Αυτοτεμνόμενες μή προσανατολισμένες Επιφάνειες .

Φιάλη του Κλάϊν (Bouteille de Klein).

Μία εκδοχή του Robert Israel .

Παραμετροποίηση της Αυτοτεμνόμενης μή προσανατολισμένης Επιφάνειας :

$$x(u,v) = -\frac{2}{15}\cdot\cos(u)\cdot\left(3\cdot\cos(v) - 30\cdot\sin(u) + 90\cdot\cos^4(u)\cdot\sin(u) - 60\cdot\cos^6(u)\cdot\sin(u) + 5\cdot\cos(u)\cdot\cos(v)\cdot\sin(u)\right)$$

$$y(u,v) = -\frac{1}{15}\cdot\sin(u)\cdot\left(3\cdot\cos(v) - 3\cdot\cos^2(u)\cdot\cos(v) - 48\cdot\cos^4(u)\cdot\cos(v) + 48\cdot\cos^6(u)\cdot\cos(v) - 60\cdot\sin(u) + 5\cdot\cos(u)\cdot\cos(v)\cdot\sin(u) - 5\cdot\cos^3(u)\cdot\cos(v)\cdot\sin(u) - 80\cdot\cos^5(u)\cos(v)\cdot\sin(u) + 80\cdot\cos^7(u)\cdot\cos(v)\cdot\sin(u)\right) :$$

$$z(u,v) = \frac{2}{15}\cdot(3 + 5\cdot\cos(u)\cdot\sin(u))\cdot\sin(v) :$$

όπου : $u = 0..\text{Pi},\ v = 0..2\cdot\text{Pi}$

```
>
> with(plots) :
> with(Student[VectorCalculus]) :
>
> OPTIONS := labels = [x, y, z], labelfont = [arial, bold, 14], title
      = "Φιάλη Klein\nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ", titlefont = [arial, 16, bold], orientation
      = [80, 20, 150], scaling = constrained :
>
```

$$> X := -\frac{2}{15}\cdot\cos(u)\cdot\left(3\cdot\cos(v) - 30\cdot\sin(u) + 90\cdot\cos^4(u)\cdot\sin(u) - 60\cdot\cos^6(u)\cdot\sin(u) + 5\cdot\cos(u)\cdot\cos(v)\cdot\sin(u)\right) :$$

$$> Y := -\frac{1}{15}\cdot\sin(u)\cdot\left(3\cdot\cos(v) - 3\cdot\cos^2(u)\cdot\cos(v) - 48\cdot\cos^4(u)\cdot\cos(v) + 48\cdot\cos^6(u)\cdot\cos(v) - 60\cdot\sin(u) + 5\cdot\cos(u)\cdot\cos(v)\cdot\sin(u) - 5\cdot\cos^3(u)\cdot\cos(v)\cdot\sin(u) - 80\cdot\cos^5(u)\cos(v)\cdot\sin(u) + 80\cdot\cos^7(u)\cdot\cos(v)\cdot\sin(u)\right) :$$

$$> Z := \frac{2}{15}\cdot(3 + 5\cdot\cos(u)\cdot\sin(u))\cdot\sin(v) :$$

```
> BS := plot3d([X, Y, Z], u = 0..Pi, v = 0..2·Pi) :
> display(BS, OPTIONS, transparency = 0.5, style = surface)
```

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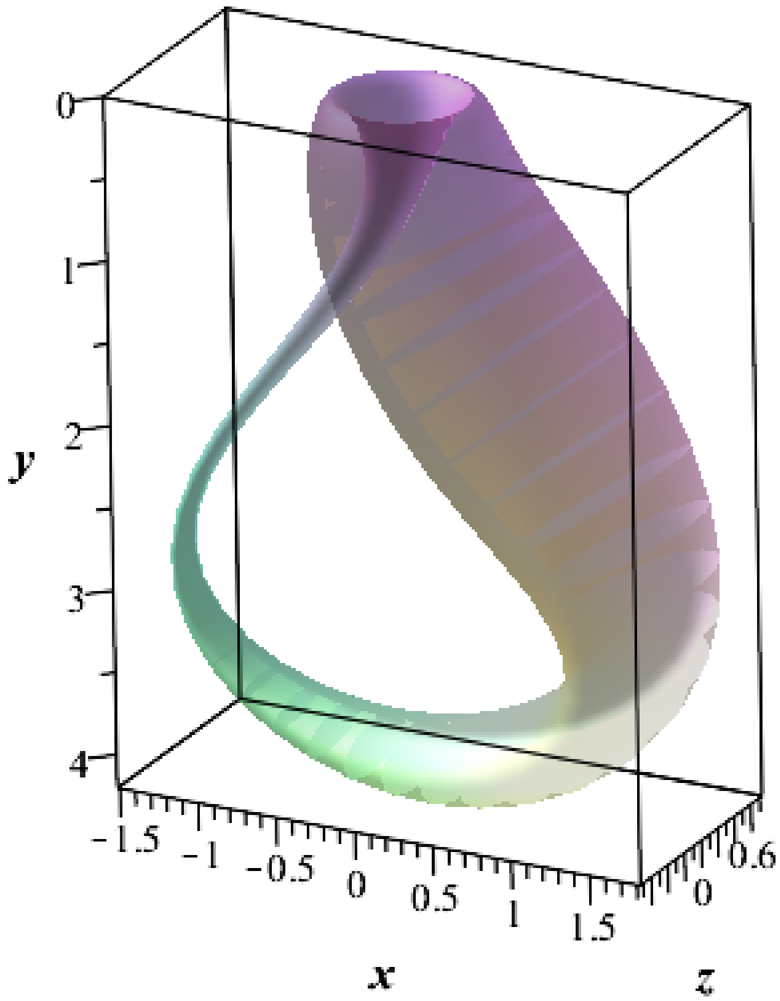

\>

\> $Ppoint0 := [subs(\{u=0, v=0\}, X), subs(\{u=0, v=0\}, Y), subs(\{u=0, v=0\}, Z)]:$

\> $Ppoint := \left[subs\left(\left\{u=\frac{\mathrm{Pi}}{4}, v=\mathrm{Pi}\right\}, X\right), subs\left(\left\{u=\frac{\mathrm{Pi}}{4}, v=\mathrm{Pi}\right\}, Y\right), subs\left(\left\{u=\frac{\mathrm{Pi}}{4}, v=\mathrm{Pi}\right\}, Z\right)\right]:$

\> $Ppoint1 := [subs(\{u=\mathrm{Pi}, v=2\cdot \mathrm{Pi}\}, X), subs(\{u=\mathrm{Pi}, v=2\cdot \mathrm{Pi}\}, Y), subs(\{u=\mathrm{Pi}, v=2\cdot \mathrm{Pi}\}, Z)]:$

\> $Ppoint2 := [subs(\{u=0, v=2\cdot \mathrm{Pi}\}, X), subs(\{u=0, v=2\cdot \mathrm{Pi}\}, Y), subs(\{u=0, v=2\cdot \mathrm{Pi}\}, Z)]:$

\>

\> $P0 := pointplot3d(Ppoint0, color=yellow, symbol=solidcircle, symbolsize=20):$

```
> P := pointplot3d(Ppoint, color = blue, symbol = solidcircle, symbolsize = 20) :
> P1 := pointplot3d(Ppoint1, color = red, symbol = solidcircle, symbolsize = 20) :
> P2 := pointplot3d(Ppoint2, color = green, symbol = solidcircle, symbolsize = 20) :
> display(BS, P0, P, P1, P2, OPTIONS, transparency = 0.5, style = surface)
```

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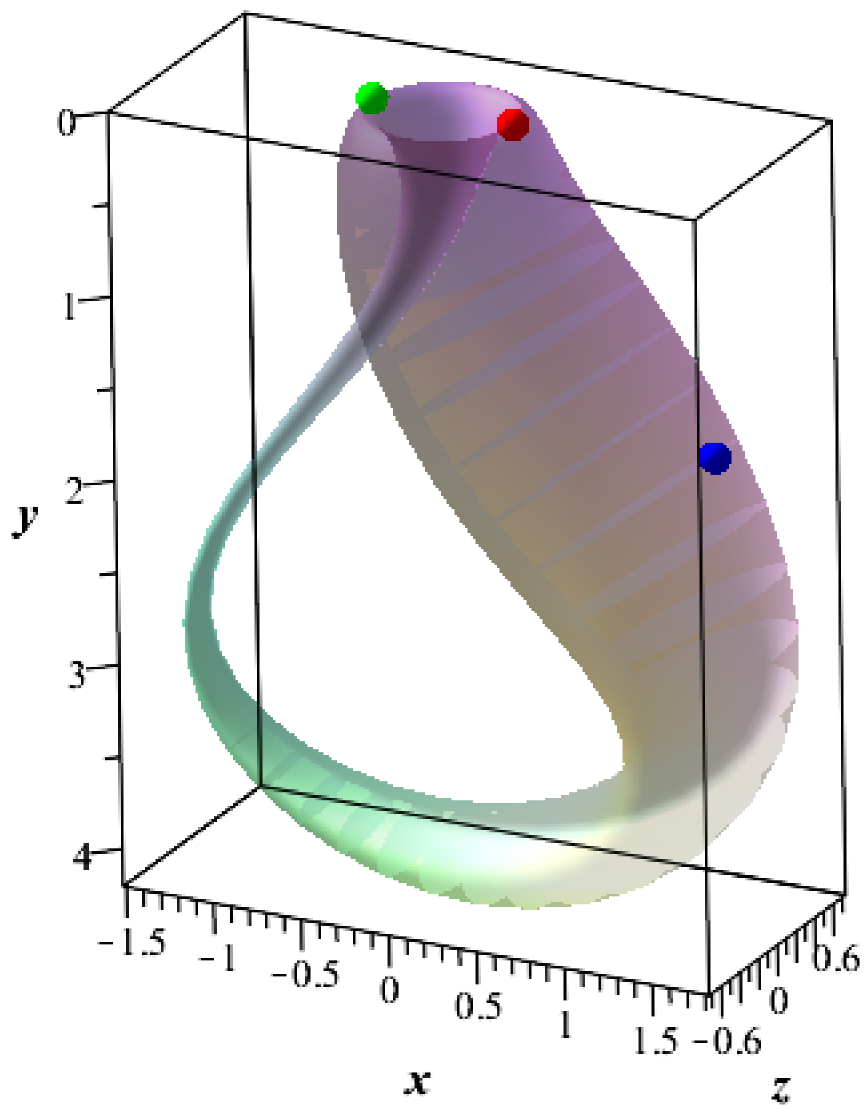


```
>
```

> $animP := animate\left(pointplot3d, \left[\left[subs\left(u = \frac{\mathrm{Pi}}{4}, X\right), subs\left(u = \frac{\mathrm{Pi}}{4}, Y\right), subs\left(u = \frac{\mathrm{Pi}}{4}, Z\right)\right],\right.\right.$

```
color = blue, symbol = solidcircle, symbolsize = 10], v = 0 ..2·Pi, frames = 80, trace = 80)
:
> animP1 := animate(pointplot3d, [ [subs(v = 2· Pi, X), subs(v = 2· Pi, Y), subs(v = 2· Pi,
Z) ], color = red, symbol = solidcircle, symbolsize = 10 ], u = 0 ..2· Pi, frames = 80, trace
= 80) :
>
> display(BS, P, P1, P2, animP, animP1, OPTIONS, transparency = 0.5)
```

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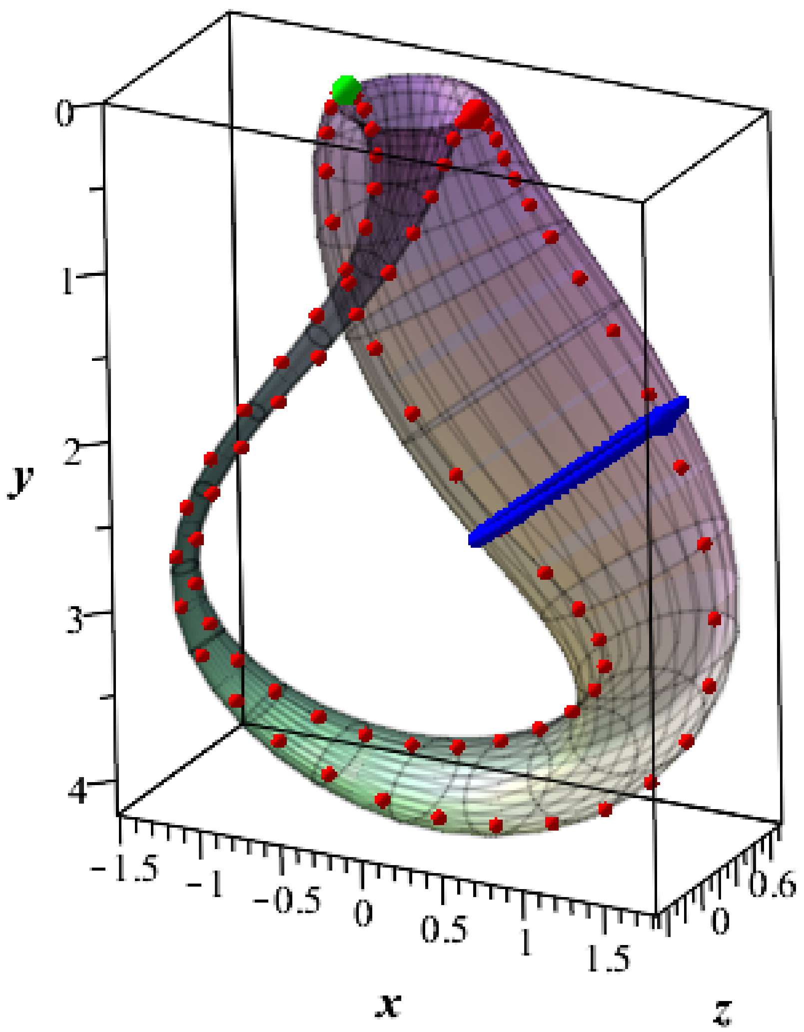


```
>
> TRIEDRO := TNBFrame(⟨subs(v = 2· Pi, X), subs(v = 2· Pi, Y), subs(v = 2· Pi, Z)⟩, range
= 0 ..2·Pi, output = animation, binormal = false, frames = 80) :
```

> *display*(*BS*, *P*, *P1*, *P2*, *animP*, *animP1*, *TRIEDRO*, *OPTIONS*, *transparency* = 0.5)

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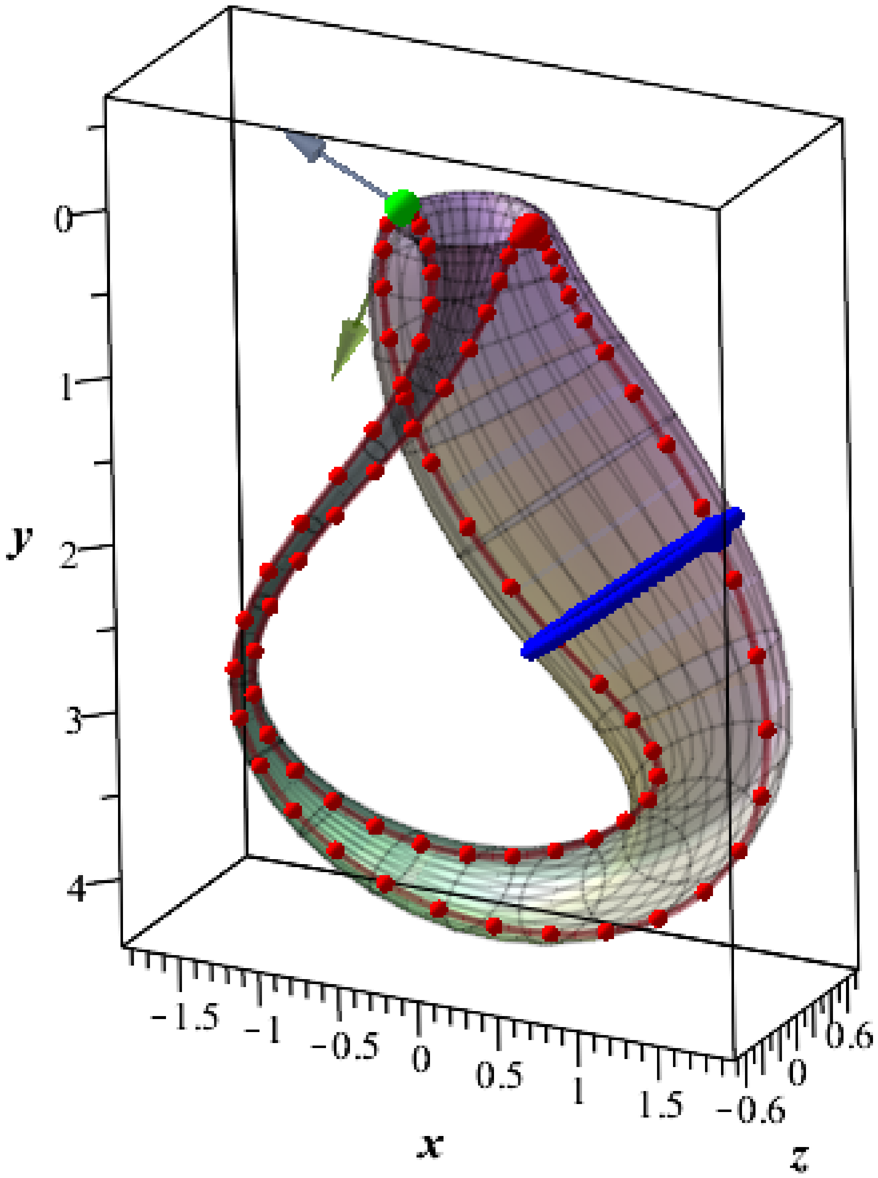

Animation de(s) vecteur(s) de Frenet-Serret: normale principal, tangent.

>