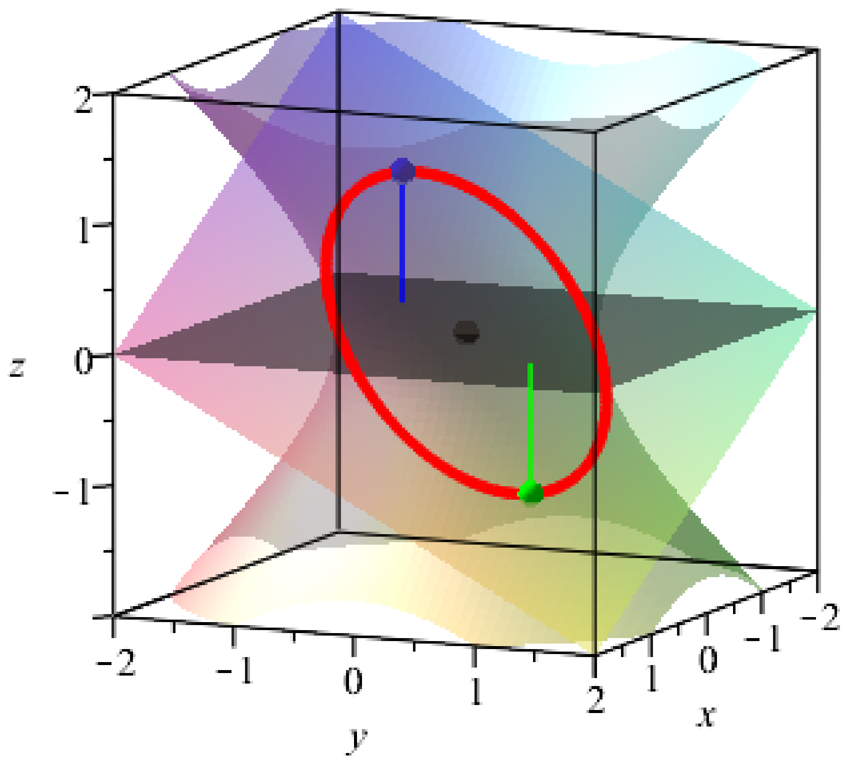
```
> 
> with(Optimization)
   [ImportMPS, Interactive, LPSolve, LSSolve, Maximize, Minimize, NLPSolve, QPSolve]      (1)
> with(plots)
[animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d,      (2)
    conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d,
    densityplot, display, dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d,
    implicitplot, implicitplot3d, inequal, interactive, interactiveparams, intersectplot,
    listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot,
    matrixplot, multiple, odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot,
    polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, rootlocus,
    semilogplot, setcolors, setoptions, setoptions3d, shadebetween, spacecurve,
    sparsematrixplot, surfdata, textplot, textplot3d, tubeplot]
> with(plottools)
[annulus, arc, arrow, circle, cone, cuboid, curve, cutin, cutout, cylinder, disk, dodecahedron,      (3)
    ellipse, ellipticArc, exportplot, extrude, getdata, hemisphere, hexahedron, homothety,
    hyperbola, icosahedron, importplot, line, octahedron, parallelepiped, pieslice, point,
    polygon, prism, project, rectangle, reflect, rotate, scale, sector, semitorus, sphere,
    stellate, tetrahedron, torus, transform, translate]
> p1 := implicitplot3d(x^2 + y^2 - z^2 - 1 = 0, x = -2 ..2, y = -2 ..2, z = -2 ..2, style = surface,
      scaling = constrained, numpoints = 100000, transparency = 0.50) :
> p2 := implicitplot3d(x + y + 2·z = 0, x = -2 ..2, y = -2 ..2, z = -2 ..2, style = surface, scaling
      = constrained, transparency = 0.50) :
> pxy := implicitplot3d(z = 0, x = -2 ..2, y = -2 ..2, z = -2 ..2, style = surface, scaling
      = constrained, transparency = 0.50, color = black) :
> p3 := intersectplot(x^2 + y^2 - z^2 - 1 = 0, x + y + 2·z = 0, x = -2 ..2, y = -2 ..2, z = -2 ..2,
      thickness = 4) :
> p4 := pointplot3d([-1, -1, 1], symbol = solidcircle, symbolsize = 25, color = blue) :
> p5 := pointplot3d([1, 1, -1], symbol = solidcircle, symbolsize = 25, color = green) :
> p0 := pointplot3d([0, 0, 0], symbol = solidcircle, symbolsize = 25, color = black) :
> p6 := line([-1, -1, 1], [-1, -1, 0], color = blue) :
> p7 := line([1, 1, -1], [1, 1, 0], color = green) :
> 
> display(p1, p2, pxy, p3, p4, p5, p0, p6, p7, orientation = [25, 80, 0], title
      = "Μέγιστη και ελάχιστη κατηγμένη\nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ", titlefont = [arial,
      bold, 14])
```

Μέγιστη και ελάχιστη κατηγμένη
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ



>

>

Θεωρούμε την έλλειψη που είναι τομή του υπερβολοειδούς με εξίσωση $x^2 + y^2 - z^2 = 1$ με το επίπεδο $x + y + 2 \cdot z = 0$.
Να βρεθούν , αν υπάρχουν , τα σημεία της έλλειψης με την μεγίστη και με την ελαχίστη **κατηγμένη** .

>

>

Ακολουθώντας τα βήματα της θεωρίας Πολλαπλασιαστές Lagrange .

>

Η τομή είναι έλλειψη σε πεπλεγμένη μορφή και οι κατηγμένες της (z) συναληθεύουν τις δύο εξισώσεις .

> $expand\left(subs\left(z = -\frac{1}{2} \cdot (x+y), x^2 + y^2 - z^2 - 1 = 0\right)\right)$

$$\frac{3}{4}x^2 + \frac{3}{4}y^2 - \frac{1}{2}xy - 1 = 0 \quad (4)$$

> $f := -\frac{1}{2} \cdot (x+y)$

$$f := -\frac{x}{2} - \frac{y}{2} \quad (5)$$

> g[1] := lhs((4))

$$g_1 := \frac{3}{4}x^2 + \frac{3}{4}y^2 - \frac{1}{2}xy - 1 \quad (6)$$

> G := f + λ[1]·g[1]

$$G := \left(\frac{3}{4}x^2 + \frac{3}{4}y^2 - \frac{1}{2}xy - 1\right)\lambda_1 - \frac{x}{2} - \frac{y}{2} \quad (7)$$

> diff(G, x) = 0

$$\left(\frac{3x}{2} - \frac{y}{2}\right)\lambda_1 - \frac{1}{2} = 0 \quad (8)$$

> diff(G, y) = 0

$$\left(\frac{3y}{2} - \frac{x}{2}\right)\lambda_1 - \frac{1}{2} = 0 \quad (9)$$

> diff(G, λ[1]) = 0

$$\frac{3}{4}x^2 + \frac{3}{4}y^2 - \frac{1}{2}xy - 1 = 0 \quad (10)$$

> ***solve*({(8), (9), (10)}, {x, y, λ[1]})**

$$\left\{x = 1, y = 1, \lambda_1 = \frac{1}{2}\right\}, \left\{x = -1, y = -1, \lambda_1 = -\frac{1}{2}\right\} \quad (11)$$

>

> solve({(8), (9)}, {x, y})

$$\left\{x = \frac{1}{2\lambda_1}, y = \frac{1}{2\lambda_1}\right\} \quad (12)$$

> solve(subs((12), (10)), λ[1])

$$-\frac{1}{2}, \frac{1}{2} \quad (13)$$

> subs(λ[1] = (13)[1], (12))

$$\{x = -1, y = -1\} \quad (14)$$

> subs(λ[1] = (13)[2], (12))

$$\{x = 1, y = 1\} \quad (15)$$

> maxZ := subs((14), (5))

$$maxZ := 1 \quad (16)$$

> minZ := subs((15), (5))

$$minZ := -1 \quad (17)$$

>

>