

ΝΑ μελετηθεί η παρακάτω Απεικόνιση :

$$z_{n+1} = f_c(z_n) \iff z_n^2 + c : \text{όπου}$$

$z_n = x_n + I \cdot y_n, z_{n+1} = x_{n+1} + I \cdot y_{n+1} \quad \& \quad c = a + I \cdot b :$

Mandelbrot : Γιά $z_0 = 0 + 0 \cdot I$ με την ανωτέρω απεικόνιση
έχουμε την Τροχιά :

> $f_c := z \rightarrow z^2 + c$

$$f_c := z \mapsto z^2 + c \quad (1)$$

> $f0 := 0 + 0 \cdot I$

$$f0 := 0 \quad (2)$$

> $f1 := f_c(0)$

$$f1 := c \quad (3)$$

> $f2 := f_c(f1)$

$$f2 := c^2 + c \quad (4)$$

> $f3 := f_c(f2)$

$$f3 := (c^2 + c)^2 + c \quad (5)$$

> $f4 := f_c(f3)$

$$f4 := ((c^2 + c)^2 + c)^2 + c \quad (6)$$

> $f5 := f_c(f4)$

$$f5 := (((c^2 + c)^2 + c)^2 + c)^2 + c \quad (7)$$

>

> `with(plots) :`

> `with(plottools) :`

Η Μιγαδική Απεικόνιση $z_{n+1} = f_c(z_n) \iff z_n \rightarrow z_n^2 + c$ αντικαθιστώντας
 $z_n = x_n + I \cdot y_n, z_{n+1} = x_{n+1} + I \cdot y_{n+1}, c = a + I \cdot b$, είναι ισοδύναμη με τις πραγματικές
 Απεικονίσεις :

$$\begin{aligned} x_{n+1} &= u \iff (x_n, y_n) \rightarrow x_n^2 - y_n^2 + a \\ y_{n+1} &= v \iff (x_n, y_n) \rightarrow 2 \cdot x_n \cdot y_n + b \end{aligned}$$

και λύνοντας Αντίστροφα ως προς ΠΡΑΓΜΑΤΙΚΑ x_n, y_n παίρνει την μορφή :

$$\left\{ \begin{array}{l} x_n = -\frac{b - y_{n+1}}{\sqrt{2 \sqrt{a^2 - 2 a x_{n+1} + b^2} - 2 b y_{n+1} + x_{n+1}^2 + y_{n+1}^2} + 2 a - 2 x_{n+1}}, \\ y_n \end{array} \right.$$

$$\begin{aligned}
&= \frac{\sqrt{2 \sqrt{a^2 - 2 a x_{n+1} + b^2 - 2 b y_{n+1} + x_{n+1}^2 + y_{n+1}^2} + 2 a - 2 x_{n+1}}}{2}, \\
x_n &= \frac{b - y_{n+1}}{\sqrt{2 \sqrt{a^2 - 2 a x_{n+1} + b^2 - 2 b y_{n+1} + x_{n+1}^2 + y_{n+1}^2} + 2 a - 2 x_{n+1}}}, y_n = \\
&\quad - \frac{\sqrt{2 \sqrt{a^2 - 2 a x_{n+1} + b^2 - 2 b y_{n+1} + x_{n+1}^2 + y_{n+1}^2} + 2 a - 2 x_{n+1}}}{2}.
\end{aligned}$$

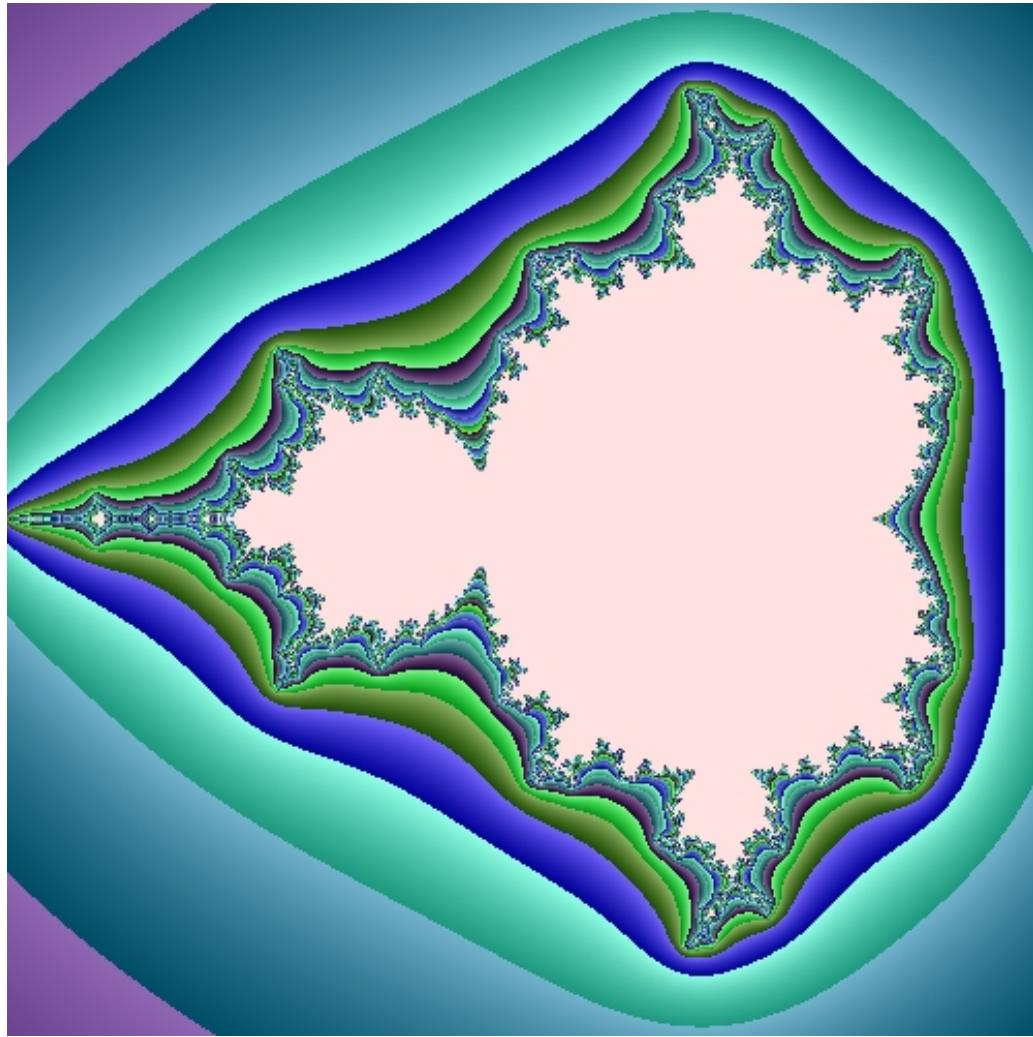
> `with(Fractals:-EscapeTime)`
`[BurningShip, Colorize, HSVColorize, Julia, LColorize, Lyapunov, Mandelbrot, Newton]` (8)

> `with(ImageTools) :`

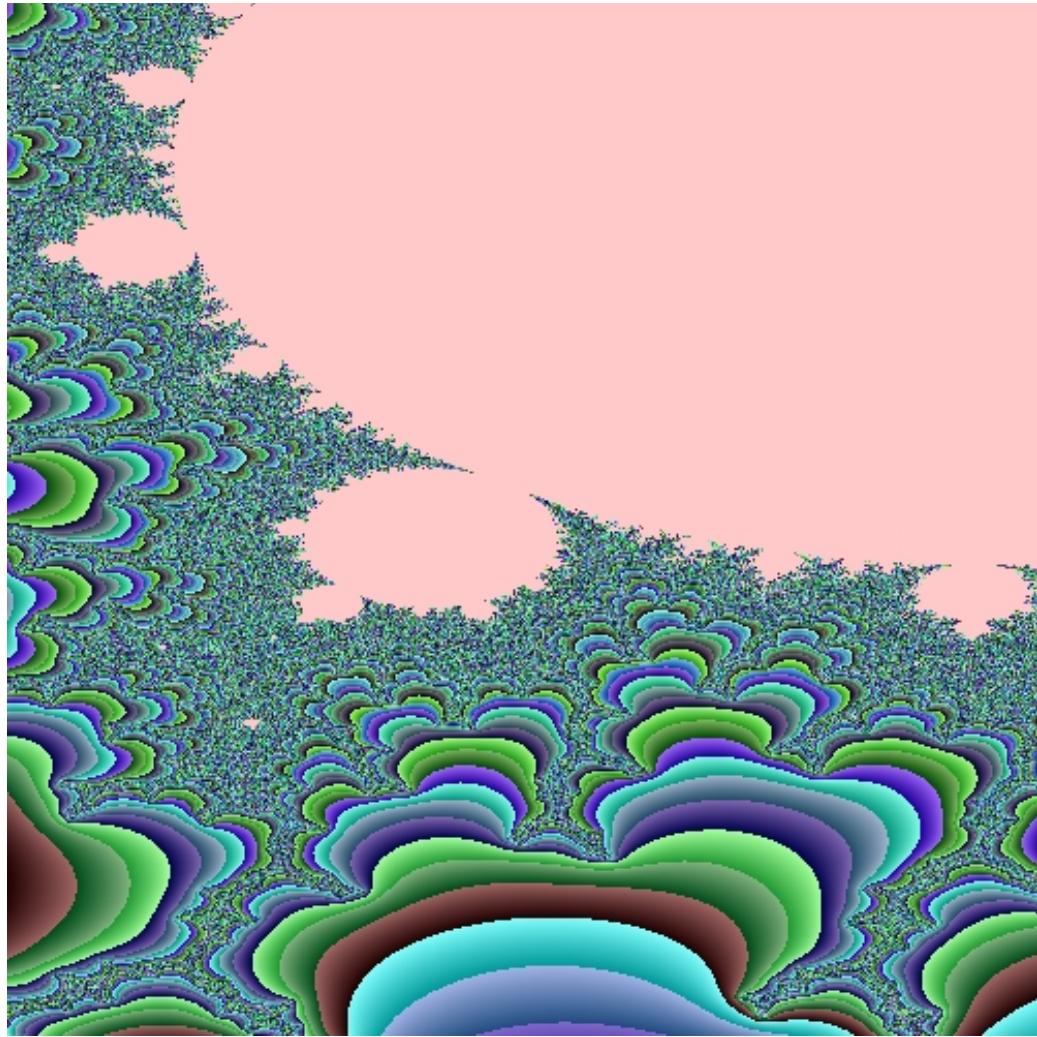
> `M := Mandelbrot(500, -2.0 - 1.35 I, 0.7 + 1.35 I)`

$$M := \left[\begin{array}{l} 1..500 \times 1..500 \times 1..3 \text{ Array} \\ \text{Data Type: } \text{float}_8 \\ \text{Storage: } \text{rectangular} \\ \text{Order: } \text{C_order} \end{array} \right] \quad (9)$$

> `Embed(M)`



```
[> Embed(Mandelbrot(500, -2.0 - 1.35 I, 0.7 + 1.35 I, output=layer1)) :  
> Embed([Mandelbrot(200, -2.0 - 1.35 I, 0.7 + 1.35 I, iterationlimit = 10, output=layer1),  
         Mandelbrot(200, -2.0 - 1.35 I, 0.7 + 1.35 I, iterationlimit = 25, output=layer1),  
         Mandelbrot(200, -2.0 - 1.35 I, 0.7 + 1.35 I, iterationlimit = 125, output=layer1)]) :  
> Embed(Mandelbrot(500, -0.1515 + 1.032 I, -0.1575 + 1.043 I, iterationlimit = 300, cutoff  
                  = 50.0))
```



```
[> Embed([Mandelbrot(200, -2.0 - 1.35 I, 0.7 + 1.35 I, cutoff=4.0, output=layer2),
          Mandelbrot(200, -2.0 - 1.35 I, 0.7 + 1.35 I, cutoff=100.0, output=layer2),
          Mandelbrot(200, -2.0 - 1.35 I, 0.7 + 1.35 I, cutoff=10000.0, output=layer2)]) :
=> R := Array(1 ..500, 1 ..500, 1 ..2, datatype=float8) :
=> Mandelbrot(500, -0.1025 + 0.95 I, -0.095 + 0.96 I, iterationlimit=200, cutoff=50.0, output
              =raw, container=R) :
=> Embed([FitIntensity(R( ) ..( ), ( ) ..( ), 1), FitIntensity(R( ) ..( ), ( ) ..( ), 2)]) :
=> P := Colorize(500, R, Array([11, 3, 2], datatype=integer4), Array([0.256, 0.859, 0.256],
                  datatype=float8), rgb=0, mode=4, layer=2) :
=> Embed(P)
```

