

ΘΕΜΑ :

Βλήμα P εκτοξεύεται από τόπο **Γεωγραφικού Πλάτους** $(\frac{\pi}{2} - \theta)$ και ύψος H έτσι ώστε η αρχική του ταχύτητα u_0 να σχηματίζει **γωνία α** με το οριζόντιο επίπεδο $(\hat{\theta} \hat{B} \hat{\phi})$ και **β** με το κατακόρυφο επίπεδο $(\hat{\theta} \hat{B} \hat{r})$

και να διευθύνεται προς ΒΑ .

Να βρεθεί η θέση πρόσκρουσης στην Επιφάνεια της ΓΗΣ .

Παραδοχές :

Η τιμή g της επιτάχυνσης της βαρύτητας διατηρείται σταθερή .

Λαμβάνεται υπόψη η Coriolis η κάθετη στο επίπεδο του Μεσημβρινού .

ΠΑΡΑΤΗΡΗΣΗ :

Για οποιοδήποτε Διάνυσμα A (Θέσεως, Ταχύτητας, Ροπής, Επιταχύνσεως, κλπ.) που είναι συνάρτηση του χρόνου t ισχύει η σχέση :

$$O \equiv \Omega \left[\frac{d\vec{A}}{dt} \Big|_{\Omega} = \frac{d\vec{A}}{dt} \Big|_O + \vec{\omega} \times \vec{A} \right]$$

Ω Αδρανειακό Σύστημα, Αδρανειακός παρατηρητής
 O ΜΗ Αδρανειακό Σύστημα, ΜΗ Αδρανειακός παρατηρητής

ΒΑΣΙΚΟ : Για ΑΔΡΑΝΕΙΑΚΟ παρατηρητή τα Μοναδιαία Διανύσματα Μεταβάλλονται Συναρτήσει του Χρόνου . Επομένως πρέπει να Υπολογίζουμε τις Παραγώγους των Μοναδιαίων Διανυσμάτων Συναρτήσει του Χρόνου .

Το Maple Υπολογίζει τις παραγώγους ΜΟΝΟ των Μοναδιαίων Διανυσμάτων : $\hat{r}(t)$, $\hat{\theta}(t)$, $\hat{\phi}(t)$.!

with(Physics[Vectors]) :
 Setup(mathematicalnotation = true) :
 $r_ := r(t) \cdot \hat{r}(t)$

$$\dot{\vec{r}} := \dot{r}(t) \hat{r}(t)$$

$\text{diff}(r_ , t)$

$$\dot{r}(t) \hat{r}(t) + r(t) (\dot{\theta}(t) \hat{\theta}(t) + \sin(\theta(t)) \dot{\phi}(t) \hat{\phi}(t))$$

$R_ := x(t) \cdot \hat{i}(t) + y(t) \cdot \hat{j}(t) + z(t) \cdot \hat{k}(t)$

$$\dot{\vec{R}} := \dot{x}(t) \hat{i}(t) + \dot{y}(t) \hat{j}(t) + \dot{z}(t) \hat{k}(t)$$

$\text{diff}(R_ , t)$

$$\dot{x}(t) \hat{i}(t) + \dot{x}(t) \dot{\hat{i}}(t) + \dot{y}(t) \hat{j}(t) + \dot{y}(t) \dot{\hat{j}}(t) + \dot{z}(t) \hat{k}(t) + \dot{z}(t) \dot{\hat{k}}(t)$$

ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ

$$\vec{\omega} = |\omega| \cos(\theta) \hat{r} - |\omega| \sin(\theta) \hat{\theta}$$

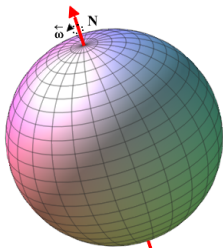
$$\dot{\phi}(t) = \text{diff}(\phi(t), t) = |\omega|$$

$$\dot{\phi}(t) = \text{diff}(\phi(t), tS2) = 0$$

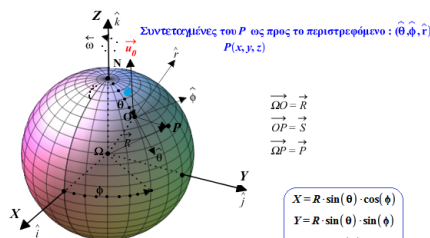
ΒΕΡΟΙΑ :

Γεωγραφικό Πλάτος : $(\frac{\pi}{2} - \theta) = 40^\circ 31' 23''$ N (Βορράς)

Γεωγραφικό Μήκος : $\phi = 22^\circ 12' 12''$ E (Ανατολή)



Ακτίνα Γης : $R = 6378, 750 \sim 6378, 135 \approx 6367, 4425 \text{ km}$
 Γωνιακή ταχύτητα της Γης : $|\omega| = 7, 292 \cdot 10^{-5} \text{ rad} \cdot \text{s}^{-1}$



$$\begin{aligned} \vec{OP} &= \vec{R} \\ \vec{OP} &= \vec{S} \\ \vec{OP} &= \vec{P} \end{aligned}$$

$$\begin{aligned} X &= R \cdot \sin(\theta) \cdot \cos(\phi) \\ Y &= R \cdot \sin(\theta) \cdot \sin(\phi) \\ Z &= R \cdot \cos(\theta) \end{aligned}$$

$$\hat{r} = \sin(\theta) \cos(\phi) \hat{i} + \sin(\theta) \sin(\phi) \hat{j} + \cos(\theta) \hat{k}$$

$$\hat{\theta} = \cos(\theta) \cos(\phi) \hat{i} + \cos(\theta) \sin(\phi) \hat{j} - \sin(\theta) \hat{k}$$

$$\hat{\phi} = -\hat{i} \sin(\phi) + \hat{j} \cos(\phi)$$

$$\begin{bmatrix} \vec{e}_1 \\ \vec{e}_2 \\ \vec{e}_3 \end{bmatrix} = \begin{bmatrix} \cos(a) \hat{i} + \cos(b) \hat{j} + \cos(c) \hat{k} \\ \cos(d) \hat{i} + \cos(e) \hat{j} + \cos(f) \hat{k} \\ \cos(g) \hat{i} + \cos(h) \hat{j} + \cos(i) \hat{k} \end{bmatrix}$$

$$\vec{e}_1 = \cos(a) \hat{i} + \cos(b) \hat{j} + \cos(c) \hat{k}$$

$$\vec{e}_2 = \cos(d) \hat{i} + \cos(e) \hat{j} + \cos(f) \hat{k}$$

$$\vec{e}_3 = \cos(g) \hat{i} + \cos(h) \hat{j} + \cos(i) \hat{k}$$

$$\begin{bmatrix} \hat{i} \\ \hat{j} \\ \hat{k} \end{bmatrix} = \begin{bmatrix} \cos(a) \vec{e}_1 + \cos(d) \vec{e}_2 + \cos(g) \vec{e}_3 \\ \cos(b) \vec{e}_1 + \cos(e) \vec{e}_2 + \cos(h) \vec{e}_3 \\ \cos(c) \vec{e}_1 + \cos(f) \vec{e}_2 + \cos(i) \vec{e}_3 \end{bmatrix}$$

$$\hat{i} = \cos(a) \vec{e}_1 + \cos(d) \vec{e}_2 + \cos(g) \vec{e}_3$$

$$\hat{j} = \cos(b) \vec{e}_1 + \cos(e) \vec{e}_2 + \cos(h) \vec{e}_3$$

$$\hat{k} = \cos(c) \vec{e}_1 + \cos(f) \vec{e}_2 + \cos(i) \vec{e}_3$$

ΜΕΤΑΣΧΗΜΑΤΙΣΜΟΣ ΣΥΣΤΗΜΑΤΩΝ

$$A_x = A_{e_1} \cos(a) + A_{e_2} \cos(d) + A_{e_3} \cos(g)$$

$$A_y = A_{e_1} \cos(b) + A_{e_2} \cos(e) + A_{e_3} \cos(h)$$

$$A_z = A_{e_1} \cos(c) + A_{e_2} \cos(f) + A_{e_3} \cos(i)$$

$$PX := x(t) \cdot \cos a + y(t) \cdot \cos d + z(t) \cdot \cos g \quad PX := x(t) \cos(\theta) \cos(\phi) - y(t) \sin(\phi) + z(t) \sin(\theta) \cos(\phi)$$

$$PY := x(t) \cdot \cos b + y(t) \cdot \cos e + z(t) \cdot \cos h \quad PY := x(t) \cos(\theta) \sin(\phi) + y(t) \cos(\phi) + z(t) \sin(\theta) \sin(\phi)$$

$$PZ := x(t) \cdot \cos c + y(t) \cdot \cos f + z(t) \cdot \cos i \quad PZ := -x(t) \sin(\theta) + z(t) \cos(\theta)$$

ΕΠΑΛΗΘΕΥΣΗ: Έστωσαν (z, x, y) οι συντεταγμένες σημείου L ως προς το σύστημα (r, θ, ϕ) .

$$\vec{L}_- := zt \cdot \vec{r}_- + xt \cdot \vec{\theta}_- + yt \cdot \vec{\phi}_- \quad \vec{L}_- := yz \hat{\phi} + zt \hat{r} + xt \hat{\theta}$$

ΩΣ ΠΡΟΣ ΤΟ ΩXYZ ΓΙΝΟΝΤΑΙ:

ChangeBasis(L_-, 1)

$$(zt \sin(\theta) \cos(\phi) + xt \cos(\theta) \cos(\phi) - yt \sin(\phi)) \hat{i} + (zt \sin(\theta) \sin(\phi) + xt \cos(\theta) \sin(\phi) + yt \cos(\phi)) \hat{j} + (-\sin(\theta) xt + \cos(\theta) zt) \hat{k}$$

$$LX := \text{Component}((18), 1) \quad LX := zt \sin(\theta) \cos(\phi) + xt \cos(\theta) \cos(\phi) - yt \sin(\phi)$$

$$LY := \text{Component}((18), 2) \quad LY := zt \sin(\theta) \sin(\phi) + xt \cos(\theta) \sin(\phi) + yt \cos(\phi)$$

$$LZ := \text{Component}((18), 3) \quad LZ := -\sin(\theta) xt + \cos(\theta) zt$$

> with(Physics[Vectors]) :
 > Setup(mathematicalnotation = true) :
 > with(plots) :
 > with(plottools) :

Δεδομένα :

> $R_ := R \cdot _r$ $\vec{R} := R \hat{r}$ (1)

> $S_ := x(t) \cdot _ \theta + y(t) \cdot _ \phi + z(t) \cdot _ r$ $\vec{S} := x(t) \hat{\theta} + y(t) \hat{\phi} + z(t) \hat{r}$ (2)

> $P_ := R_ + S_$ $\vec{P} := \hat{r} (R + z(t)) + x(t) \hat{\theta} + y(t) \hat{\phi}$ (3)

> $RI := 6367442$ $RI := 6367442$ (4)

> $\Omega := 7.292 \cdot 10^{-5}$ $\Omega := 0.00007292000000$ (5)

> $H := 10000$ $H := 10000$ (6)

> $\alpha := \frac{\text{Pi}}{4}$ $\alpha := \frac{\pi}{4}$ (7)

> $\beta := 0.25$ $\beta := 0.25$ (8)

> $u[0] := 2000$ $u_0 := 2000$ (9)

> $g := 9.80$ $g := 9.80$ (10)

> $GPI := 0.7072607424$ $GPI := 0.7072607424$ (11)

> $GMI := 0.3875212718$ $GMI := 0.3875212718$ (12)

> $\omega_ := \Omega \cdot \cos\left(\frac{\pi}{2} - GPI\right) \cdot _ r - \Omega \cdot \sin\left(\frac{\pi}{2} - GPI\right) \cdot _ \theta$ $\vec{\omega} := 0.00004738006019 \hat{r} - 0.00005542974199 \hat{\theta}$ (13)

> $u0_ := -u[0] \cdot \cos(\alpha) \cos(\beta) \cdot _ \theta + u[0] \cdot \cos(\alpha) \sin(\beta) \cdot _ \phi + u[0] \cdot \sin(\alpha) \cdot _ r$ $\vec{u0} := 247.4039593 \sqrt{2} \hat{\phi} - 968.9124217 \sqrt{2} \hat{\theta} + 1000. \sqrt{2} \hat{r}$ (14)

> $F_ := -m \cdot g \cdot _ r$ $\vec{F} := -9.80 m \hat{r}$ (15)

$$\left[\begin{array}{l} > \text{evalf}\left(\frac{\pi}{2} - \text{GPI}\right) \\ \\ \end{array} \right. \quad 0.8635355846 \quad (16)$$

Ανάλυση :

$$\left[\begin{array}{l} > u_ := \text{diff}(P_ , t) + \omega_ \times P_ \\ \vec{u} := \widehat{r} (\dot{z}(t) - 0.00005542974199 y(t)) + \widehat{\theta} (\dot{x}(t) - 0.00004738006019 y(t)) + \widehat{\phi} (\dot{y}(t) \\ + 0.00004738006019 x(t) + 0.00005542974199 R + 0.00005542974199 z(t)) \end{array} \right. \quad (17)$$

$$\left[\begin{array}{l} > a_ := \text{diff}(u_ , t) + \omega_ \times u_ \\ \vec{a} := \widehat{r} (\ddot{z}(t) - 0.0001108594840 \dot{y}(t) - 2.626264512 \cdot 10^{-9} x(t) - 3.072456297 \cdot 10^{-9} R \\ - 3.072456297 \cdot 10^{-9} z(t)) + \widehat{\theta} (\ddot{x}(t) - 0.00009476012038 \dot{y}(t) - 2.244870104 \cdot 10^{-9} x(t) \\ - 2.626264512 \cdot 10^{-9} R - 2.626264512 \cdot 10^{-9} z(t)) + \widehat{\phi} (\ddot{y}(t) + 0.00009476012038 \dot{x}(t) \\ + 0.0001108594840 \dot{z}(t) - 5.317326401 \cdot 10^{-9} y(t)) \end{array} \right. \quad (18)$$

$$\left[\begin{array}{l} > \text{FYGOKENTROS} := \omega_ \times (\omega_ \times P_) \\ \text{FYGOKENTROS} := -5.317326401 \cdot 10^{-9} y(t) \widehat{\phi} + (-2.626264512 \cdot 10^{-9} x(t) \\ - 3.072456297 \cdot 10^{-9} R - 3.072456297 \cdot 10^{-9} z(t)) \widehat{r} + (-2.244870104 \cdot 10^{-9} x(t) \\ - 2.626264512 \cdot 10^{-9} R - 2.626264512 \cdot 10^{-9} z(t)) \widehat{\theta} \end{array} \right. \quad (19)$$

$$\left[\begin{array}{l} > \text{CORIOLIS} := 2 \cdot \omega_ \times (\text{diff}(P_ , t)) \\ \text{CORIOLIS} := -0.0001108594840 \widehat{r} \dot{y}(t) - 0.00009476012038 \widehat{\theta} \dot{y}(t) + (0.00009476012038 \\ \dot{x}(t) + 0.0001108594840 \dot{z}(t)) \widehat{\phi} \end{array} \right. \quad (20)$$

Αν αγνοηθεί η φυγόκεντρος $\vec{\omega} \times (\vec{\omega} \times \vec{P})$:

$$\left[\begin{array}{l} > a_ := (18) - (19) \\ \vec{a} := \widehat{r} (\ddot{z}(t) - 0.0001108594840 \dot{y}(t)) + \widehat{\theta} (\ddot{x}(t) - 0.00009476012038 \dot{y}(t)) + \widehat{\phi} (\ddot{y}(t) \\ + 0.00009476012038 \dot{x}(t) + 0.0001108594840 \dot{z}(t)) \end{array} \right. \quad (21)$$

ΑΝ αγνοηθεί η Coriolis κατά : $\widehat{\theta} :-2 \dot{y}(t) |\omega| \cos(\theta) \widehat{\theta}$, $\widehat{r} :-2 \dot{y}(t) |\omega| \sin(\theta) \widehat{r}$
θα έχουμε :

$$\left[\begin{array}{l} > a_ := (21) - \left(-2 \dot{y}(t) \Omega \cos\left(\frac{\pi}{2} - \text{GPI}\right) \widehat{\theta} \right) - \left(-2 \dot{y}(t) \Omega \sin\left(\frac{\pi}{2} - \text{GPI}\right) \widehat{r} \right) \\ \vec{a} := \widehat{r} \ddot{z}(t) + \widehat{\theta} \ddot{x}(t) + \widehat{\phi} (\ddot{y}(t) + 0.00009476012038 \dot{x}(t) + 0.0001108594840 \dot{z}(t)) \end{array} \right. \quad (22)$$

ΤΕΛΙΚΗ ΕΚΦΡΑΣΗ ΤΗΣ ΕΠΙΤΑΧΥΝΣΕΩΣ ΤΟΥ ΒΑΗΜΑΤΟΣ Ρ ΩΣ ΠΡΟΣ ΤΟ ΑΔΡΑΝΕΙΑΚΟ ΣΥΣΤΗΜΑ :

$$\vec{a} := 2 \dot{z}(t) \Omega \sin(\theta) \widehat{\phi} + 2 \dot{x}(t) \Omega \cos(\theta) \widehat{\phi} + \ddot{z}(t) \widehat{r} + \ddot{x}(t) \widehat{\theta} + \ddot{y}(t) \widehat{\phi}$$

Μπορούμε να γράψουμε :

$$\left[\begin{array}{l} > -g \cdot r = (22) \\ -9.80 \widehat{r} = \widehat{r} \ddot{z}(t) + \widehat{\theta} \ddot{x}(t) + \widehat{\phi} (\ddot{y}(t) + 0.00009476012038 \dot{x}(t) + 0.0001108594840 \dot{z}(t)) \end{array} \right. \quad (23)$$

$$\left[\begin{array}{l} > \text{lhs}((23)) - \text{rhs}((23)) = 0 \end{array} \right.$$

$$(-\ddot{y}(t) - 0.00009476012038 \dot{x}(t) - 0.0001108594840 \dot{z}(t)) \hat{\phi} + (-9.80 - \ddot{z}(t)) \hat{r} - \hat{\theta} \ddot{x}(t) = 0 \quad (24)$$

$$\begin{aligned} > \text{Component}(\text{lhs}((24)), 1) = 0 \\ & -9.80 - \ddot{z}(t) = 0 \end{aligned} \quad (25)$$

$$\begin{aligned} > \text{Component}(\text{lhs}((24)), 2) = 0 \\ & -\ddot{x}(t) = 0 \end{aligned} \quad (26)$$

$$\begin{aligned} > \text{Component}(\text{lhs}((24)), 3) = 0 \\ & -\ddot{y}(t) - 0.00009476012038 \dot{x}(t) - 0.0001108594840 \dot{z}(t) = 0 \end{aligned} \quad (27)$$

$$\begin{aligned} > \text{Eq1} := (26) \cdot (-1) \\ & \text{Eq1} := \ddot{x}(t) = 0 \end{aligned} \quad (28)$$

$$\begin{aligned} > \text{Eq2} := (27) \cdot (-1) \\ & \text{Eq2} := \ddot{y}(t) + 0.00009476012038 \dot{x}(t) + 0.0001108594840 \dot{z}(t) = 0 \end{aligned} \quad (29)$$

$$\begin{aligned} > \text{Eq3} := (25) \cdot (-1) \\ & \text{Eq3} := 9.80 + \ddot{z}(t) = 0 \end{aligned} \quad (30)$$

$$\begin{aligned} > \text{ics1} := x(0) = 0, D(x)(0) = -u[0] \cdot \cos(\alpha) \cdot \cos(\beta) \\ & \text{ics1} := x(0) = 0, D(x)(0) = -968.9124217 \sqrt{2} \end{aligned} \quad (31)$$

$$\begin{aligned} > \text{ics3} := z(0) = \mathbf{H}, D(z)(0) = u[0] \cdot \sin(\alpha) \\ & \text{ics3} := z(0) = 10000, D(z)(0) = 1000 \sqrt{2} \end{aligned} \quad (32)$$

$$\begin{aligned} > \text{ics2} := y(0) = 0, D(y)(0) = u[0] \cdot \cos(\alpha) \cdot \sin(\beta) \\ & \text{ics2} := y(0) = 0, D(y)(0) = 247.4039593 \sqrt{2} \end{aligned} \quad (33)$$

$$\begin{aligned} > \text{sol1} := \text{dsolve}(\{\text{Eq1}, \text{ics1}\}) \\ & \text{sol1} := x(t) = -\frac{9689124217 \sqrt{2} t}{10000000} \end{aligned} \quad (34)$$

$$\begin{aligned} > \text{sol3} := \text{dsolve}(\{\text{Eq3}, \text{ics3}\}) \\ & \text{sol3} := z(t) = -\frac{49 t^2}{10} + 1000 \sqrt{2} t + 10000 \end{aligned} \quad (35)$$

$$\begin{aligned} > \dot{x}(t) = \text{diff}(\text{rhs}((34)), t) \\ & \dot{x}(t) = -\frac{9689124217 \sqrt{2}}{10000000} \end{aligned} \quad (36)$$

$$\begin{aligned} > \dot{z}(t) = \text{diff}(\text{rhs}((35)), t) \\ & \dot{z}(t) = -\frac{49 t}{5} + 1000 \sqrt{2} \end{aligned} \quad (37)$$

$$\begin{aligned} > \text{subs}(\{(36), (37)\}, \text{Eq2}) \\ & \ddot{y}(t) + 0.01904522628 \sqrt{2} - 0.001086422943 t = 0 \end{aligned} \quad (38)$$

$$\begin{aligned} > \text{sol2} := \text{dsolve}(\{(38), \text{ics2}\}) \\ & \text{sol2} := y(t) = \frac{362140981 t^3}{2000000000000} - \frac{476130657 \sqrt{2} t^2}{50000000000} + \frac{2474039593 \sqrt{2} t}{10000000} \end{aligned} \quad (39)$$

Χρόνος απαιτούμενος *toriz* για άφιξη του Βλήματος στο ΟΡΙΖΟΝΤΙΟ επίπεδο Οθφρ .

$$\begin{aligned} > \text{toriz} := \text{fsolve}(\text{subs}(z(t) = 0, (35)), t = 0.1 .. \infty) \\ & \text{toriz} := 295.5208414 \end{aligned} \quad (40)$$

$$\begin{aligned} > \text{xt} := \text{evalf}(\text{subs}(t = \text{toriz}, \text{rhs}((34)))) \\ & \text{xt} := -404937.1632 \end{aligned} \quad (41)$$

$$\begin{aligned} > \text{yt} := \text{evalf}(\text{subs}(t = \text{toriz}, \text{rhs}((39)))) \\ & \text{yt} := 106894.4996 \end{aligned} \quad (42)$$

$$\begin{aligned} > \text{zt} := \text{evalf}(\text{subs}(t = \text{toriz}, \text{rhs}((35)))) \\ & \text{zt} := 0.0001 \end{aligned} \quad (43)$$

ΑΡΧΙΚΗ ΤΑΧΥΤΗΤΑ ΕΚΤΟΞΕΥΣΗΣ $u_0 := ???$

ΓΩΝΙΑ ΕΚΤΟΞΕΥΣΗΣ ΩΣ ΠΡΟΣ ΤΟ ΟΡΙΖΟΝΤΙΟ ΕΠΙΠΕΔΟ : $\alpha := ???$

ΓΩΝΙΑ ΕΚΤΟΞΕΥΣΗΣ ΩΣ ΠΡΟΣ ΤΟ ΚΑΤΑΚΟΡΥΦΟ ΕΠΙΠΕΔΟ : $\beta := ???$

ΕΠΤΑΧΥΝΣΗ ΒΑΡΥΤΗΤΑΣ ΣΤΑΘΕΡΗ : $g = 9.80 \frac{[m]}{[s]^2}$

ΓΩΝΙΑΚΗ ΤΑΧΥΤΗΤΗΤΑ ΠΕΡΙΣΤΡΟΦΗΣ ΤΗΣ ΓΗΣ : $\Omega = 7.292 \cdot 10^{-5} \frac{[rad]}{[s]}$

ΝΕΕΣ ΘΕΣΕΙΣ ΜΕΤΑ ΤΗΝ ΣΤΡΟΦΗ $D\phi = \Omega \cdot Tpr$ [rad]

$$\begin{aligned} > xBn := RI \cdot \sin\left(\frac{\pi}{2} - GP1\right) \cdot \cos(GM1 + D\phi) \\ & xBn := 4.840176456 \cdot 10^6 \cos(0.3875212718 + D\phi) \end{aligned} \quad (44)$$

$$\begin{aligned} > yBn := RI \cdot \sin\left(\frac{\pi}{2} - GP1\right) \cdot \sin(GM1 + D\phi) \\ & yBn := 4.840176456 \cdot 10^6 \sin(0.3875212718 + D\phi) \end{aligned} \quad (45)$$

$$\begin{aligned} > zBn := RI \cdot \cos\left(\frac{\pi}{2} - GP1\right) \\ & zBn := 4.137270779 \cdot 10^6 \end{aligned} \quad (46)$$

$$\begin{aligned} > xMOn := RI \cdot \sin\left(\frac{\pi}{2} - GPMO\right) \cdot \cos(GMMO + D\phi) \\ & xMOn := 6367442 \cos(GPMO) \cos(GMMO + D\phi) \end{aligned} \quad (47)$$

$$\begin{aligned} > yMOn := RI \cdot \sin\left(\frac{\pi}{2} - GPMO\right) \cdot \sin(GMMO + D\phi) \\ & yMOn := 6367442 \cos(GPMO) \sin(GMMO + D\phi) \end{aligned} \quad (48)$$

$$\begin{aligned} > zMOn := RI \cdot \cos\left(\frac{\pi}{2} - GPMO\right) \\ & zMOn := 6367442 \sin(GPMO) \end{aligned} \quad (49)$$

$$> \text{DistanceBMon} := \text{sqrt}((xBn - xMOn)^2 + (yBn - yMOn)^2 + (zBn - zMOn)^2) :$$

ΣΥΝΤΕΤΑΓΜΕΝΕΣ ΘΕΣΗΣ ΕΚΤΟΞΕΥΣΗΣ ΩΣ ΠΡΟΣ ΤΟ

ΑΔΡΑΝΕΙΑΚΟ ΣΥΣΤΗΜΑ ΩΧΥΖ :

$$\begin{aligned} > XTE := R1 \cdot \sin\left(\frac{\pi}{2} - GP1\right) \cdot \cos(GM1 + \Omega \cdot t) \\ XTE := 4.840176456 \cdot 10^6 \cos(0.00007292000000 t + 0.3875212718) \end{aligned} \quad (50)$$

$$\begin{aligned} > YTE := R1 \cdot \sin\left(\frac{\pi}{2} - GP1\right) \cdot \sin(GM1 + \Omega \cdot t) \\ YTE := 4.840176456 \cdot 10^6 \sin(0.00007292000000 t + 0.3875212718) \end{aligned} \quad (51)$$

$$\begin{aligned} > ZTE := R1 \cdot \cos\left(\frac{\pi}{2} - GP1\right) \\ ZTE := 4.137270779 \cdot 10^6 \end{aligned} \quad (52)$$

$$\begin{aligned} > evalf(subs(t=0, (50))) \\ 4.481270602 \cdot 10^6 \end{aligned} \quad (53)$$

$$\begin{aligned} > evalf(subs(t=0, (51))) \\ 1.829076794 \cdot 10^6 \end{aligned} \quad (54)$$

ΣΥΝΤΕΤΑΓΜΕΝΕΣ ΒΛΗΜΑΤΟΣ ΩΣ ΠΡΟΣ ΤΟ ΑΔΡΑΝΕΙΑΚΟ ΣΥΣΤΗΜΑ ΩΧΥΖ : (ΠΡΟΣΟΧΗ ΣΤΙΣ ΑΝΤΙΣΤΟΙΧΙΕΣ)

$$PX := x(t) \cdot \cos\alpha + y(t) \cdot \cos\delta + z(t) \cdot \cos\sigma \quad PX := x(t) \cos(\theta) \cos(\phi) - y(t) \sin(\phi) + z(t) \sin(\theta) \cos(\phi)$$

$$PY := x(t) \cdot \cos\beta + y(t) \cdot \cos\epsilon + z(t) \cdot \cos\eta \quad PY := x(t) \cos(\theta) \sin(\phi) + y(t) \cos(\phi) + z(t) \sin(\theta) \sin(\phi)$$

$$PZ := x(t) \cdot \cos\gamma + y(t) \cdot \cos\zeta + z(t) \cdot \cos\iota \quad PZ := -x(t) \sin(\theta) + z(t) \cos(\theta)$$

ΕΠΙΛΗΘΕΥΣΗ : Έστωσαν (zt, xt, yt) οι συντεταγμένες σημείου L ως προς το σύστημα (r, θ, ϕ) .

$$\vec{L}_- := zt \cdot \vec{r} + xt \cdot \vec{\theta} + yt \cdot \vec{\phi} \quad \vec{L}_- := yz \hat{\phi} + zt \hat{r} + xt \hat{\theta}$$

ΩΣ ΠΡΟΣ ΤΟ ΩΧΥΖ ΓΙΝΟΝΤΑΙ :

$ChangeBasis(L_-, 1)$

$$(zt \sin(\theta) \cos(\phi) + xt \cos(\theta) \cos(\phi) - yt \sin(\phi)) \hat{i} + (zt \sin(\theta) \sin(\phi) + xt \cos(\theta) \sin(\phi) + yt \cos(\phi)) \hat{j} + (-\sin(\theta) xt + \cos(\theta) zt) \hat{k}$$

$$LX := Component((18), 1) \quad LX := zt \sin(\theta) \cos(\phi) + xt \cos(\theta) \cos(\phi) - yt \sin(\phi)$$

$$LY := Component((18), 2) \quad LY := zt \sin(\theta) \sin(\phi) + xt \cos(\theta) \sin(\phi) + yt \cos(\phi)$$

$$LZ := Component((18), 3) \quad LZ := -\sin(\theta) xt + \cos(\theta) zt$$

$$\begin{aligned} > LX := \sin\left(\frac{\pi}{2} - GP1\right) \cdot \cos(GM1 + \Omega \cdot t) \cdot rhs(sol3) + \cos\left(\frac{\pi}{2} - GP1\right) \cdot \cos(GM1 + \Omega \cdot t) \\ \cdot rhs(sol1) - \sin(GM1 + \Omega \cdot t) \cdot rhs(sol2) : \end{aligned}$$

$$\begin{aligned} > LY := \sin\left(\frac{\pi}{2} - GP1\right) \cdot \sin(GM1 + \Omega \cdot t) \cdot rhs(sol3) + \cos\left(\frac{\pi}{2} - GP1\right) \cdot \sin(GM1 + \Omega \cdot t) \\ \cdot rhs(sol1) + \cos(GM1 + \Omega \cdot t) \cdot rhs(sol2) : \end{aligned}$$

$$\begin{aligned} > LZ := -\sin\left(\frac{\text{Pi}}{2} - \text{GP1}\right) \cdot rhs(\text{sol1}) + \cos\left(\frac{\text{Pi}}{2} - \text{GP1}\right) \cdot rhs(\text{sol3}) : \\ > XBL := LX + RI \cdot \sin\left(\frac{\pi}{2} - \text{GP1}\right) \cdot \cos(\text{GM1} + \Omega \cdot t) \\ XBL := 0.7601445692 \cos(0.00007292000000 t + 0.3875212718) \left(-\frac{49 t^2}{10} + 1000 \sqrt{2} t \right. \end{aligned} \quad (55)$$

$$\begin{aligned} & \left. + 10000 \right) - 629.5547019 \cos(0.00007292000000 t + 0.3875212718) \sqrt{2} t \\ & - \sin(0.00007292000000 t + 0.3875212718) \left(\frac{362140981 t^3}{2000000000000} - \frac{476130657 \sqrt{2} t^2}{50000000000} \right. \\ & \left. + \frac{2474039593 \sqrt{2} t}{10000000} \right) + 4.840176456 10^6 \cos(0.00007292000000 t + 0.3875212718) \end{aligned}$$

$$\begin{aligned} > YBL := LY + RI \cdot \sin\left(\frac{\pi}{2} - \text{GP1}\right) \cdot \sin(\text{GM1} + \Omega \cdot t) \\ YBL := 0.7601445692 \sin(0.00007292000000 t + 0.3875212718) \left(-\frac{49 t^2}{10} + 1000 \sqrt{2} t \right. \end{aligned} \quad (56)$$

$$\begin{aligned} & \left. + 10000 \right) - 629.5547019 \sin(0.00007292000000 t + 0.3875212718) \sqrt{2} t \\ & + \cos(0.00007292000000 t + 0.3875212718) \left(\frac{362140981 t^3}{2000000000000} - \frac{476130657 \sqrt{2} t^2}{50000000000} \right. \\ & \left. + \frac{2474039593 \sqrt{2} t}{10000000} \right) + 4.840176456 10^6 \sin(0.00007292000000 t + 0.3875212718) \end{aligned}$$

$$\begin{aligned} > ZBL := LZ + RI \cdot \cos\left(\frac{\pi}{2} - \text{GP1}\right) \\ ZBL := 1386.267495 \sqrt{2} t - 3.183794500 t^2 + 4.143768319 10^6 \end{aligned} \quad (57)$$

ΣΥΝΤΕΤΑΓΜΕΝΕΣ ΣΤΟΧΟΥ ΩΣ ΠΡΟΣ ΤΟ ΑΔΡΑΝΕΙΑΚΟ ΣΥΣΤΗΜΑ ΩΧΥΖ :

$$\begin{aligned} > XST := RI \cdot \sin\left(\frac{\pi}{2} - \text{GPMO}\right) \cdot \cos(\text{GMMO} + \Omega \cdot t) : \\ > YST := RI \cdot \sin\left(\frac{\pi}{2} - \text{GPMO}\right) \cdot \sin(\text{GMMO} + \Omega \cdot t) : \\ > ZST := RI \cdot \cos\left(\frac{\pi}{2} - \text{GPMO}\right) : \end{aligned}$$

ΠΡΟΣΔΙΟΡΙΣΜΟΣ ΤΩΝ ΣΥΝΤΕΤΑΓΜΕΝΩΝ θ,φ, ΤΟΥ ΣΗΜΕΙΟΥ ΠΡΟΣΚΡΟΥΣΗΣ :

ΩΣ ΠΡΟΣ ΤΟ ΑΔΡΑΝΕΙΑΚΟ ΣΥΣΤΗΜΑ ΩΧΥΖ

$$\begin{aligned} > XPROSKR := R1 \cdot \sin(\theta) \cdot \cos(\phi) \\ & \quad XPROSKR := 6367442 \sin(\theta) \cos(\phi) \end{aligned} \quad (58)$$

$$\begin{aligned} > YPROSKR := R1 \cdot \sin(\theta) \cdot \sin(\phi) \\ & \quad YPROSKR := 6367442 \sin(\theta) \sin(\phi) \end{aligned} \quad (59)$$

$$\begin{aligned} > ZPROSKR := R1 \cdot \cos(\theta) \\ & \quad ZPROSKR := 6367442 \cos(\theta) \end{aligned} \quad (60)$$

> *sys* := *simplify*({*XBL* - *XPROSKR* = 0, *YBL* - *YPROSKR* = 0, *ZBL* - *ZPROSKR* = 0},
symbolic) :

> *sys*[1] :

> *sys*[2] :

> *sys*[3] :

$$\begin{aligned} > fsolve(*sys*, \{t = 0.1 .. 1000, \theta = 0.1 .. 0.88, \phi = 0.387 .. 0.5\}) \\ & \quad \{\phi = 0.4340475217, t = 305.1388970, \theta = 0.7979609382\} \end{aligned} \quad (61)$$

$$\begin{aligned} > (61)[1] \\ & \quad \phi = 0.4340475217 \end{aligned} \quad (62)$$

$$\begin{aligned} > (61)[2] \\ & \quad t = 305.1388970 \end{aligned} \quad (63)$$

$$\begin{aligned} > (61)[3] \\ & \quad \theta = 0.7979609382 \end{aligned} \quad (64)$$

>

ΔΙΑΡΚΕΙΑ ΠΤΗΣΗΣ : *Tpr* .

$$\begin{aligned} > Tpr := rhs((61)[2]) \\ & \quad Tpr := 305.1388970 \end{aligned} \quad (65)$$

ΚΑΡΤΕΣΙΑΝΕΣ ΣΥΝΤΕΤΑΓΜΕΝΕΣ X,Y,Z, ΤΟΥ ΣΗΜΕΙΟΥ ΠΡΟΣΚΡΟΥΣΗΣ

ΩΣ ΠΡΟΣ ΤΟ ΑΔΡΑΝΕΙΑΚΟ ΣΥΣΤΗΜΑ ΩΧΥΖ

$$\begin{aligned} > X := evalf(subs(\{(61)[1], (61)[2], (61)[3]\}, (58))) \\ & \quad X := 4.135947386 \cdot 10^6 \end{aligned} \quad (66)$$

$$\begin{aligned} > Y := evalf(subs(\{(61)[1], (61)[2], (61)[3]\}, (59))) \\ & \quad Y := 1.917131601 \cdot 10^6 \end{aligned} \quad (67)$$

$$\begin{aligned} > Z := evalf(subs(\{(61)[1], (61)[2], (61)[3]\}, (60))) \\ & \quad Z := 4.445544204 \cdot 10^6 \end{aligned} \quad (68)$$

ΣΥΝΤΕΤΑΓΜΕΝΕΣ ΘΕΣΗΣ ΕΚΤΟΞΕΥΣΗΣ
ΩΣ ΠΡΟΣ ΤΟ ΑΔΡΑΝΕΙΑΚΟ ΣΥΣΤΗΜΑ ΩΧΥΖ

$$XTE := 4.481270602 \cdot 10^6$$

$$YTE := 1.829076794 \cdot 10^6$$

$$ZTE := 4.137270779 \cdot 10^6$$

$$> D\phi := \Omega \cdot Tpr$$

$$D\phi := 0.02225072837$$

(69)

ΕΦΑΡΜΟΓΗ

ΒΕΡΟΙΑ : (

$$0.7072607424 \text{ [[rad]] N , } 0.3875212718 \text{ [[rad]] E)}$$

ΣΚΟΠΙΑ : (

$$0.7329911619 \text{ [[rad]] N , } 0.3739891520 \text{ [[rad]] E)}$$

ΜΟΣΧΑ :

$$(0.9731222871 \text{ [[rad]] N , } 0.6565457408 \text{ [[rad]] E)}$$

GREENWICH :GMT: (

$$0.8984470177 \text{ [[rad]] N , } 0 \text{ [[rad]])}$$

$$> GMTP := evalf\left(\text{convert}\left(51 + \frac{28}{60} + \frac{38}{60 \cdot 60}, \text{units, deg, rad}\right)\right) \text{ [[rad]]$$

$$GMTP := 0.8984470177 \text{ [[rad]]$$

(70)

$$> GMTM := 0 \text{ [[rad]]$$

$$GMTM := 0$$

(71)

$$> GMTP1 := 0.8984470177$$

$$GMTP1 := 0.8984470177$$

(72)

$$> GMTMI := 0$$

$$GMTMI := 0$$

(73)

$$> Ra := 6367442 \text{ [[m]]$$

$$Ra := 6367442 \text{ [[m]]$$

(74)

$$\begin{aligned} > GP := evalf\left(\text{convert}\left(40 + \frac{31}{60} + \frac{23}{60 \cdot 60}, \text{units, deg, rad}\right)\right) \llbracket \text{rad} \rrbracket \\ & GP := 0.7072607424 \llbracket \text{rad} \rrbracket \end{aligned} \quad (75)$$

$$\begin{aligned} > GP1 := 0.7072607424 \\ & GP1 := 0.7072607424 \end{aligned} \quad (76)$$

$$\begin{aligned} > GM := evalf\left(\text{convert}\left(22 + \frac{12}{60} + \frac{12}{60 \cdot 60}, \text{units, deg, rad}\right)\right) \llbracket \text{rad} \rrbracket \\ & GM := 0.3875212718 \llbracket \text{rad} \rrbracket \end{aligned} \quad (77)$$

$$\begin{aligned} > GM1 := 0.3875212718 \\ & GM1 := 0.3875212718 \end{aligned} \quad (78)$$

$$\begin{aligned} > GPsk := \text{convert}(41.9973, \text{units, deg, rad}) \\ & GPsk := 0.7329911619 \end{aligned} \quad (79)$$

$$\begin{aligned} > GMsk := \text{convert}(21.4280, \text{units, deg, rad}) \\ & GMsk := 0.3739891520 \end{aligned} \quad (80)$$

$$\begin{aligned} > GPMO := \text{convert}(55.7558, \text{units, deg, rad}) \\ & GPMO := 0.9731222871 \end{aligned} \quad (81)$$

$$\begin{aligned} > GMMO := \text{convert}(37.6173, \text{units, deg, rad}) \\ & GMMO := 0.6565457408 \end{aligned} \quad (82)$$

$$\begin{aligned} > GP1 := 0.7072607424 \\ & GP1 := 0.7072607424 \end{aligned} \quad (83)$$

$$\begin{aligned} > GM1 := 0.3875212718 \\ & GM1 := 0.3875212718 \end{aligned} \quad (84)$$

$$\begin{aligned} > xB := RI \cdot \sin\left(\frac{\pi}{2} - GP1\right) \cdot \cos(GM1) \\ & xB := 4.481270602 \cdot 10^6 \end{aligned} \quad (85)$$

$$\begin{aligned} > yB := RI \cdot \sin\left(\frac{\pi}{2} - GP1\right) \cdot \sin(GM1) \\ & yB := 1.829076794 \cdot 10^6 \end{aligned} \quad (86)$$

$$\begin{aligned} > zB := RI \cdot \cos\left(\frac{\pi}{2} - GP1\right) \\ & zB := 4.137270779 \cdot 10^6 \end{aligned} \quad (87)$$

$$> xSK := RI \cdot \sin\left(\frac{\pi}{2} - GPsk\right) \cdot \cos(GMsk) :$$

$$> ySK := RI \cdot \sin\left(\frac{\pi}{2} - GPsk\right) \cdot \sin(GMsk) :$$

$$> zSK := RI \cdot \cos\left(\frac{\pi}{2} - GPsk\right) :$$

$$\begin{aligned} > \text{DistanceBSK} := \text{sqrt}\left((xB - xSK)^2 + (yB - ySK)^2 + (zB - zSK)^2\right) \\ & \text{DistanceBSK} := 176168.1254 \end{aligned} \quad (88)$$

$$\begin{aligned} > xMO := RI \cdot \sin\left(\frac{\pi}{2} - GPMO\right) \cdot \cos(GMMO) \\ & xMO := 2.838188760 \cdot 10^6 \end{aligned} \quad (89)$$

$$\begin{aligned} > y_{MO} := R1 \cdot \sin\left(\frac{\pi}{2} - GPMO\right) \cdot \sin(GMMO) \\ & \qquad \qquad \qquad y_{MO} := 2.187065106 \cdot 10^6 \end{aligned} \tag{90}$$

$$\begin{aligned} > z_{MO} := R1 \cdot \cos\left(\frac{\pi}{2} - GPMO\right) \\ & \qquad \qquad \qquad z_{MO} := 5.263625026 \cdot 10^6 \end{aligned} \tag{91}$$

$$\begin{aligned} > DistanceBMO := \text{sqrt}\left((x_B - x_{MO})^2 + (y_B - y_{MO})^2 + (z_B - z_{MO})^2\right) \\ & \qquad \qquad \qquad DistanceBMO := 2.023992950 \cdot 10^6 \end{aligned} \tag{92}$$

$$\begin{aligned} > \text{evalf}\left(\frac{45 \cdot \text{Pi}}{180}\right) \\ & \qquad \qquad \qquad 0.7853981635 \end{aligned} \tag{93}$$

$$\begin{aligned} > PLATOS := \text{spacecurve}\left(\left[R1 \cdot \sin\left(\frac{\pi}{2} - GP1\right) \cdot \cos(\phi), R1 \cdot \sin\left(\frac{\pi}{2} - GP1\right) \cdot \sin(\phi), R1 \cdot \cos\left(\frac{\pi}{2} - GP1\right) \right], \phi = 0 .. 2 \cdot \text{Pi}, \text{color} = \text{red}, \text{thickness} = 2\right) : \end{aligned}$$

$$\begin{aligned} > MHKOS := \text{spacecurve}\left([R1 \cdot \sin(\theta) \cdot \cos(GM1), R1 \cdot \sin(\theta) \cdot \sin(GM1), R1 \cdot \cos(\theta)], \theta = 0 .. \text{Pi}, \text{color} = \text{blue}, \text{thickness} = 2\right) : \end{aligned}$$

$$\begin{aligned} > GRIN := \text{spacecurve}\left([R1 \cdot \sin(\theta) \cdot \cos(0), R1 \cdot \sin(\theta) \cdot \sin(0), R1 \cdot \cos(\theta)], \theta = 0 .. \text{Pi}, \text{color} = \text{black}, \text{thickness} = 5, \text{linestyle} = 1\right) : \end{aligned}$$

$$\begin{aligned} > GMT := \text{pointplot3d}\left(\left[R1 \cdot \sin\left(\frac{\pi}{2} - GMTP1\right) \cdot \cos(GMTM1), R1 \cdot \sin\left(\frac{\pi}{2} - GMTP1\right) \cdot \sin(GMTM1), R1 \cdot \cos\left(\frac{\pi}{2} - GMTP1\right) \right], \text{symbol} = \text{solidcircle}, \text{symbolsize} = 20, \text{color} = \text{red}\right) : \end{aligned}$$

$$\begin{aligned} > GMT1 := \text{textplot3d}\left(\left[1.1 \cdot R1 \cdot \sin\left(\frac{\pi}{2} - (GMTP1 + 0.1)\right) \cdot \cos(GMTM1 - 0.2), 1.1 \cdot R1 \cdot \sin\left(\frac{\pi}{2} - (GMTP1 + 0.1)\right) \cdot \sin(GMTM1 - 0.2), R1 \cdot \cos\left(\frac{\pi}{2} - (GMTP1 + 0.1)\right) \right], \text{"G"}, \text{font} = [\text{arial}, \text{bold}, 15]\right) : \end{aligned}$$

$$\begin{aligned} > ISHMER := \text{spacecurve}\left(\left[R1 \cdot \sin\left(\frac{\pi}{2}\right) \cdot \cos(\phi), R1 \cdot \sin\left(\frac{\pi}{2}\right) \cdot \sin(\phi), R1 \cdot \cos\left(\frac{\pi}{2}\right) \right], \phi = 0 .. 2 \cdot \text{Pi}, \text{color} = \text{yellow}, \text{thickness} = 2\right) : \end{aligned}$$

$$\begin{aligned} > GH := \text{plot3d}\left([R1 \cdot \sin(\theta) \cdot \cos(\phi), R1 \cdot \sin(\theta) \cdot \sin(\phi), R1 \cdot \cos(\theta)], \theta = 0 .. \text{Pi}, \phi = 0 .. 2 \cdot \text{Pi}\right) : \end{aligned}$$

$$\begin{aligned} > B := \text{pointplot3d}\left(\left[R1 \cdot \sin\left(\frac{\pi}{2} - GP1\right) \cdot \cos(GM1), R1 \cdot \sin\left(\frac{\pi}{2} - GP1\right) \cdot \sin(GM1), R1 \cdot \cos\left(\frac{\pi}{2} - GP1\right) \right], \text{symbol} = \text{solidcircle}, \text{symbolsize} = 20, \text{color} = \text{green}\right) : \end{aligned}$$

$$\begin{aligned} > MOSXA := \text{pointplot3d}\left(\left[R1 \cdot \sin\left(\frac{\pi}{2} - GPMO\right) \cdot \cos(GMMO), R1 \cdot \sin\left(\frac{\pi}{2} - GPMO\right) \cdot \sin(GMMO), R1 \cdot \cos\left(\frac{\pi}{2} - GPMO\right) \right], \text{symbol} = \text{solidcircle}, \text{symbolsize} = 20, \text{color} = \text{red}\right) : \end{aligned}$$

```
·sin(GMMO), R1·cos( $\frac{\pi}{2} - \text{GPMO}$ )]], symbol = solidcircle, symbolsize = 20, color = yellow):
```

```
> MOSXA1 := textplot3d([ [1.1·R1·sin( $\frac{\pi}{2} - (\text{GPMO} - 0.1)$ )·cos(GMMO + 0.2), 1.1·R1·sin( $\frac{\pi}{2} - (\text{GPMO} - 0.1)$ )·sin(GMMO + 0.2), 1.1·R1·cos( $\frac{\pi}{2} - (\text{GPMO} - 0.1)$ )], "M"], font = [arial, bold, 15]):
```

```
> AKSON := spacecurve([0, 0, R1·1.2 - m·(R1·2·1.2)], m = 0..1, color = black, thickness = 5):
```

```
> BORRAS := textplot3d([0, 0, 1.4 R1, "Nord"], font = [arial, bold, 14]):
```

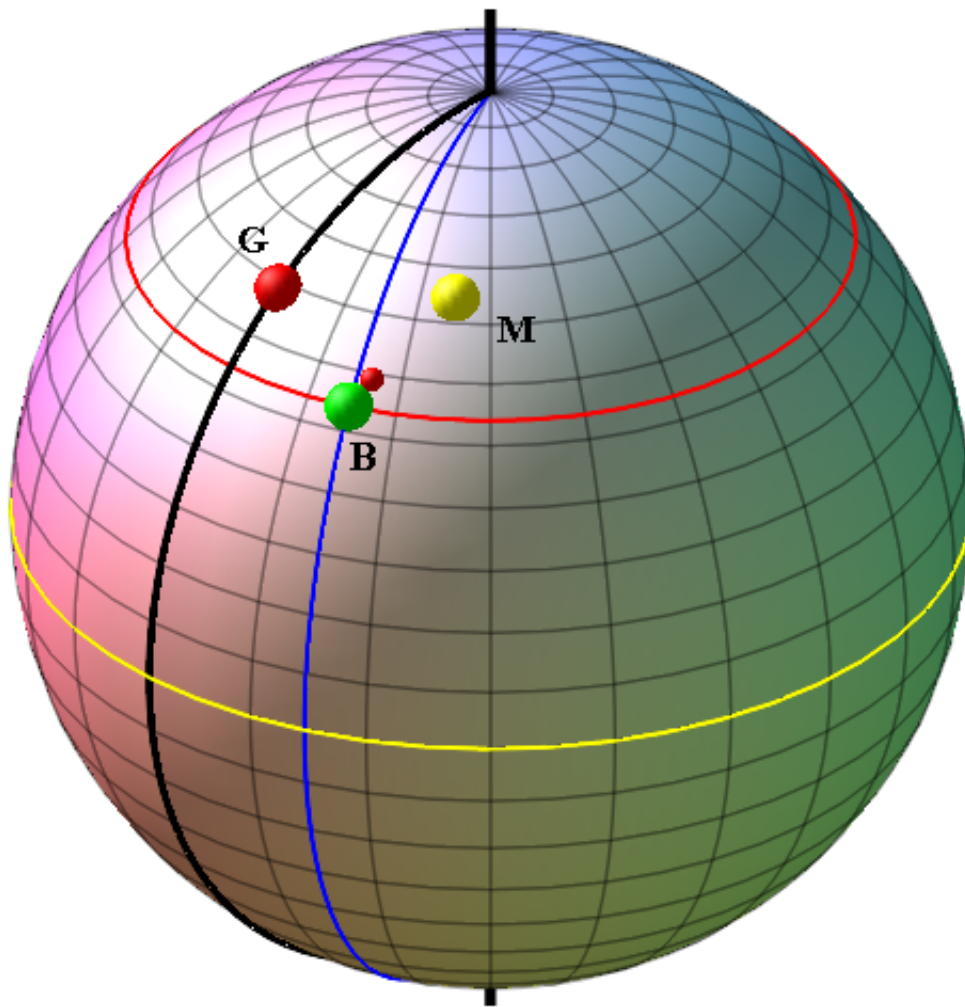
```
> BEROIA := textplot3d([ [1.1·R1·sin( $\frac{\pi}{2} - (\text{GP1} - 0.1)$ )·cos(GM1 + 0.1), 1.1·R1·sin( $\frac{\pi}{2} - (\text{GP1} - 0.1)$ )·sin(GM1 + 0.1), 1.1·R1·cos( $\frac{\pi}{2} - (\text{GP1} - 0.1)$ )], "B"], font = [arial, bold, 15]):
```

```
> BLHMA := pointplot3d([X, Y, Z], symbol = solidcircle, symbolsize = 10, color = red):
```

```
> SYNOLO := display(GMT, GMT1, PLATOS, MHKOS, GRIN, ISHMER, GH, B, MOSXA, AKSON, BORRAS, BEROIA, MOSXA1, BLHMA, orientation = [45, 60, 0], axes = none):
```

```
> display(SYNOLO, scaling = constrained)
```

Nord



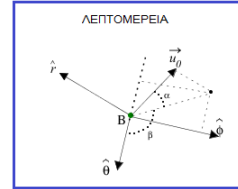
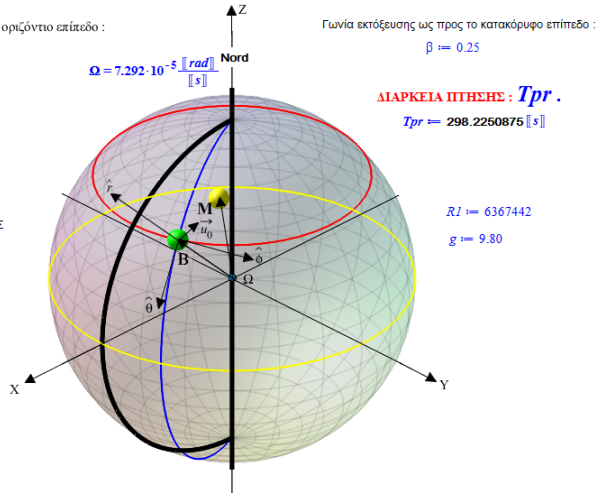
Γωνία εκτόξευσης ως προς το οριζόντιο επίπεδο :

$$\alpha := \frac{\pi}{4}$$

Ταχύτητα Εκτόξευσης :

$$v_0 := \frac{2000}{[s]} [m]$$

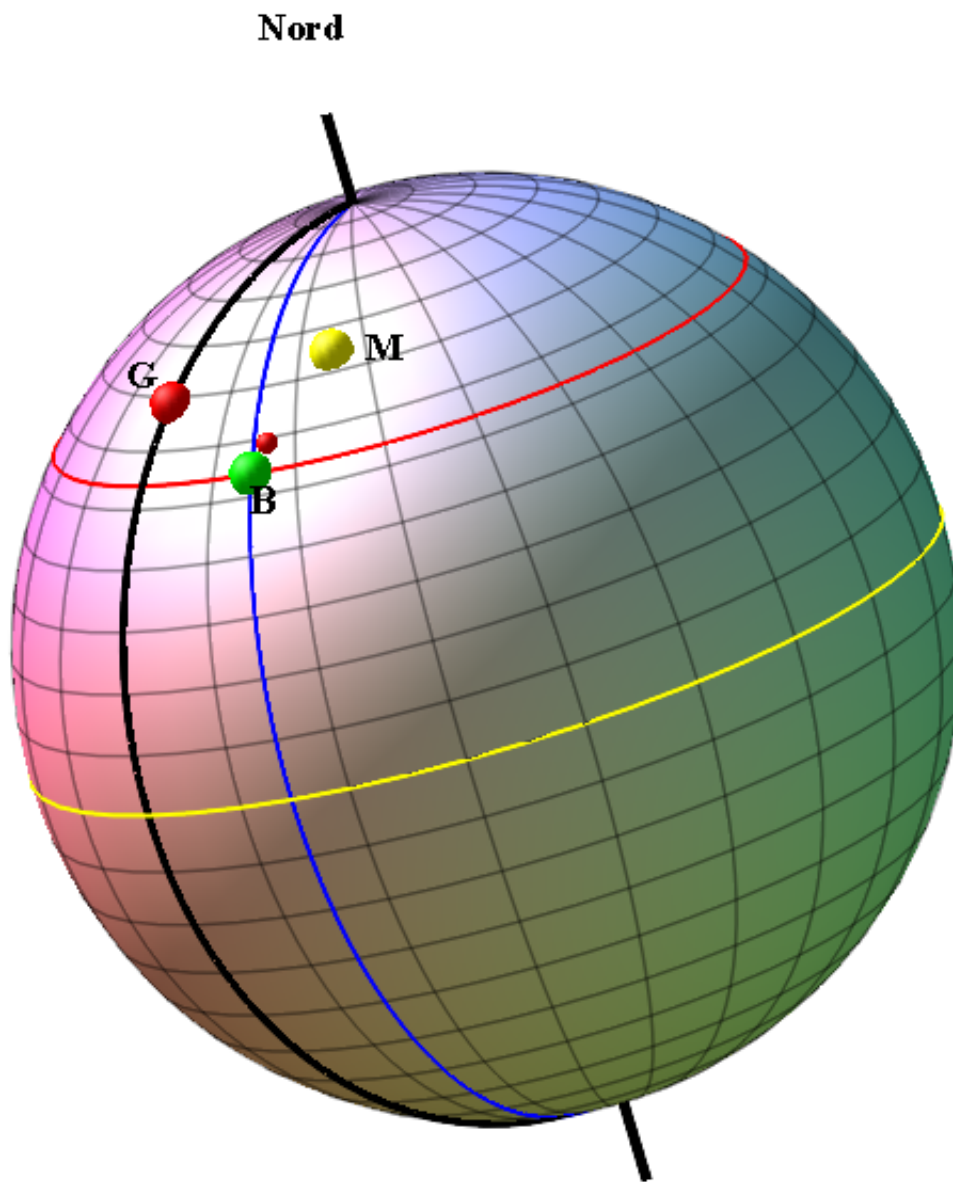
ΣΑΒΒΑΣ Π.ΓΑΒΡΙΗΛΙΔΗΣ



```
> ROT := rotate(SYNOLO,  $\frac{23.439247}{180} \cdot \text{Pi}$ , [[0, 0, 0], [1, 0, 0]]):
```

```
> display(ROT, scaling = constrained, title  
= "ΘΕΣΕΙΣ GREENWICH – ΒΕΡΟΙΑΣ – ΣΤΟΧΟΥ - ΜΟΣΧΑΣ\nΣΑΒΒΑΣ Π.  
ΓΑΒΡΙΗΛΙΔΗΣ", titlefont = [arial, bold, 12])
```

ΘΕΣΕΙΣ GREENWICH-ΒΕΡΟΙΑΣ-ΣΤΟΧΟΥ -ΜΟΣΧΑΣ
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ



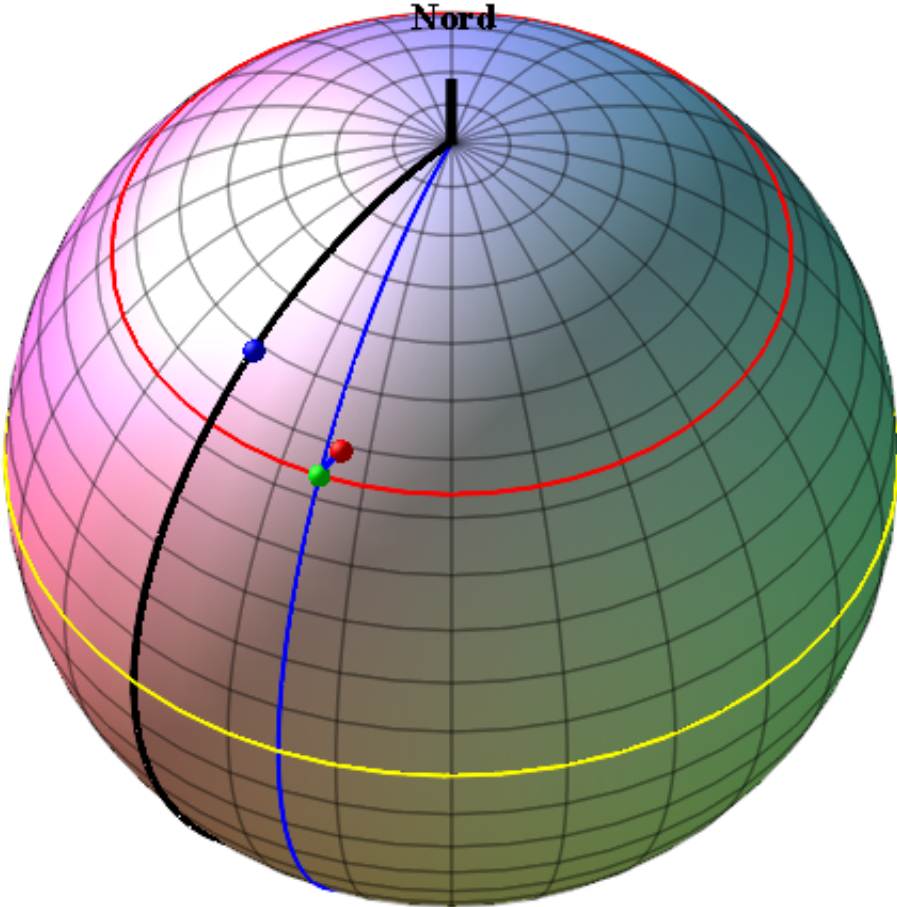
ANIMATION

- > $AGH := \text{animate}(\text{plot3d}, [[R1 \cdot \sin(\theta) \cdot \cos(\phi + \Omega \cdot t), R1 \cdot \sin(\theta) \cdot \sin(\phi + \Omega \cdot t), R1 \cdot \cos(\theta)], \theta = 0 .. \text{Pi}, \phi = 0 .. 2 \cdot \text{Pi}], t = 0 .. 86165, \text{frames} = 20) :$
- > $AGRIN1 := \text{animate}(\text{pointplot3d}, \left[\left[R1 \cdot \sin\left(\frac{\pi}{2} - \text{GMTP1}\right) \cdot \cos(\text{GMTM1} + \Omega \cdot t), R1 \cdot \sin\left(\frac{\pi}{2} - \text{GMTP1}\right) \cdot \sin(\text{GMTM1} + \Omega \cdot t), R1 \cdot \cos\left(\frac{\pi}{2} - \text{GMTP1}\right) \right], \text{symbol} = \text{solidcircle}, \text{symbolsize} = 10, \text{color} = \text{blue} \right], t = 0 .. 86165, \text{frames} = 20) :$
- > $ABER := \text{animate}(\text{pointplot3d}, \left[\left[R1 \cdot \sin\left(\frac{\pi}{2} - \text{GP1}\right) \cdot \cos(\text{GM1} + \Omega \cdot t), R1 \cdot \sin\left(\frac{\pi}{2} - \text{GP1}\right) \cdot \sin(\text{GM1} + \Omega \cdot t), R1 \cdot \cos\left(\frac{\pi}{2} - \text{GP1}\right) \right], \text{symbol} = \text{solidcircle}, \text{symbolsize} = 10, \text{color} = \text{green} \right], t = 0 .. 86165, \text{frames} = 20) :$
- > $APLATOS := \text{animate}(\text{spacecurve}, \left[\left[R1 \cdot \sin\left(\frac{\pi}{2} - \text{GP1}\right) \cdot \cos(\phi + \Omega \cdot t), R1 \cdot \sin\left(\frac{\pi}{2} - \text{GP1}\right) \cdot \sin(\phi + \Omega \cdot t), R1 \cdot \cos\left(\frac{\pi}{2} - \text{GP1}\right) \right], \phi = 0 .. 2 \cdot \text{Pi}, \text{color} = \text{red}, \text{thickness} = 2 \right], t = 0 .. 86165, \text{frames} = 20) :$
- > $AMHKOS := \text{animate}(\text{spacecurve}, [[R1 \cdot \sin(\theta) \cdot \cos(\text{GM1} + \Omega \cdot t), R1 \cdot \sin(\theta) \cdot \sin(\text{GM1} + \Omega \cdot t), R1 \cdot \cos(\theta)], \theta = 0 .. \text{Pi}, \text{color} = \text{blue}, \text{thickness} = 2], t = 0 .. 86165, \text{frames} = 20) :$
- > $AGRIN := \text{animate}(\text{spacecurve}, [[R1 \cdot \sin(\theta) \cdot \cos(0 + \Omega \cdot t), R1 \cdot \sin(\theta) \cdot \sin(0 + \Omega \cdot t), R1 \cdot \cos(\theta)], \theta = 0 .. \text{Pi}, \text{color} = \text{black}, \text{thickness} = 5, \text{linestyle} = 1], t = 0 .. 86165, \text{frames} = 20) :$
- > $AISHMER := \text{animate}(\text{spacecurve}, \left[\left[R1 \cdot \sin\left(\frac{\pi}{2}\right) \cdot \cos(\phi + \Omega \cdot t), R1 \cdot \sin\left(\frac{\pi}{2}\right) \cdot \sin(\phi + \Omega \cdot t), R1 \cdot \cos\left(\frac{\pi}{2}\right) \right], \phi = 0 .. 2 \cdot \text{Pi}, \text{color} = \text{yellow}, \text{thickness} = 2 \right], t = 0 .. 86165, \text{frames} = 20) :$
- > $TROXIA := \text{spacecurve}([XBL, YBL, ZBL], t = 0 .. Tpr, \text{color} = \text{blue}, \text{thickness} = 3) :$
- > $BLHMA := \text{pointplot3d}([X, Y, Z], \text{symbol} = \text{solidcircle}, \text{symbolsize} = 10, \text{color} = \text{red}) :$
- > $BLHMA1 := \text{animate}(\text{pointplot3d}, [[R1 \cdot \sin(\text{rhs}((64))) \cdot \cos(\text{rhs}((62)) + \Omega \cdot t), R1 \cdot \sin(\text{rhs}((64))) \cdot \sin(\text{rhs}((62)) + \Omega \cdot t), R1 \cdot \cos(\text{rhs}((64)))], \text{symbol} = \text{solidcircle}, \text{symbolsize} = 10, \text{color} = \text{red}], t = 0 .. 86165, \text{frames} = 20) :$
- > $B := \text{pointplot3d}\left(\left[R1 \cdot \sin\left(\frac{\pi}{2} - \text{GP1}\right) \cdot \cos(\text{GM1}), R1 \cdot \sin\left(\frac{\pi}{2} - \text{GP1}\right) \cdot \sin(\text{GM1}), R1 \cdot \cos\left(\frac{\pi}{2} - \text{GP1}\right) \right], \text{symbol} = \text{solidcircle}, \text{symbolsize} = 10, \text{color} = \text{green}\right) :$
- > $AKSON := \text{spacecurve}([0, 0, R1 \cdot 1.2 - m \cdot (R1 \cdot 2 \cdot 1.2)], m = 0 .. 1, \text{color} = \text{black}, \text{thickness} = 5) :$
- > $\text{display}(AGH, AGRIN1, AKSON, ABER, APLATOS, AMHKOS, AGRIN, AISHMER, TROXIA, BLHMA, B, BORRAS, BLHMA1, \text{scaling} = \text{constrained}, \text{axes} = \text{none}, \text{orientation} = [45, 45,$

0], *title*

= "ΘΕΣΕΙΣ GREENWICH-ΒΕΡΟΙΑΣ-ΣΤΟΧΟΥ ΓΙΑ ΑΔΡΑΝΕΙΑΚΟ ΠΑΡΑΤΗΡΗΤΗ ΣΤΗ
ΘΕΣΗ ΕΚΤΟΞΕΥΣΗΣ\nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ-ANIMATION", *titlefont* = [*arial*, *bold*,
12])

**ΘΕΣΕΙΣ GREENWICH-ΒΕΡΟΙΑΣ-ΣΤΟΧΟΥ ΓΙΑ ΑΔΡΑΝΕΙΑΚΟ
ΠΑΡΑΤΗΡΗΤΗ ΣΤΗ ΘΕΣΗ ΕΚΤΟΞΕΥΣΗΣ
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ-ANIMATION**



ΤΡΟΧΙΑ ΤΟΥ ΒΛΗΜΑΤΟΣ ΟΠΩΣ ΤΗΝ ΒΛΕΠΕΙ ΠΑΡΑΤΗΡΗΤΗΣ (ΠΕΡΙΣΤΡΕΦΟΜΕΝΟΣ) ΑΠΟ ΤΗΝ ΘΕΣΗ ΕΚΤΟΞΕΥΣΗΣ .

> rhs(sol1)

$$-\frac{9689124217 \sqrt{2} t}{10000000} \quad (94)$$

> rhs(sol2)

$$\frac{362140981 t^3}{2000000000000} - \frac{476130657 \sqrt{2} t^2}{50000000000} + \frac{2474039593 \sqrt{2} t}{10000000} \quad (95)$$

> rhs(sol3)

$$-\frac{49 t^2}{10} + 1000 \sqrt{2} t + 10000 \quad (96)$$

> TBL := spacecurve([(94), (95), (96)], t=0..Tpr, thickness=2, linestyle=3, color=blue) :

> BL := animate(pointplot3d, [(94), (95), (96)], symbol=solidcircle, symbolsize=25, color=red,], t=0..Tpr, frames=80) :

> BL1 := pointplot3d([subs(t=0, (94)), subs(t=0, (95)), subs(t=0, (96))], symbol=solidcircle, symbolsize=25, color=red) :

> PAR := pointplot3d([0, 0, 0], symbol=solidbox, symbolsize=25, color=green) :

> PAR1 := textplot3d([0, 0, -12000, "ΒΕΡΟΙΑ"], font=[arial, bold, 12]) :

> AFIKSI := pointplot3d([subs(t=Tpr, (94)), subs(t=Tpr, (95)), subs(t=Tpr, (96))], symbol=solidcircle, symbolsize=30, color=yellow) :

> TEX := textplot3d([subs(t=Tpr, (94)), subs(t=Tpr, (95)), subs(t=Tpr, (96)) - 12000, "ΣΤΟΧΟΣ"], font=[arial, 12, bold]) :

> display(TBL, BL, BL1, PAR, PAR1, AFIKSI, TEX, orientation=[95, 75, 0], scaling=unconstrained, labels=[x, y, z], labelfont=[arial, bold, 14], title="ΤΡΟΧΙΑ ΒΛΗΜΑΤΟΣ ΓΙΑ ΠΑΡΑΤΗΡΗΤΗ ΣΤΗ ΘΕΣΗ ΕΚΤΟΞΕΥΣΗΣ\nΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ-ANIMATION", titlefont=[arial, 12, bold], axes=boxed)

**ΤΡΟΧΙΑ ΒΛΗΜΑΤΟΣ ΓΙΑ ΠΑΡΑΤΗΡΗΤΗ ΣΤΗ ΘΕΣΗ
ΕΚΤΟΞΕΥΣΗΣ
ΣΑΒΒΑΣ Π. ΓΑΒΡΙΗΛΙΔΗΣ-ANIMATION**

