbf{45}))$

$$-6.096270690 \tag{65}$$

> $subs((\mathbf{48})[6], (\mathbf{45}))$

$$-6.096270690 \tag{66}$$

>

Προσδιορισμός τών τιμών $\left.\frac{\partial^2}{\partial \Xi\, \partial \mathrm{H}} f\right|_{(48)[i]}$

>

> $subs((\mathbf{48})[1], (\mathbf{46}))$

$$0. \tag{67}$$

> $subs((\mathbf{48})[2], (\mathbf{46}))$

$$0. \tag{68}$$

> $subs((\mathbf{48})[3], (\mathbf{46}))$

$$0. \tag{69}$$

> $subs((\mathbf{48})[4], (\mathbf{46}))$

$$0. \tag{70}$$

> $subs((\mathbf{48})[5], (\mathbf{46}))$

$$0. \tag{71}$$

> $subs((\mathbf{48})[6], (\mathbf{46}))$

$$0. \tag{72}$$

>

Προσδιορισμός των τιμών

$$D2i := \left.\frac{\partial^2}{\partial \Xi^2} f \cdot \frac{\partial}{\partial Z} f \right|_{realsolMAXMIN[i]} :$$

>

> $D21 := subs((48)[1], (44) \cdot lhs((43)))$

$$D21 := 1.284372855 \quad (73)$$

> $D22 := subs((48)[2], (44) \cdot lhs((43)))$

$$D22 := -1.284372855 \quad (74)$$

> $D23 := subs((48)[3], (44) \cdot lhs((43)))$

$$D23 := 22.49541407 \quad (75)$$

> $D24 := subs((48)[4], (44) \cdot lhs((43)))$

$$D24 := -22.49541407 \quad (76)$$

> $D25 := subs((48)[5], (44) \cdot lhs((43)))$

$$D25 := -1.129182937 \quad (77)$$

> $D26 := subs((48)[6], (44) \cdot lhs((43)))$

$$D26 := 1.129182937 \quad (78)$$

>

>

ΠΡΟΣΔΙΟΡΙΣΜΟΣ ΤΩΝ ΑΚΡΟΤΑΤΩΝ . (maxZ,minZ) .

>

$$D1i := \left. Determinant\left(\begin{bmatrix} \frac{\partial^2}{\partial \Xi^2} f & \frac{\partial^2}{\partial \Xi \, \partial H} f \\ \frac{\partial^2}{\partial \Xi \, \partial H} f & \frac{\partial^2}{\partial H^2} f \end{bmatrix}\right)\right|_{realsolMAXMIN[i]} :$$

$$D2i := \left.\frac{\partial^2}{\partial \Xi^2} f \cdot \frac{\partial}{\partial Z} f \right|_{realsolMAXMIN[i]} :$$

>

>

>

> $with(LinearAlgebra):$

> $D11 := Determinant\left(\begin{bmatrix} \mathbf{(55)} & 0 \\ 0 & \mathbf{(61)} \end{bmatrix}\right)$

$$D11 := 0.03640577834 \quad \mathbf{(79)}$$

> $D12 := Determinant\left(\begin{bmatrix} \mathbf{(56)} & 0 \\ 0 & \mathbf{(62)} \end{bmatrix}\right)$

$$D12 := 0.03640577834 \quad \mathbf{(80)}$$

> $D13 := Determinant\left(\begin{bmatrix} \mathbf{(57)} & 0 \\ 0 & \mathbf{(63)} \end{bmatrix}\right)$

$$D13 := -31.76441422 \quad \mathbf{(81)}$$

> $D14 := Determinant\left(\begin{bmatrix} \mathbf{(58)} & 0 \\ 0 & \mathbf{(64)} \end{bmatrix}\right)$

$$D14 := -31.76441422 \quad \mathbf{(82)}$$

> $D15 := Determinant\left(\begin{bmatrix} \mathbf{(59)} & 0 \\ 0 & \mathbf{(65)} \end{bmatrix}\right)$

$$D15 := -73.92931774 \quad \mathbf{(83)}$$

> $D16 := Determinant\left(\begin{bmatrix} \mathbf{(60)} & 0 \\ 0 & \mathbf{(66)} \end{bmatrix}\right)$

$$D16 := -73.92931774 \quad \mathbf{(84)}$$

>

>

Γιά τις αρνητικές τιμές *D1i* έχουμε ΣΑΓΜΑΤΙΚΑ ΣΗΜΕΙΑ .

1.Περίπτωση : *D1i >0, D2i >0* έχουμε Ελάχιστο Z .

>

> $minZ := (solve(subs(\{\Xi = rhs(\mathbf{(48)}[1][2]), H = rhs(\mathbf{(48)}[1][1])\}, \mathbf{(40)}), Z))[1]$

$$minZ := -0.6608651503 \quad \mathbf{(85)}$$

>

>

2.Περίπτωση : *D1i > 0 , D2i < 0* έχουμε Μέγιστο Z .

>

> $maxZ := (solve(subs(\{\Xi = rhs(\mathbf{(47)}[2][2]), H = rhs(\mathbf{(47)}[2][1])\}, \mathbf{(40)}), Z)[2])$

$$maxZ := 0.6608651503 \quad \mathbf{(86)}$$

>

>

> $L1 := [0.8369278491, 0, 0]:$

> $L2 := [1.155672220, 0, 0]:$

> $L3 := [-1.005061569, 0, 0]:$

> $L4 := [0.4878520000, 0.8660254038, 0.]:$

> $L5 := [0.4878520000, -0.8660254038, 0.]:$

> $Terre := [-\mu[7], 0, 0]$

$$Terre := [-0.01214800000, 0, 0] \quad \textbf{(87)}$$

> $Lune := [1 - \mu[7], 0, 0]$

$$Lune := [0.9878520000, 0, 0] \quad \textbf{(88)}$$

>

>

>

> $subs(\{\Xi = L1[1], \mathrm{H} = 0\}, \textbf{(34)})$

$$-2.309551775 + \frac{1.975704000}{\sqrt{0.7209297975 + Z^2}} + \frac{0.02429600000}{\sqrt{0.02277809932 + Z^2}} = 0 \quad \textbf{(89)}$$

> $solve(\textbf{(89)}, \mathrm{Z})$

$$-0.2560351199, 0.2560351199 \quad \textbf{(90)}$$

> $subs(\{\Xi = L2[1], \mathrm{H} = 0\}, \textbf{(34)})$

$$-1.674421720 + \frac{1.975704000}{\sqrt{1.363804066 + Z^2}} + \frac{0.02429600000}{\sqrt{0.02816362624 + Z^2}} = 0 \quad \textbf{(91)}$$

> $solve(\textbf{(91)}, \mathrm{Z})$

$$-0.3665299110, 0.3665299110 \quad \textbf{(92)}$$

> $subs(\{\Xi = L3[1], \mathrm{H} = 0\}, \textbf{(34)})$

$$-1.999851243 + \frac{1.975704000}{\sqrt{0.9858773555 + Z^2}} + \frac{0.02429600000}{\sqrt{3.971704494 + Z^2}} = 0 \quad \textbf{(93)}$$

> $solve(\textbf{(93)}, \mathrm{Z})$

$$-0.04610110073, 0.04610110073 \quad \textbf{(94)}$$

>

>

> $subs(\{\Xi = 0, \mathrm{H} = 0, \}, \textbf{(34)})$

$$-3.010 + \frac{1.975704000}{\sqrt{0.0001475739040 + Z^2}} + \frac{0.02429600000}{\sqrt{0.9758515739 + Z^2}} = 0 \quad \textbf{(95)}$$

> $solve(\textbf{(95)}, \mathrm{Z})$

$$-0.6607568579, 0.6607568579 \quad \textbf{(96)}$$

> $subs(\mathrm{H} = 0, \textbf{(34)})$

$$\Xi^2 - 3.010 + \frac{1.975704000}{\sqrt{(\Xi + 0.01214800000)^2 + Z^2}} + \frac{0.02429600000}{\sqrt{(\Xi - 0.9878520000)^2 + Z^2}} = 0 \quad \textbf{(97)}$$

> $TOMH\Xi0Z := implicitplot(\textbf{(97)}, \Xi = -1.3..1.3, \mathrm{Z} = \textbf{(96)}[1] - 0.1..\textbf{(96)}[2] + 0.1, numpoints = 10000, gridlines):$

> $diff(\textbf{(97)}, \Xi)$

$$2\,\Xi - \frac{0.9878520000\,(2\,\Xi + 0.02429600000)}{\left((\Xi + 0.01214800000)^2 + Z^2\right)^{3/2}} - \frac{0.01214800000\,(2\,\Xi - 1.975704000)}{\left((\Xi - 0.9878520000)^2 + Z^2\right)^{3/2}} = 0 \quad \textbf{(98)}$$

> *diff*((**97**), Z)

$$-\frac{1.975704000\,Z}{\left((\Xi + 0.01214800000)^2 + Z^2\right)^{3/2}} - \frac{0.02429600000\,Z}{\left((\Xi - 0.9878520000)^2 + Z^2\right)^{3/2}} = 0 \quad \textbf{(99)}$$

> *solve*({(**98**), (**99**)}, {Ξ, Z})

{Ξ = 0.8369278491, Z = 0.}, {Ξ = 1.155672220, Z = 0.}, {Ξ = −1.005061569, Z = 0.}, {Ξ = 0.9978986774 − 0.01709062021 I, Z = 0.1147295074 + 0.1998751729 I}, {Ξ = 0.9978986774 − 0.01709062021 I, Z = −0.1147295074 − 0.1998751729 I}, {Ξ = −0.2948290742 + 0.3506388343 I, Z = 0.3761024464 + 1.260733219 I}, {Ξ = −0.2948290742 + 0.3506388343 I, Z = −0.3761024464 − 1.260733219 I}, {Ξ = −0.2948290742 − 0.3506388343 I, Z = 0.3761024464 − 1.260733219 I}, {Ξ = −0.2948290742 − 0.3506388343 I, Z = −0.3761024464 + 1.260733219 I}, {Ξ = 0.9978986774 + 0.01709062021 I, Z = 0.1147295074 − 0.1998751729 I}, {Ξ = 0.9978986774 + 0.01709062021 I, Z = −0.1147295074 + 0.1998751729 I} **(100)**

> ***solve***(***subs***({**Ξ = *Terre*[1], H = 0**}, (**34**)), **Z**)

0.6608627800, −0.6608627800 **(101)**

> ***solve***(***subs***({**Ξ = *Lune*[1], H = 0**}, (**34**)), **Z**)

0.2265155571, −0.2265155571 **(102)**

>

> *L1*[1]

0.8369278491 **(103)**

> *L2*[1]

1.155672220 **(104)**

> *L3*[1]

−1.005061569 **(105)**

> *subs*(Ξ = 0, (**34**))

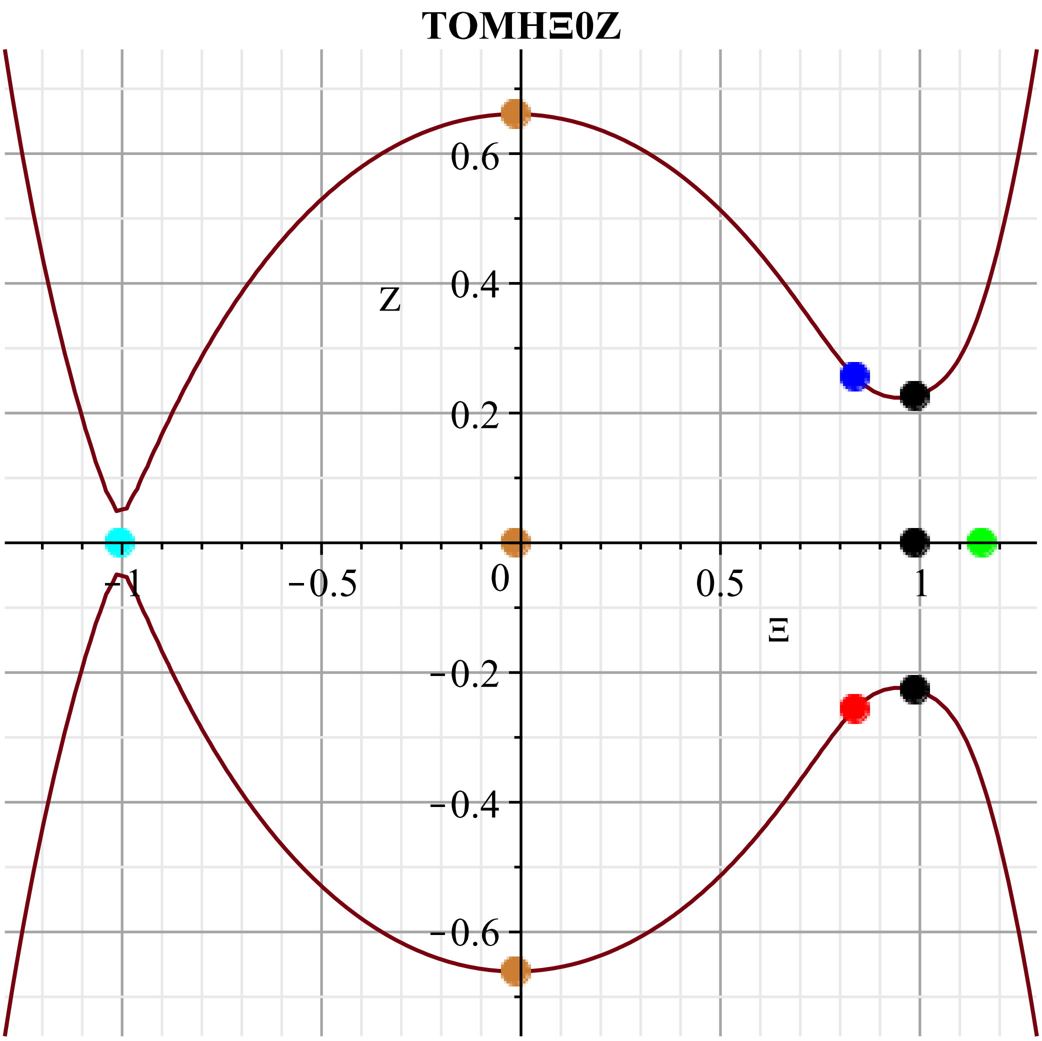
$$-3.010 + H^2 + \frac{1.975704000}{\sqrt{0.0001475739040 + H^2 + Z^2}} + \frac{0.02429600000}{\sqrt{0.9758515739 + H^2 + Z^2}} = 0 \quad \textbf{(106)}$$

> *TOMHH0Z* := *implicitplot*((**106**), H = -1.3 ..1.3, Z = (**96**)[1] − 0.1 ..(**96**)[2] + 0.1, *numpoints* = 10000, *gridlines*, *title* = "TOMHH0Z", *titlefont* = [*arial*, *bold*, 12]) :

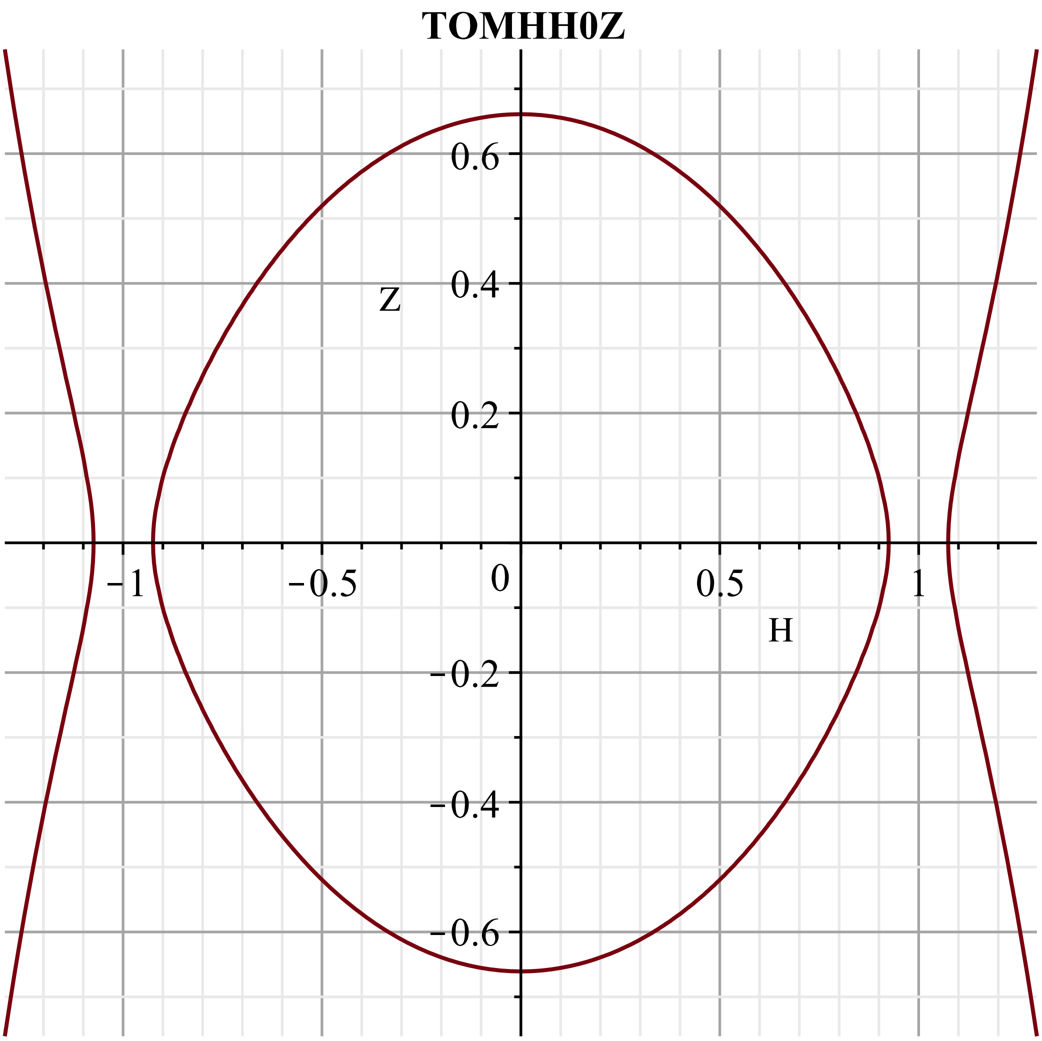
>

> *pp* := *pointplot*([[*L1*[1], (**90**)[1]], [*L1*[1], (**90**)[2]], [*L2*[1], 0], [*L3*[1], 0], [*Terre*[1], 0], [*Terre*[1], (**101**)[1]], [*Terre*[1], (**101**)[2]], [*Lune*[1], 0], [*Lune*[1], (**102**)[1]], [*Lune*[1], (**102**)[2]]], *color* = [*red*, *blue*, *green*, *cyan*, *gold*, *gold*, *gold*, *black*, *black*, *black*], *symbol* = *solidcircle*, *symbolsize* = 20) :

> *display*(*TOMHΞ0Z*, *pp*, *title* = "TOMHΞ0Z", *titlefont* = [*arial*, *bold*, 12])



> *display*(*TOMHH0Z*)

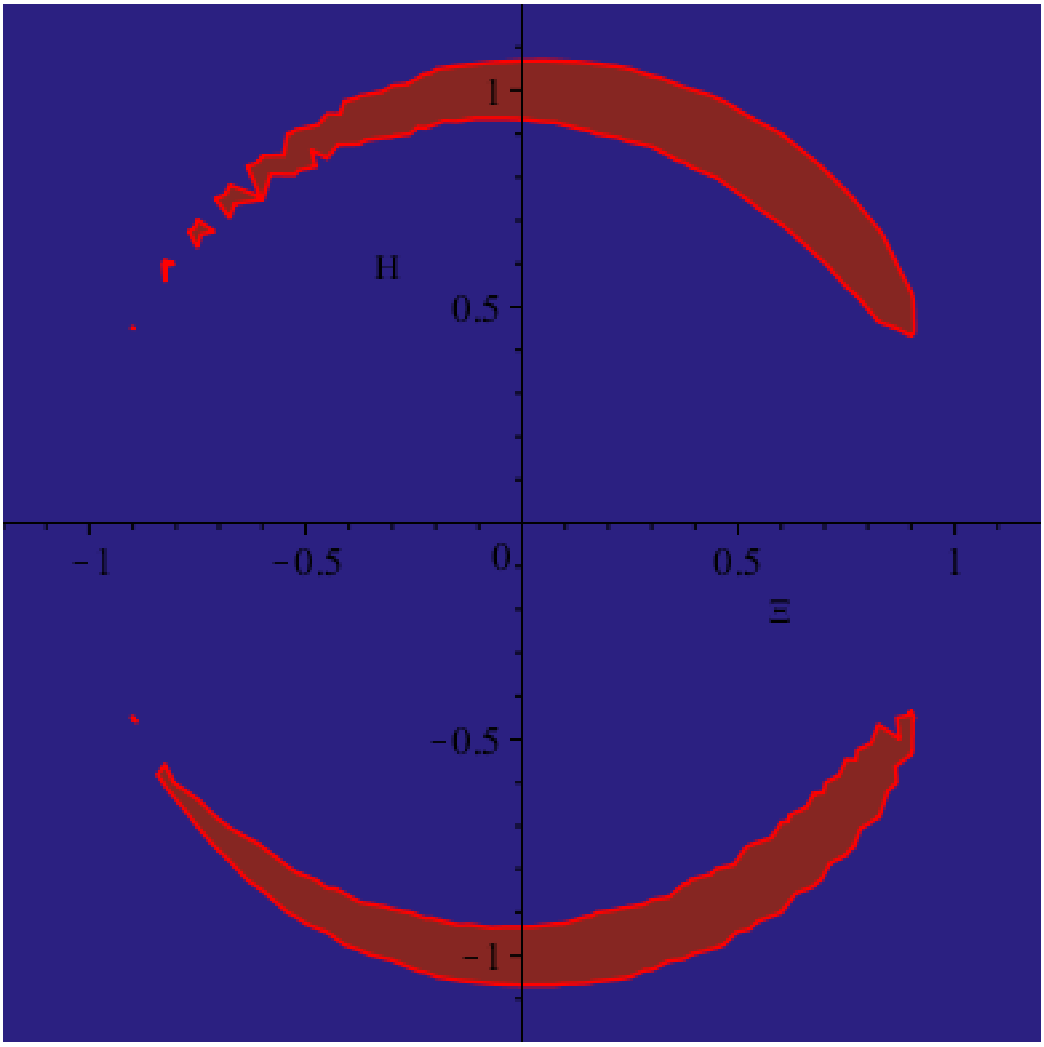


>

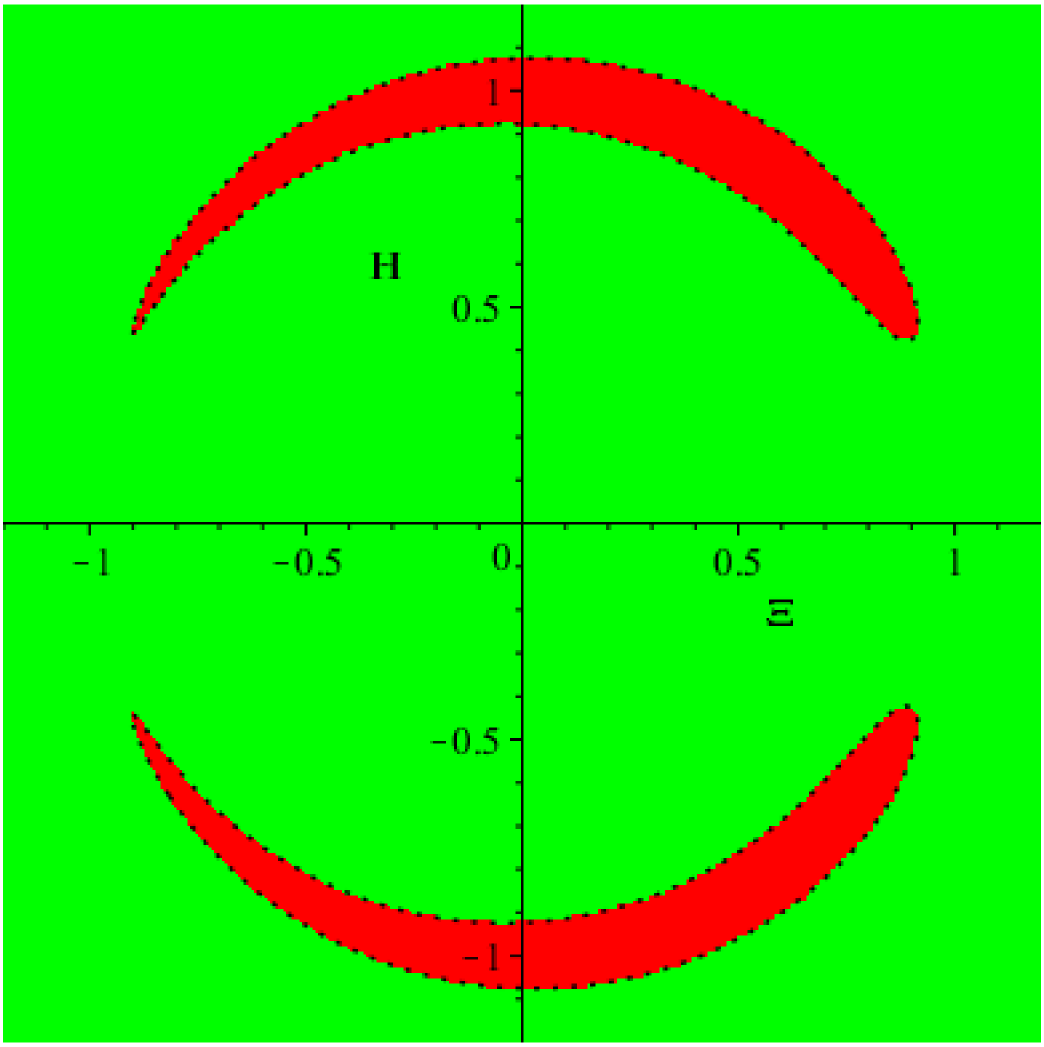
Η ΚΟΚΚΙΝΗ ΠΕΡΙΟΧΗ ΕΙΝΑΙ ΑΠΑΓΟΡΕΥΡΙΚΗ ΓΙΑ ΤΗΝ ΚΙΝΗΣΗ ΤΟΥ ΤΡΙΤΟΥ ΣΩΜΑΤΟΣ ΓΙΑ ΤΗΝ ΔΕΔΟΜΕΝΗ ΣΤΑΘΜΗ ΕΝΕΡΓΕΙΑΣ C[i]

>

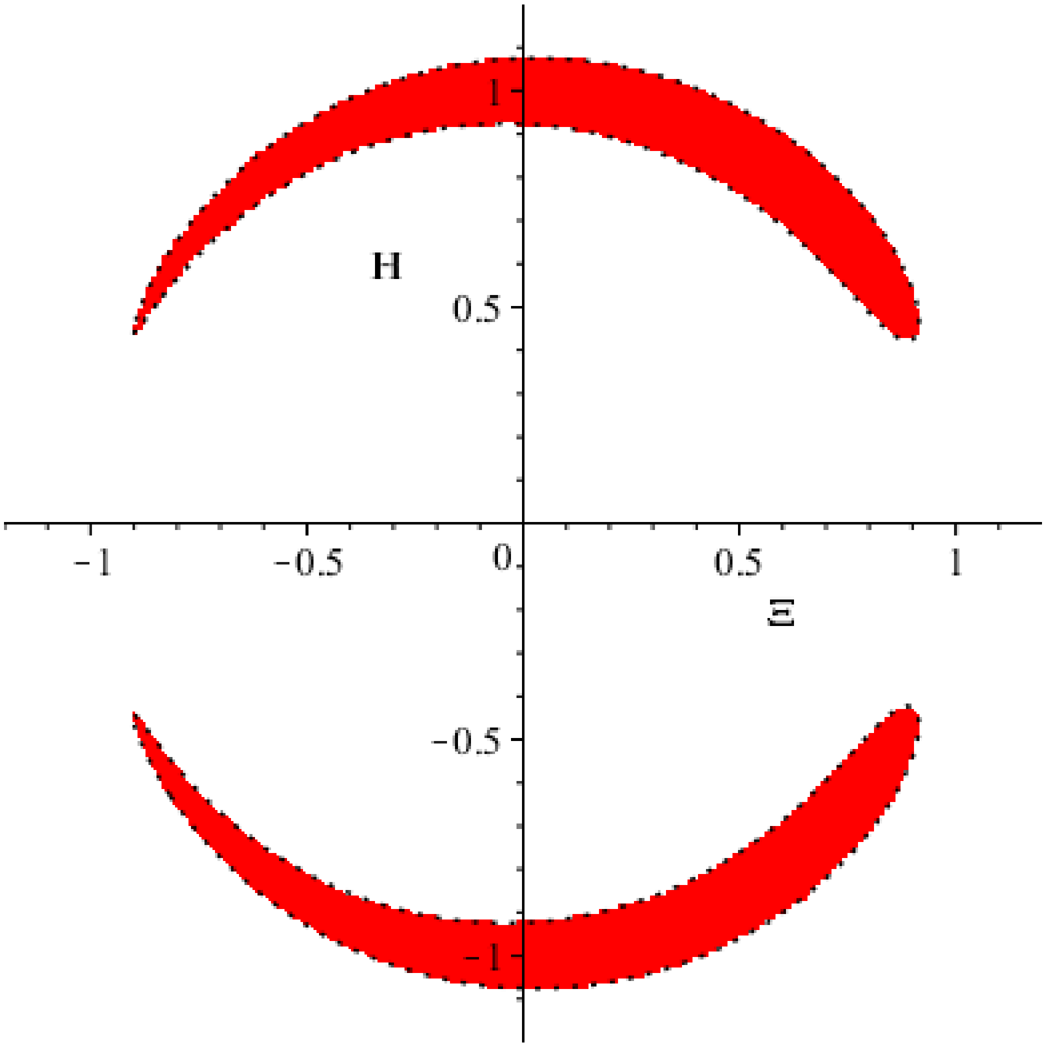
> *implicitplot*(**(38)**, Ξ = −1.20 ..1.20, Η = −1.20 ..1.20, *scaling* = *constrained*, *numpoints* = 1000, *color* = *red*, *filled* = *true*)



> *inequal*((**39**), Ξ = −1.20 ..1.20, H = −1.20 ..1.20, *scaling* = *constrained*, *optionsfeasible* = [*color* = *GREEN*], *optionsexcluded* = [*color* = *RED*], *labels* = [Ξ, H], *labelfont* = [*arial*, *bold*, 12])



> $inequal\big(\mathbf{(39)}, \Xi = -1.20\,..1.20, \mathrm{H} = -1.20\,..1.20, scaling = constrained, optionsfeasible = [color = white], optionsexcluded = [color = RED], labels = [\Xi, \mathrm{H}], labelfont = [arial, bold, 12]\big)$



>

>

ΣΥΝΤΕΤΑΓΜΕΝΕΣ ΣΤΟ ΠΕΡΙΣΤΡΕΦΟΜΕΝΟ ΣΥΣΤΗΜΑ :

> $L1 := [0.8369278491, 0, 0]:$

> $L2 := [1.155672220, 0, 0]:$

> $L3 := [-1.005061569, 0, 0]:$

> $L4 := [0.4878520000, 0.8660254038, 0.]:$

> $L5 := [0.4878520000, -0.8660254038, 0.]:$

> $Terre := [-\mu[7], 0, 0]$

$$Terre := [-0.01214800000, 0, 0] \quad \textbf{(107)}$$

> $Lune := [1 - \mu[7], 0, 0]$

$$Lune := [0.9878520000, 0, 0] \quad \textbf{(108)}$$

>

>

ΜΕΤΑΤΡΟΠΗ ΣΥΝΤΕΤΑΓΜΕΝΩΝ ΑΠΟ ΤΟ ΠΕΡΙΣΤΡΕΦΟΜΕΝΟ

ΣΥΣΤΗΜΑ ΣΤΟ ΑΔΡΑΝΕΙΑΚΟ ΣΥΣΤΗΜΑ ΣΥΝΤΕΤΑΓΜΕΝΩΝ .

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \cos(\tau) & -\sin(\tau) \\ \sin(\tau) & \cos(\tau) \end{bmatrix} \cdot \begin{bmatrix} \Xi \\ \mathrm{H} \end{bmatrix}$$

> $XL1 := \cos(\tau) \cdot L1[1] - \sin(\tau) \cdot L1[2]$

$$XL1 := 0.8369278491 \cos(\tau) \quad (109)$$

> $YL1 := \sin(\tau) \cdot L1[1] + \cos(\tau) \cdot L1[2]$

$$YL1 := 0.8369278491 \sin(\tau) \quad (110)$$

>

> $XL2 := \cos(\tau) \cdot L2[1] - \sin(\tau) \cdot L2[2]$

$$XL2 := 1.155672220 \cos(\tau) \quad (111)$$

> $YL2 := \sin(\tau) \cdot L2[1] + \cos(\tau) \cdot L2[2]$

$$YL2 := 1.155672220 \sin(\tau) \quad (112)$$

>

> $XL3 := \cos(\tau) \cdot L3[1] - \sin(\tau) \cdot L3[2]$

$$XL3 := -1.005061569 \cos(\tau) \quad (113)$$

> $YL3 := \sin(\tau) \cdot L3[1] + \cos(\tau) \cdot L3[2]$

$$YL3 := -1.005061569 \sin(\tau) \quad (114)$$

>

> $XL4 := \cos(\tau) \cdot L4[1] - \sin(\tau) \cdot L4[2]$

$$XL4 := 0.4878520000 \cos(\tau) - 0.8660254038 \sin(\tau) \quad (115)$$

> $YL4 := \sin(\tau) \cdot L4[1] + \cos(\tau) \cdot L4[2]$

$$YL4 := 0.4878520000 \sin(\tau) + 0.8660254038 \cos(\tau) \quad (116)$$

>

> $XL5 := \cos(\tau) \cdot L5[1] - \sin(\tau) \cdot L5[2]$

$$XL5 := 0.4878520000 \cos(\tau) + 0.8660254038 \sin(\tau) \quad (117)$$

> $YL5 := \sin(\tau) \cdot L5[1] + \cos(\tau) \cdot L5[2]$

$$YL5 := 0.4878520000 \sin(\tau) - 0.8660254038 \cos(\tau) \quad (118)$$

>

> $XTerre := \cos(\tau) \cdot Terre[1] - \sin(\tau) \cdot Terre[2]$

$$XTerre := -0.01214800000 \cos(\tau) \quad (119)$$

> $YTerre := \sin(\tau) \cdot Terre[1] + \cos(\tau) \cdot Terre[2]$

$$YTerre := -0.01214800000 \sin(\tau) \quad (120)$$

>

> $XLune := \cos(\tau)\cdot Lune[1] - \sin(\tau)\cdot Lune[2]$

$$XLune := 0.9878520000\cos(\tau) \quad (121)$$

> $YLune := \sin(\tau)\cdot Lune[1] + \cos(\tau)\cdot Lune[2]$

$$YLune := 0.9878520000\sin(\tau) \quad (122)$$

>

> $$UA := X^2 + Y^2 + \frac{1.975704}{\sqrt{(X + 0.012148\cos(\tau))^2 + (Y + 0.012148\sin(\tau))^2 + Z^2}} + \frac{0.024296}{\sqrt{(X - 0.987852\cos(\tau))^2 + (Y - 0.987852\sin(\tau))^2 + Z^2}} - C[3] = 0 :$$

>

ΑΠΕΙΚΟΝΙΣΕΙΣ ΣΤΟ ΑΔΡΑΝΕΙΑΚΟ ΣΥΣΤΗΜΑ ΣΥΝΤΕΤΑΓΜΕΝΩΝ .

>

> $P1 := [XL1, YL1, 0]$:

> $P2 := [XL2, YL2, 0]$:

> $P3 := [XL3, YL3, 0]$:

> $P4 := [XL4, YL4, 0]$:

> $P5 := [XL5, YL5, 0]$:

> $P6 := [XTerre, YTerre, 0]$:

> $P7 := [XLune, YLune, 0]$:

> $points := [P1, P2, P3, P4, P5, P6, P7]$:

> animP := animate(pointplot3d, [points, color = [green, yellow, olive, maroon, coral, blue, red], symbol = solidcircle, symbolsize = 10], τ = 0 ..2·Pi, frames = 2, trace = 0) :

> T1 := [XL1, YL1 − 0.1, 0, "L1"] :

> T2 := [XL2, YL2 − 0.1, 0, "L2"] :

> T3 := [XL3, YL3 − 0.1, 0, "L3"] :

> T4 := [XL4, YL4 + 0.1, 0.2, "L4"] :

> T5 := [XL5, YL5 − 0.1, 0.15, "L5"] :

> T6 := [XTerre − 0.05, YTerre + 0.05, 0, "Γ"] :

> T7 := [XLune, YLune + 0.10, 0, "Σ"] :

> T := [T1, T2, T3, T4, T5, T6, T7] :

> animT := animate(textplot3d, [T, font = [arial, bold, 10]], τ = 0 ..2·Pi, frames = 2, trace = 0) :

> ARXH := pointplot3d([0, 0, 0], color = yellow, symbol = solidcircle, symbolsize = 5) :

>

> axonX := spacecurve([x, 0, 0], x = -1.2 ..0.0, color = blue, thickness = 2, linestyle = 4) :

> axonY := spacecurve([0, y, 0], y = -1.2 ..0.0, color = blue, thickness = 2, linestyle = 4) :

> axonZ := spacecurve([0, 0, z], z = -0.5 ..0.0, color = blue, thickness = 2, linestyle = 4) :

>

> axX := arrow(⟨1.4, 0, 0⟩, width = 0.02, head_length = 0.1, head_width = 0.1, color = blue) :

```
> axY := arrow(⟨0, 1.4, 0⟩, width = 0.02, head_length = 0.1, head_width = 0.1, color = blue) :
> axZ := arrow(⟨0, 0, 0.5⟩, width = 0.02, head_length = 0.1, head_width = 0.1, color = blue) :
> TaxX := textplot3d([1.45, 0, 0, "X"], font = [arial, bold, 14]) :
> TaxY := textplot3d([0, 1.45, 0, "Y"], font = [arial, bold, 14]) :
> TaxZ := textplot3d([0, 0, 0.6, "Z"], font = [arial, bold, 14]) :
> line1 := animate(spacecurve, [[XL3 + λ·(XL2 − XL3), YL3 + λ·(YL2 − YL3), 0], λ = 0
      ..1, color = red, linestyle = 4], τ = 0 ..2·Pi, frames = 2, trace = 0) :
> line2 := animate(spacecurve, [[XL4 + λ·(XL5 − XL4), YL4 + λ·(YL5 − YL4), 0], λ = 0
      ..1, color = red, linestyle = 4], τ = 0 ..2·Pi, frames = 2, trace = 0) :
> line3 := animate(spacecurve, [[XTerre + λ·(XL4 − XTerre), YTerre + λ·(YL4 − YTerre),
      0], λ = 0 ..1, color = red, linestyle = 4], τ = 0 ..2·Pi, frames = 2, trace = 0) :
> line4 := animate(spacecurve, [[XTerre + λ·(XL5 − XTerre), YTerre + λ·(YL5 − YTerre),
      0], λ = 0 ..1, color = red, linestyle = 4], τ = 0 ..2·Pi, frames = 2, trace = 0) :
> line5 := animate(spacecurve, [[XLune + λ·(XL4 − XLune), YLune + λ·(YL4 − YLune),
      0], λ = 0 ..1, color = red, linestyle = 4], τ = 0 ..2·Pi, frames = 2, trace = 0) :
> line6 := animate(spacecurve, [[XLune + λ·(XL5 − XLune), YLune + λ·(YL5 − YLune),
      0], λ = 0 ..1, color = red, linestyle = 4], τ = 0 ..2·Pi, frames = 2, trace = 0) :
>
> animUA := animate(implicitplot3d, [UA, X = -1.5 ..1.5, Y = -1.5 ..1.5, Z = -0.7 ..0, style
      = surfacecontour, numpoints = 1000, transparency = 0.0], τ = 0 ..2·Pi, frames = 2) :
>
> display(animUA, axX, axY, axZ, TaxX, TaxY, TaxZ, ARXH, axonX, axonY, axonZ, animP,
      animT, line1, line2, line3, line4, line5, line6, title
      = "ΑΔΡΑΝΕΙΑΚΟ  ΣΥΣΤΗΜΑ ΤΡΟΧΙΑ ΣΥΣΤΗΜΑΤΟΣ\nΣΑΒΒΑΣ Π.
      ΓΑΒΡΙΗΛΙΔΗΣ\nΔΙΑΡΚΕΙΑ ΚΙΝΗΣΗΣ : t=27.32 ΗΜΕΡΕΣ", titlefont = [arial, 14, bold],
      labels = [X, Y, Z], labelfont = [arial, 14, bold], orientation = [45, 45, 0], axes = boxed,
      scaling = constrained, animUA) :
>
```